SCATTERING FROM A RANDOM INTERFACE

by

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ABSTRACT

The problem of scattering from a random interface separating two fluids with different densities and sound speeds is considered. The method is to write coupled integral equations in coordinate space connecting the surface and volume values of the Green's function for the deterministic problem. In Fourier transform space the equations simplify, and it is possible to write a single integral equation for the Fourier transform of the surface value of the Green's function. Feynman-diagram methods can be used to aid the construction of both the Dyson equation for the mean of this Green's function and the Bethe-Salpeter equation for the mean of its second moment. These are derived assuming a Gaussian distribution of surface heights and using the accompanying cluster decomposition. As an example, a simple integral equation for the scattering amplitude corresponding to multiple scattering using the Kirchhoff approximation is also derived. It is analogous to the smoothing approximation used in random volume scattering theory. Its numerical solution for the special case of a Neumann surface is presented and, for large values of the Rayleigh roughness parameter, yields more coherent specular intensity than the Kirchhoff approximation. Other examples and the relation of our formalism to other methods are also discussed. In the limiting cases the general formalism reduces to the standard results. In particular, in the flat surface limit we get the result in Officer's book.

1. Introduction and Notation

We describe the scattering of a scalar wave from a random two fluid interface in Fig. 1. Regions V_1 and V_2 are semi-infinite fluids with sound speeds, wavenumbers, and densities given by c_1, k_1, ρ_1 and c_2, k_2, ρ_2 respectively. Coordinate three-vectors are specified by $\underset{\sim}{x} = (x,y,z) = (x_\perp, z)$ and the random interface by $z = h(x_\perp)$. The latter is assumed to be a Gaussian distributed random variable. A three-vector on the surface is $\underset{\sim}{x}_s = (x_\perp, h(x_\perp))$. The method is to derive integral equations for the Green's function of the problem, which is composed of two parts, G_1 defined in V_1, and G_2 defined in V_2. They satisfy the equations ($x \in V_j$)

$$(\partial_m \partial_m + k_{1,2}^2) G_{1,2}(\underset{\sim}{x}, \underset{\sim}{x}'') = -\delta(\underset{\sim}{x} - \underset{\sim}{x}'') \tag{1}$$

where $\partial_m = \partial/\partial x_m$ is the derivative (repeated subscripts are summed from 1 to 3), appropriate radiation conditions for large $|\underset{\sim}{x}|$, and continuity conditions at the interface. The free-space Green's functions $G_j^{o\pm}(\underset{\sim}{x}, \underset{\sim}{x}'')$ ($j = 1,2$) satisfy similar equations except that $\underset{\sim}{x}$ is in all space. Explicitly they are

$$G_j^{o\pm}(\underset{\sim}{x}, \underset{\sim}{x}'') = [4\pi|\underset{\sim}{x}-\underset{\sim}{x}''|]^{-1} \exp[\pm ik_j|\underset{\sim}{x}-\underset{\sim}{x}''|] \tag{2}$$

where the $\pm$ indicate the radiation condition. We now drop this $\pm$ notation for simplicity, resurrecting it only when necessary.

Next, apply Green's theorem to G_1 and G_2^o in V_1 and to G_2 and G_2^o in V_2, that is use the same free space Green's function in both regions. The results are evaluated in V_1 and combined to yield

$$\begin{aligned} &G_1(\underset{\sim}{x}', \underset{\sim}{x}'')\theta(z' - h(x_\perp')) \\ &= G_2^o(\underset{\sim}{x}', \underset{\sim}{x}'') + (k_1^2 - k_2^2)\int G_2^o(\underset{\sim}{x}', \underset{\sim}{x}