ACOUSTIC SCATTERING FROM ROUGH SURFACES

B. G. Hurdle, K. D. Flowers & J. A. DeSanto

Naval Research Laboratory
Washington, D.C. 20375

ABSTRACT

A review of scattering from the ocean bottom and ocean ice cover is given. Areas of deficiency are cited.

INTRODUCTION

NATO has an interest in acoustic scattering for several reasons. Among these are the support of navigation, communication, target detection, target classification, tracking, and fire control. In each of these operational functions, underwater scattering plays a major role in the degree of success that can be obtained. Antisubmarine warfare units whether they be active or passive are subjected to the environment of the ocean with a limited number of propagation paths over which the sound is transmitted. Scattering occurs when the paths intersect the rough boundaries and when discontinuities or inhomogeneities are encountered in the volume.

In practice all acoustic waves propagating in the ocean are scattered to some degree. There is less effective scattering at low frequencies and for propagation over certain paths. The frequency effect is caused by the relative size of the scatterers with respect to the wavelength of the sound. The path effect is caused by the type, amount, and quantity of scatterers encountered.

Some deep water paths of interest to NATO are: RAP, SOFAR, surface duct, bottom bounce, and convergence zone forming types. In shallow water, wave guide type paths are, in general, the only propagation modes available. All of the paths involve volume scattering which is caused by inhomogeneities in the volume, such as, fish, internal wave fields, and random fluctuations of the physical parameters of the medium. Those paths concerned solely with volume scattering are: SOFAR and possibly convergence zone forming types. Surface duct paths have multiple interactions with the sea surface or ice cover and energy is continuously scattered out of the duct. Generally a sloping bottom and large topographic features are additional complications in this type of problem.

We discuss scattering from surfaces, specifically, from the bottom and ice cover, in more detail in subsequent sections.

SCATTERING FROM SURFACES

When acoustic waves intersect a rough surface, several effects are observed. These effects are a spatial redistribution of the scattered energy, a transmission of energy through the surface, frequency smearing of the signal, and time smearing of the signal. These effects will be discussed below for the specific surface types: sea bottom, and under ice. Since surface topography is common to both rough surfaces, we discuss its effect first.

Since it is impractical to measure precisely the topography of those scattering surfaces met in practice, it is required, in some way, to model these surfaces in order to do scattering problems.

Scattering theories based on these models, to be of use, must be verified by actual measurements or shown to predict known solutions adequately. Here, essentially, is the reason why surface scattering has remained a major unsolved problem; the complete lack of an exact, tractable solution for a realistic surface. Exact must be qualified with tractable since it is not difficult to write formally the exact solution [1-3] to the surface scattering problem. The real difficulty arises in its evaluation.

Scattering from rough surfaces can be treated either deterministically or statistically. Deterministric surfaces can be further divided into separable and nonseparable surfaces, where the category separable means that the surface fits into a natural coordinate scheme for which the solution of the Helmholtz equation is separable.[4] Examples of the separable type include the half plane, [2-5] rectangularly corrugated surfaces, [6-19] and sawtooth corrugations.[20-26] A classic example of the nonseparable type of surface is the sinusoidal surface. [27-43] In addition to surfaces which are geometrically rough, some authors [44-45] deal with a flat surface having a periodic impedance modulation.

The solution for the scattered velocity potential (or pressure) is often found by writing certain assumed forms of the fields with unknown (complex) amplitude coefficients in the various geometric regions of the problem. Continuity of pressure and velocity across the common boundaries yield linear equations relating the amplitude coefficients. These linear equations are usually solved by matrix inversion [11,12,34,35,39] although in certain cases matrix inversion can be avoided. This is true if it is possible to relate the sets of linear equations to the residue series of integrals of certain meromorphic functions.[8,13-19] In this case, the amplitude coefficients are given in terms of the function. Other methods of solving linear equations are also employed. For example, continued fractions are used on the linear equations for a periodic impedance modulated surface.[45] If the surface is periodic, then it can be shown that the energy is scattered in discrete directions (grating effect) and the problem can be collapsed to a consideration of only a single period of the surface.[35]

Note, however, that the series expansions assumed must converge to the boundary conditions imposed on the surface. That all assumed expansions do not converge is well illustrated by the so-called Rayleigh hypothesis, i.e., that the field in the wells of a sinusoidal surface could be represented as a sum of upgoing plane waves combined with the single downgoing incident plane wave.[27,34-36] The Rayleigh hypothesis, for example, has been shown to be either true or false depending on whether $kb \leq 0.448$ or $kb > 0.448$ where the surface is given as $y = b \cos(kx)$.[46-48] The number 0.448 arises as the solution of a transcendental equation, which solution yields the singularities of the field expansion.

Solutions can be found using the Helmholtz integral formula, which simply yields an integral representation of the solution of the Helmholtz equation in terms of single and double layers on the surface.[49-53] In addition to the known free space Green's function, both the velocity potential (or the pressure) and its normal derivative are required to be known on the surface. For a rigorous mathematical problem, only one of the two, either velocity potential or its normal derivative, can be specified.[3] An integral equation is then constructed for the surface value of the unspecified other quantity. Its solution is then substituted into the original Helmholtz integral formula to give, along with the assumed boundary value, an integral representation for the velocity potential (or pressure).

A mathematically rigorous variation of the procedure involves choosing a Green's function which itself satisfies a specified boundary condition and then writing Rayleigh-Sommerfeld integral formulas for the velocity potential.[54-56] Now only one surface value is involved, either velocity potential or its normal derivative (depending on which of the Rayleigh-Sommerfeld integral formulas one chooses), producing a mathematically rigorous problem. In this case the additional boundary condition assumption involves the specification of only one boundary value. Although the formulation of the problem is mathematically rigorous, the practical difficulty of specifying a boundary Green's function for arbitrary boundaries remains. The latter problem is as difficult as the original problem of calculating the velocity potential.

Other methods of solution are imaging, as has been applied to symmetrical bosses on a perfectly reflecting plane,[57-61] and generalized harmonic analysis [62-65] which may be applied to any surface.

We now consider approximate types of solutions, some of which have been comparatively reviewed.[66-68] The Helmholtz integral formula is an integral over the scattering surface and requires a knowledge of the field, ψ, and its normal derivative, $\frac{\partial\psi}{\partial n}$ everywhere on the surface. Since these two boundary conditions are not generally known, they are approximated by assuming that the surface is smooth enough to replace the field at a point by the field that would be present on the tangent plane at the point (Kirchhoff or geometrical acoustics approximation).[6,54-56,69-70] That is $\psi = (1 + V)\psi_i$ and $\frac{\partial\psi}{\partial n} = (1-V)\vec{n}\cdot\vec{k}_i\psi_i$, where V = local Fresnel reflection coefficient, ψ_i = incident field, and k_i = incident wave number. The scattered field is now completely specified; however, in general, the integration cannot be performed. Additional assumptions usually made are: V = constant, and the source and observation point removed to the Fraunhofer zone. The approximations made are: the surface is smooth, i.e., does not change appreciably within a wavelength, and the surface is completely illuminated (no shadowing). Surfaces that are shadowed are treated by making further assumptions and are discussed by several authors.[68-73]

The method of series expansion of the fields, where allowable, does not suffer from the restrictions noted in the Helmholtz formulation. Allowable refers to the necessity that the expansion in elementary solutions to the Helmholtz equation must converge to the boundary conditions imposed on the surface.[47,48] The problem encountered here is that infinite sets of equations must be solved for the expansion coefficients. The coefficients can, for some surfaces, be determined exactly by various methods discussed above. However, these exact solutions are not easily evaluated and parameter dependencies are not obvious. In general, the approximations are: using finite sets of equations and proceeding by matrix inversion,[11,12,35,36,41] and using variational methods which choose coefficients such that the mean square error in the boundary conditions is minimized.[20]

If the roughness of the surface can be treated as small, various perturbation methods become available.[3,34,70] Mathematical techniques involving boundary perturbations and perturbations of the solutions of known canonical problems exist.[3] For example, the latter problem could consist of a surface with two scales of roughness.[14] The larger roughness part of the problem is canonical in the sense that the solution can be written down, whereas the smaller roughness induces small geometrical variations in the surface, and a corresponding variation in the scattered field.

Since the surfaces cannot realistically be measured, it is convenient to model them using statistical methods. This is done by considering random surfaces[6,77-84] or random point sets with given statistical properties.[85-90] Thus, when the statistical properties of the surface are determined, different moments of the scattered field can be calculated. Another approach is to consider smooth surfaces with random impedence[91-92] or smoothed boundary conditions.[93-95]

Although there exist many different statistical models of surface scattering in the literature, we attempt to classify and describe only a few.

A diagrammatic solution for scattering from single valued multivariate Gaussian surface has been derived.[96-98] Evaluation of the diagrams present numerical difficulties and only one first order diagram has been completed.[99]

There is a large amount of work which is based on the Helmholtz integral formula using the Kirchhoff approximation where both source and receiver are moved to the Fraunhofer zone.[6,69-70,78-84,100-102] The surface $z = \zeta(x,y)$ is then treated as a random variable and suitable moments of the field calculated. These models require knowledge of the joint probability distribution of any two surface points.

At least one model[103] addresses non Gaussian surface statistics whereas most others use Gaussian statistics for ease of evaluation.

One model[34] expands $\zeta(x,y)$ in a harmonic series with coefficients related to the surface power spectrum. This model requires the availability of the power spectrum of the surface and is extremely difficult to evaluate.

Two models, randomly spaced half planes,[104,105] and randomly spaced bosses on a plane,[60,61] are based on exact solutions where the spacing of elements has been randomized.

Models based on random facets,[106-108] and random point sets[85-89] have been developed. These incorporate such features as random spacings or slopes, random scattering strengths, and various directivity functions.

Some of these models have been used to describe experimental results with limited success. A basic problem here is the determination of the surface statistics and choice of parameters.

Surfaces with specified statistics have been constructed, one[109] physical and not yet used in an experiment, the other numerical[110] used in a computer experiment. Those models based on nonphysical surfaces[95-89 104,105] are especially difficult since there appears to be no way to relate the surface parameters to the parameters of the solution.

SCATTERING FROM THE SEA BOTTOM

The sea bottom, in general, has discontinuities, thereby limiting the use of the Kirchhoff approximation. In addition, in most places the bottom does not present a large impedance contrast. Thus the impedance of the bottom material and its distribution becomes extremely important in the description of the scattered field. Because of its remoteness, both the topography and impedance characteristics of the bottom are difficult to obtain.

The bottom, in places, is stratified and has been modeled[111-116] as such. In other places, however, the bottom composition is much more complicated and requires a statistical treatment. Existing models and detailed experimental data[117-124] on this aspect of the problem are insufficient. However, two collections, MGS and NAVADO, of data yield average acoustic scattering information for large ocean bottom regions. The scattering data are classified according to bottom characteristics. Frequency smearing of signals scattered from the bottom, due entirely to source-receiver motion, has not been measured. Further, rough surface time smearing and time smearing caused by penetration into the bottom have not been adequately studied.

Measurement of the bottom relief and composition presents a major unsolved problem. Use of precision fathometers and various other acoustic devices aid in the determination of surface relief and sub-bottom reflectivity along a track, but have limited resolution. At least one paper[125] has dealt with the problem of measuring bottom surface statistics from scattering experiments. There are many papers[111-122, 126-129] presenting experimental data on scattering from the sea bottom for the backscatter and specular directions. Data for other directions are extremely limited.[111,117,130-133]

A recent Russian reference text[134] has nine chapters devoted to scattering and reflection from the ocean bottom and a bibliography through 1969.

SCATTERING FROM AN ICE COVER

The problem of predicting scattering from the under surface of an ice cover is extremely difficult because it is inaccessible, has extremes in roughness, has extensive entrapped air, and the ice cover itself is discontinuous. All of the problems encountered in sea bottom scattering are here. The great majority of our data on under ice scattering is obtained from multiple scattering in the forward direction during propagation experiments.

Wave motion on the ice has been detected and measured.[135-136] In the central Arctic this motion is very small and should have little effect on the acoustic scattering problems. However, in fringe Arctic regions it may be more important. Most of the experimental scattering data from under ice have been for the backscatter case,[137–144] although at least one paper deals with specular reflection.[145] Recent transmission studies under ice[146–149] indicate that for sufficiently low frequencies the scattering loss becomes insignificant. There is very little knowledge about the acoustical properties of sea ice and even less is known about the surface topography necessary for scatter prediction. The method for measuring the latter using a submarine and obtaining related information is the subject of other papers.[150,151] Currently one of the hopes is that the measurement of characteristics of the upper surface by aircraft can be correlated with the roughness of below surface ice for acoustic purposes.

CONCLUDING REMARKS

Most theoretical treatments of scattering consider an incident plane wave or at least a locally plane wave. This choice is made in order to simplify the problem to manageable proportions. By assuming an incident plane wave, an approximation to the problem has been made. The fact that we are considering a linear problem and any incident wave may be represented by a suitable superposition of plane waves is in many instances not much comfort.

The assumption of an incident plane wave is a good approximation where the size of a scatterer is small compared to the acoustic wavelength. In the case of a surface, the projection of the effective insonified region normal to the incident direction should deviate from a wave front by a small amount compared to a wavelength.

More difficult theoretically is the addition of source and receiver directivities. Since directivity is a <u>far field</u> term as derived in the free field it is more proper to refer to the difficulty as the finite size of sources and receivers in a non-free-field environment.

Scattering from realistic surfaces is approached by breaking down the allowable surfaces into three categories. The first category comprises those surfaces that have little roughness compared to an acoustic wavelength, second, those that have roughness comparable to a wavelength, and third, those that are very rough compared to an acoustic wavelength. The first and third of these have been worked on by numerous authors, while the second has been considered by only a few. Limited solutions exist for the smooth case, but no tractable results are available for the intermediate case, which is the case most often met in practice; the very rough case is approximated by the geometrical acoustics solution.

These are all surface relief type problems, if in addition the surface is penetrable, then several other complications enter the problem. Only relatively simple approximations have been made toward the solution of acoustic scattering from surfaces bounding inhomogeneous material.

Statistical scattering theories that have been developed do not predict the experimental results to a reasonable or required degree of accuracy. By parameter changes they can be brought into relatively close agreement, but, in general, the parameters cannot be associated with characteristics of the scatterers.

Adequate testing of the various approximations made in scattering theory has to be done. This will require experimentation under extremely well controlled and measured conditions.

It is proposed that the proper approach to solving problems of acoustic scattering from rough surfaces is one in which an appropriate field expansion is evaluated in conjunction with a controlled experiment. The authors feel that the ingredients for the solution lie in the theoretical diagrammatic expansions[97] and specified surface constructions.[109]

In 1968 the Russians assessed the understanding of and progress on scattering problems in the ocean as one of their major goals at the USSR Academy of Sciences and Sixth All Union Acoustic Conference.[152] As indicated by the magnitude of recent activities, the Russians are continuing to place a strong emphasis on ocean scattering problems.

REFERENCES

1. P. M. Morse and K. U. Ingard, Theoretical Acoustics (McGraw-Hill Book Co., New York, 1968).
2. B. B. Baker and E. T. Copson, The Mathematical Theory of Huygen's Principle (Clarendon Press, Oxford, 1953).
3. P. M. Morse and H. Feshback, Methods of Mathematical Physics (McGraw-Hill Book Co., New York, 1953), Pt. II., Pgs. 528-539.
4. P. Moon and D. E. Spencer, Field Theory Handbook (Springer-Verlag, Berlin, 1961).
5. I. Stakgold, Boundary Value Problems of Mathematical Physics (Macmillan Co., New York, 1967) Vols. I and II.
6. P. Beckmann and A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces (Macmillan Co., New York, 1963).
7. J. F. Carlson and A. E. Heins, "The Reflection of an Electromagnetic Plane Wave by an Infinite Set of Plates, I and II," Quart. Appl. Math. 4, 313 (1947) and 5, 82 (1947).
8. F. Berz, "Reflection and Refraction of Microwaves at a Set of Parallel Metallic Plates," Proc. IEE (London), 98, 47 (1951), Pt. III.
9. E. A. N. Whitehead, "The Theory of Parallel-Plate Media for Microwave Lenses," Proc. IEE (London) 98, 133 (1951), Pt. III.
10. R. A. Hurd, "The Propagation of an Electromagnetic Wave Along an Infinite Corrugated Surface," Can. J.Phys. 32, 727 (1954).
11. L. N. Deryugin, "The Reflection of a Laterally Polarized Plane Wave from a Surface of Rectangular Corrugations," Radio Engineering 15, 2, 15-26 (1960).

12. L. N. Deryugin, "The Reflection of a Longitudinally Polarized Plane Wave from a Surface of Rectangular Corrugations," Radio Engineering 15, 5, 9-16 (1960).

13. J. A. DeSanto, "Scattering from a Periodic Corrugated Structure: Thin Comb with Soft Boundaries," J. Math. Phys. 12, 9 (Sept. 1971).

14. J. A. DeSanto, "Scattering from a Periodic Corrugated Structure: Thin Comb with Hard Boundaries," J. Math. Phys. 13, 3 (Mar. 1972).

15. J. A. DeSanto, "Scattering from a Periodic Corrugated Surface: Semi-Infinite Inhomogeneously Filled Plates with Soft Boundaries," NRL Rept. No. 7320, (November 1971).

16. J. A. DeSanto, "Scattering from a Periodic Corrugated Surface: Semi-Infinite Inhomogeneously Filled Plates with Hard Boundaries," NRL Rept. 7321, (November 1971).

17. J. A. DeSanto, "Scattering from a Periodic Corrugated Surface: Finite-Depth Inhomogeneously Filled Plates with Soft Boundaries," NRL Report 7375, (May 1972)

18. J. A. DeSanto, "Scattering from a Periodic Corrugated Surface: Finite-Depth Inhomogeneously Filled Plates with Hard Boundaries," NRL Rept. 7377, (May 1972).

19. R. Mittra and S. W. Lee, Analytical Techniques in the Theory of Guided Waves (Macmillan Co., New York, 1971).

20. W. C. Meecham and C. W. Peters, "Reflection of Plane-Polarized Electromagnetic Radiation from an Echelette Diffraction Grating," J. Appl. Phys. 28, 216 (1957).

21. Z. Szekeley, "The Scattered Field from a Sawtooth Corrugated Surface," Symposium on Microwave Optics - Part II, edited by B. S. Karasik (McGill University, Montreal, Canada, 1953), p. 329.

22. A. N. Leporski, "Scattering of Sound Waves by Sinusoidal and Saw-Tooth Surfaces," Soviet Phys. - Acoustics 2, 177 (1956).

23. T. Itoh and R. Mittra, "An Analytical Study of the Echelette Grating with Application to Open Resonators," IEEE Trans. MTT-17, 319 (1969).

24. A. D. Lapin, "The Scattering of a Plane Wave at a Serrated Surface," Soviet Phys. - Acoustics 9, 37 (1963).

25. R. D. Hatcher and J. H. Rohrbaugh, "Theory of the Echelette Grating - Part I," J. Opt. Soc. Am. 46, 104 (1956) and - Part II, J. Opt. Soc. Am. 48, 704 (1958).

26. R. D. Hatcher, J. H. Rohrbaugh, C. Prine, and W. G. Zoellner, "Theory of the Echelette Grating - Part III," J. Opt. Soc. Am. 48, 410 (1958).

27. Lord Rayleigh (J. W. Strutt) Theory of Sound (Dover, New York 1945).

28. B. A. Lippmann, "Note on the Theory of Gratings," J. Opt. Soc. Am. 43, 5, 408 (1953) (L).

29. H. S. Heaps, "Non-Specular Reflection of Sound from a Sinusoidal Surface," J. Acoust. Soc. Am. 27, 4, 698-705 (1955).

30. W. C. Meecham, "Variational Method for the Calculation of the Distribution Energy Reflected from a Periodic Surface," J. Appl. Phys. 27, 4, 361-367 (1956).

31. W. C. Meecham, "Fourier Transform Method for the Treatment of the Problem of the Reflection of Radiation from Irregular Surfaces," J. Acoust. Soc. Am. 28, 3, 370-377 (1956).

32. L. M. Brekhovskikh, "Diffraction of Waves by a Rough Surface, Parts I and II," (in Russian), Zh. Eksper. i Theor. Fiz. 23, 275-304 (1952).

33. T. O. LaCasce and P. Tamarkin, "Underwater Sound Reflection from a Corrugated Surface," J. Appl. Phys. 27, 2, 138-148 (1956).

34. S. O. Rice, "Reflection of Electromagnetic Waves from Slightly Rough Surfaces," Comm. Pure Appl. Math. 4, 351-378 (1951).

35. J. L. Uretsky, "Reflection of a Plane Sound Wave from a Sinusoidal Surface," J. Acoust. Soc. Am. 35, 8, 1293-1294 (1963)(L).

36. J. L. Uretsky, "Scattering of Plane Waves from Periodic Surfaces," Ann. Phys. (N.Y.) 33, 400-427(1965).

37. S. R. Murphy and G. E. Lord, "Scattering from a Sinusoidal Surface-A Direct Comparison of the Results of Marsh and Uretsky," J. Acoust. Soc. Am. 36, 8, 1598-1599 (1964)(L).

38. G. R. Barnard, C. W. Horton, M. K. Miller, and F. R. Spitznogle, "Underwater-Sound Reflection from a Pressure-Release Sinusoidal Surface," J. Acoust. Soc. Am. 39, 1162 (1966).

39. H. W. Marsh, "In Defense of Rayleigh's Scattering from Corrugated Surfaces," J. Acoust. Soc. Am. 35, 11, 1835-1836 (1963)(L).

40. K. A. Zaki and A. R. Neureuther, "Scattering from a Perfectly Conducting Surface with a Sinusoidal Height Profile: TE Polarization, " IEEE Trans. AP-19, 208 (1971).

41. T. B. A. Senior, "The Scattering of Electromagnetic Waves by a Corrugated Sheet," Can. J. Phys. 37, 787 (1959).

42. T. B. A. Senior and T. C. Tong, "Scattering by a Periodic Surface," Univ. of Mich. Radiation Lab Rept. No. 7 (Sept. 1970).

43. J. G. Parker, "Reflection of Plane Sound Waves from a Sinusoidal Surface," J. Acoust. Soc. Am. 29, 3, 377-380 (1957).

44. Y. P. Lysanov, "On the Scattering of Sound by a Nonuniform Surface," Soviet Physics-Acoustic 3, 45 (1958).

45. A. Hessel and A. A. Oliner, "A New Theory of Wood's Anomalies on Optical Gratings," Appl. Opt. 4, 1275(1965).

46. J. A. DeSanto, It is possible to show that the full amplitude of the plane waves scattered from a sinusoidal surface can be naturally written as the Rayleigh amplitude plus a sum of other terms.

47. R. F. Millar, "On the Rayleigh Assumption in Scattering by a Periodic Surface. I and II," Proc. Cam. Phil. Soc. 65, 773 (1969) and 69, 217 (1971).

48. R. L. Holford, "Scattering of Sound Waves at a Periodic Pressure-Release Surface: An Exact Solution," Bell Telephone Laboratories Report under Contract No. N00014-69-C-0074 (5 Mar. 1971).

49. B. Noble, "Integral Equation Perturbation Methods in Low Frequency Diffraction," in Electromagnetic Waves, edited by R. Langer (University of Wisconsin Press, Madison, Wisc., 1962).

50. G. Chertock, "Sound Radiation from Vibrating Surfaces," J. Acoust. Soc. Am. 36, 1305 (1964).

51. L. G. Copley, "Integral Formulation in Acoustic Radiation," J. Acoust. Soc. Am. 44, 28 (1968).

52. L. G. Copley and H. A. Schenck, "Vanishing of the Surface Pressure Contribution to the Helmholtz Integral," J. Acoust. Soc. Am. 44, 228 (1968).

53. H. A. Schenck, "An Improved Integral Formulation for Acoustic Radiation Problems," J. Acoust. Soc. Am. 44, 41 (1968).

54. E. Wolf and E. W. Marchand, "Comparison of the Kirchhoff and the Rayleigh-Sommerfeld Theories of Diffraction at an Aperture," J. Opt. Soc. Am. 54, 587 (1964).

55. N. Mukunda, "Consistency of Rayleigh's Diffraction Formulas with Kirchhoff's Boundary Conditions," J. Opt. Soc. Am. 52, 336 (1962).

56. J. W. Goodman, Introduction to Fourier Optics (McGraw-Hill, New York, 1968).

57. V. Twersky, "On the Non-Specular Reflection of Plane Waves of Sound," A. Acoust. Soc. Am. 22, 539-546 (1950).

58. V. Twersky, "On the Non-Specular Reflection of Sound from Planes with Absorbent Bosses," J. Acoust. Soc. Am. 23, 336-338 (1951a).

59. V. Twersky, "Reflection Coefficients for Certain Rough Surfaces," J. Appl. Phys. 24, 569-660 (1953).

60. V. Twersky, "On the Scattering and Reflection of Electromagnetic Waves by Rough Surfaces," Trans. I.R.E. AP-5, 81-90 (1957a).

61. V. Twersky, "On Scattering and Reflection of Sound by Rough Surfaces," J. Acoust. Soc. Am. 29, 209-225 (1957b).

62. H. W. Marsh, "Exact Solution of Wave Scattering by Irregular Surfaces," J. Acoust. Soc. Am. 33, 330-333 (1961).

63. H. W. Marsh, M. Schulkin, and S. G. Kneale, "Scattering of Underwater Sound by the Sea Surface," J. Acoust. Soc. Am. 33, 3, 334-340 (1961).

64. H. W. Marsh, "Non-Specular Scattering of Underwater Sound by the Sea-Surface," in Underwater Acoustics, edited by V. M. Albers (Plenum Press, New York, 1962), Lecture 11, pp. 193-197.

65. H. W. Marsh, "Sound Reflection and Scattering from the Sea Surface," J. Acoust. Soc. Am. 35, 2, 240-244 (1963).

66. L. Fortuin, "Reflection and Scattering at the Sea Surface," J. Acoust. Soc. Am. 47, 1209-1228 (1970).

67. A. B. Shmelev, "Wave Scattering by Statistically Uneven Surfaces," Sov. Phys. Upsek 15, 173 (1972).

68. H. Trinkaus, "Fundamental Approximations for the Scattering of Acoustic Waves from a Rough Surface," Saclantcen Memorandum SM-15 (1 Aug. 1973)

69. C. Eckart, "Scattering of Sound from the Sea Surface," J. Acoust. Soc. Am. 25, 566-570 (1953).

70. W. C. Meecham, "On the Use of the Kirchhoff Approximation for the Solution of Reflection Problems," J. Rat. Mech Anal. 5, 323 (1956).

71. P. Beckmann, "Shadowing of Random Rough Surfaces," Trans. IEEE Antennas Propagation 13, 384-388 (1965).

72. R. J. Wagner, "Shadowing of Randomly Rough Surfaces," J. Acoust. Soc. Am. 41, 138-147 (1967).

73. P. J. Lynch and R. J. Wagner, "Rough-Surface Scattering: Shadowing, Multiple Scatter, and Energy Conservation," J. Math. Phys. 11, 3032 (1970).

74. B. G. Smith, "Geometrical Shadowing of a Random Rough Surface," Trans. IEEE Antennas Propagation 15, 668-671 (1967).

75. K. E. Hawker and P. J. Welton, "Shadowing of Randomly Rough Surfaces," J. Acoust. Soc. Am. 45(A), 295 (1969).

76. R. R. Gardner, "Acoustic Backscattering from a Rough Surface at Extremely Low Grazing Angles," Ph.D. Dissertation, University of California, San Diego (1970).

77. B. F. Kur'yanov, "The Scattering of Sound at a Rough Surface with Two Types of Irregularity," Soviet Physics - Acoustics 8, 3, 252-257 (1963).

78. M. A. Isakovich, "The Scattering of Waves from a Statistically Rough Surface," (in Russian), Zhurn. Eksp. Theor. Fiz. 23, 305-314 (1952).

79. W. S. Ament, "Toward a Theory of Reflection by a Rough Surface," Proc. I.R.E. 41, 142-146 (1953).

80. C. W. Horton and T. G. Muir, "Theoretical Studies on the Scattering of Acoustical Waves from a Rough Surface," J. Acoust. Soc. Am. 41, 627-634 (1967).

81. H. Davies, "The Reflection of Electromagnetic Waves from a Rough Surface," Proc. I.E.E., Pt. III, 101, 209-214 (1954). Discussion on above paper, Proc. I.E.E., Pt. III, 102.

82. J. Feinstein, "Some Stochastic Problems in Wave Propagation," Part I. Trans. I.R.E. AP-2, 23-30 (1954).

83. J. G. Parker, "Reflection of Plane Sound Waves from an Irregular Surface," J. Acoust. Soc. Am. 28, 4, 672-680 (1956).

84. Yu P. Lysanov, "Theory of the Scattering of Waves at Periodically Uneven Surfaces," Soviet Physics - Acoustics 4, 1, 1-10 (1958).

85. L. M. Spetner and I. Katz, "Two Statistical Models for Radar Terrain Return," Trans. I.R.E. AP-8, 242-246 (1960).

86. M. A. Spizzichino, "La Reflexion des ondes electromagnetiques par une surface irreguliere," Research Rept. No. 549 T, Centre National d'Etudes des Telecommunications, 4.11.1959 (1959).

87. F. Du Castel and A. Spizzichino, "Reflection en milieu Inhomogene," (Reflexions partielles dans l'atmosphere et propagation a grande distance, 3 eme partie). Ann Tele-comm. 14, 33-40 (1959).

88. D. Middleton, "A Statistical Theory of Reverberation and Similar First-Order Scattered Fields - Part I: Waveforms of the General Process," IEEE Trans. IT-13, 3, 372-392 (1967).

89. D. Middleton, "A Statistical Theory of Reverberation and Similar First-Order Scattered Fields - Part II: Moments, Spectra, and Special Distributions," IEEE Trans. IT-13, 3, 393-414 (1967).

90. S. M. Karp, R. M. Gagliardi, I. S. Reed, "Radiation Models Using Discrete Radiator Ensembles," Proc. IEEE 56, 1704 (1968).

91. K. Sobczyk, "Reflection of Scalar Wave from a Plane with Random Impedance", Acta Physica Polonica A44, 581 (1973).

92. W. A. Kuperman, "Sound Propagation in Shallow Water, II", Saclantcen Conf. Proc. 14, Edited by O. F. Hastrup, O. V. Olesen 15 Nov. 1974.

93. M. S. Howe, "Contributions to the Theory of Scattering by Randomly Irregular Surfaces," Proc. R. Soc. London A337, 413(1974).

94. A. R. Wenzel, "Smoothed boundary conditions for Randomly Rough Surfaces," J. Math. Phys. 15, 317 (1974).

95. V. D. Freilikher, I. M. Fuks, "Attenuation of the Mean Field in a Waveguide at the Critical Frequency," Radiophysics and Quantum Electronics 13, 96-99 (1970).

96. G. G. Zipfel, "Scattering of Scalar Waves From a Random Irregular Interface," J. Math. Phys. 15, 101 (1974).

97. G. G. Zipfel, J. A. DeSanto, "Scattering of a Scalar Wave from a Random Rough Surface: A Diagrammatic Approach," J. Math. Phys. 13, 1903 (1972).

98. J. A. DeSanto, "Scattering from a Random Rough Surface: Diagram Methods for Elastic Media", J. Math. Phys. 14, 1566 (1973).

99. J. A. DeSanto, O. Shisha, "Numerical Solution of a Singular Integral Equation in Random Rough Surface Scattering Theory", J. Comp. Phys. 15, 286 (1974).

100. M. L. Boyd, R. L. Deavenport, "Forward and Specular Scattering From a Rough Surface: Theory and Experiment", JASA 53, 791 (1973).

101. C. S. Clay, H. Medwin, W. M. Wright, "Specularly Scattered Sound and the Probability density function of a rough surface", JASA 53, 1678 (1973).

102. H. Medwin, J. D. Hagy, "Helmholtz-Kirchhoff Theory for Sound Transmission through a Statistically Rough Plane Interface Between Dissimilar Fluids," JASA 51, 1083 (1971).

103. P. Beckmann, "Scattering by Non-Gaussian Surfaces," IEEE Trans. AP-21, 169 (1973).

104. W. S. Ament, "Forward and Back-Scattering by Certain Rough Surfaces", Trans. I.R.E. AP-4, 369-373 (1956).

105. W. S. Ament, "Reciprocity and Scattering by Certain Rough Surfaces," Trans. I.R.E. AP-8, 167-174 (1960).

106. L. S. Ornstein and A. Van Der Berg, "Reflectivity of Corrugated Surfaces," Physica 4, 1181 (1937).

107. P. Beckmann, "A New Approach to the Problem of Reflection from a Rough Surface," Acta Techn. CSAV 2, 311-355 (1957).

108. R. B. Patterson, "Model of a Rough Boundary as a Backscatter of Wave Radiation", J. Acoust. Soc. Am. 36, 1150-1153 (1964).

109. L. S. Schuetz and G. G. Zipfel, "Theory and Construction of Multivariate Gaussian Surfaces," J. Acoust. Soc. Am., Vol. 56, No. 1, 99-109 (1974).

110. J. C. Novarini and J. W. Caruthers, "Numerical Modeling of Randomly Rough Surfaces with Application to Sea Surfaces," Texas A&M University, Dept. of Oceanography, Ref. No. 71-13-T (1971).

111. G. R. Barnard, J. L. Bardin, and W. B. Hempkins, "Underwater Sound Reflection from Layered Media," J. Acoust. Soc. Am. 36, 2119 (1964).

112. H. P. Bucker, J. A. Whitney, G. S. Yee, and R. R. Gardner, "Reflection of Low-Frequency Sonar Signals from a Smooth Ocean Bottom," J. Acoust. Soc. Am. 37, 1037 (1965).

113. B. F. Cole, "Marine Sediment Attenuation and Ocean-Bottom Reflected Sound," J. Acoust. Soc. Am. 38, 291 (1965).

114. A. H. Nuttall and B. F. Cron, "Signal-Waveform Distortion Caused by Reflection off Lossy-Layered Bottoms," J. Acoust. Soc. Am. 40, 1094 (1966).

115. R. S. Winokur and J. Bohn, "Sound Reflection from a Low Velocity Bottom," J. Acoust. Soc. Am. 44, 1130 (1968).

116. H. C. Morris, "Bottom-Reflection-Loss Model with a Velocity Gradient," J. Acoust. Soc. Am. 48, 1198 (1970).

117. A. W. Nolle, et al., "Acoustic Properties of Water-Filled Sands," J. Acoust. Soc. Am. 35, 1394-1408 (1963).

118. J. L. Jones, C. B. Leslie, and L. E. Barton, "Acoustic Characteristics of Underwater Bottoms," J. Acoust. Soc. Am. 36, 154 (1964).

119. C. M. McKinney and C. D. Anderson, "Measurements of Backscattering of Sound from the Ocean Bottom," J. Acoust. Soc. Am. 36, 158 (1964).

120. H. M. Merklinger, "Bottom Reverberation Measured with Explosive Charges Fired Deep in the Ocean," J. Acoust. Soc. Am. 44 508 (1968).

121. R. B. Patterson, "Relationships between Acoustic Backscatter and Geological Characteristics of the Deep Ocean Floor," J. Acoust. Soc. Am. 46, 756 (1969).

122. D. R. Horn, B. M. Horn, M. N. Delach, and M. Ewing, "Prediction of Sonic Properties of Deep Sea Cores from the Hatteras Abyssal Plane and Environs," Lamont-Doherty Technical Report No. 1 (NAVSHIPS Contract N00024-69-1184 of Nov. 1969).

123. Yu. Yu. Zhitkovskii, "Relationship between the Reflection and Scattering of Sound by the Ocean Bottom," Acoustics Institute, Acad. Sci., Moscow, Vol. 18, No. 4 533-6 (1972).

124. V. I. Volovov, "Fluctuation Frequency of the Envelopes of Sound Signals Reflected from the Ocean Bottom," Acoustics Institute, Acad. Sci., Moscow, Vol. 17, No. 3 466-7 (1971).

125. E. Y. T. Kuo, "Wave Scattering and Transmission at Irregular Surfaces," J. Acoust. Soc. Am. 36, 2135-2142 (1964).

126. F. R. Menotti, S. R. Santaniello, and W. R. Schumacher, "Studies of Observed and Predicted Values of Bottom Reflectivity as a Function of Incident Angle," J. Acoust. Soc. Am. 38, 707 (1965).

127. J. P. Buckley and R. J. Urick, "Backscattering from the Deep Sea Bed at Small Grazing Angles," J. Acoust. Soc. Am. 44 (L), 648 (1968).

128. K. D. Flowers and B. G. Hurdle, "Monostatic Scattering from the Ocean Bottom," J. Acoust. Soc. Am. Vol. 51, No. 3, 1109-1111 (1972).

129. C. S. Clay, "Scattering and Reflection of Acoustic Waves at the Bottom and Surface of the Ocean," University of Wisconsin, Final Report, No. 74-1 (1974).

130. R. J. Urich, "The Backscattering of Sound from a Harbor Bottom," J. Acoust. Soc. Am. 26, 231 (1954).

131. B. G. Hurdle, K. D. Flowers, and K. P. Thompson, "Bistatic Acoustic Scattering from the Ocean Bottom," NRL Rept. No. 7285 (1971).

132. B. G. Hurdle, K. D. Flowers, and K. P. Thompson, "Three-Dimensional Scattered Fields from the Ocean Bottom," NRL Rept. No. 7325 (1971).

133. P. B. Schmidt, "Monostatic and Bistatic Backscattering Measurements from the Deep Ocean Bottom," J. Acoust. Soc. Am. Vol 50, No. 1, 326-31 (1971).

134. L. M. Brekhovskikh, "Acoustics of the Ocean," U.S. Joint Publications Research Service, No. 64008-1, (1975).

135. K. Hunkins, "Seismic Studies of Sea Ice," J. Geophys. Res. 65, 3459 (1960).

136. K. Hunkins, "Waves on the Arctic Ocean," J. Geophys. Res. 67, 2477 (1962).

137. R. H. Mellen, "Underwater Acoustic Scattering from Arctic Ice," J. Acoust. Soc. Am. 40 (L) 1200 (1966).

138. R. P. Chapman, "Backscattering Strengths of Sea Ice," J. Acoust. Soc. Am. 39 (L), 1191 (1966).

139. J. R. Brown and D. W. Brown, "Reverberation under Arctic Sea-Ice," J. Acoust. Soc. Am. 40, 399 (1966).

140. J. R. Brown, "Reverberation Under Arctic Ice," J. Acoust. Soc. Am. 36 (L) 601 (1964).

141. A. R. Milne, "Underwater Backscattering Strengths of Arctic Pack Ice," J. Acoust. Soc. Am. 36, 1551 (1964).

142. R. P. Chapman and H. D. Scott, "Backscattering Strength of Young Sea Ice," J. Acoust. Soc. Am. 36 (L), 2417 (1964).

143. R. H. Mellen and H. W. Marsh, "Underwater Sound Reverberation in the Arctic Ocean," J. Acoust. Soc. Am. 35, 1645 (1963).

144. W. B. Birch, "Under-Ice Reverberation Modeling by Means of Multiple Regression," Program of the 84th Meeting of the Acoustical Society of America (1972).

145. M. P. Langleben, "Reflection of Sound at the Water-Sea Ice Interface," J. Geophys. Res. 75, 5243 (1970).

146. D. I. Diachok, "Effects of Sea-Ice Ridge Characteristics on Under Ice Reflection Loss in Arctic and Sub-arctic Waters," Proceedings of the Satellite Symposium on Underwater Acoustics (1974).

147. Diachok, D. I. and Kozo, T. L., "Measured Effects of Ice Roughness on Under-Ice Transmission Loss," Program of the 84th Meeting of the Acoustical Society of America (1972).

148. T. J. Tulko and Lindsay, R. B., "Comparison of Under-Ice and Open Water Transmission Loss Measurements in Baffin Bay," Program of the 84th Meeting of the Acoustical Society of America (1972).

149. S. K. Numrich, "Low-Frequency Sound Propagated in the Marginal Ice Zone of the Greenland Sea," JASA, Vol. 56, 550 (1974).

150. R. E. Francois, "The Unmanned Arctic Research Submersible System," Technol. Soc. U. Vol. 7, No. 1 46-8 (1973).

151. W. Lyon, "Ocean and Sea Ice Research in the Arctic Ocean via Submarine," Trans. N.Y. Acad. Sci. 23, 662-674 (1961).

152. L. M. Brekhovskikh, "Some Problems of Oceanic Acoustics," Atmos. Oceanic Phys. 4, 12, 1291-1304 (1968).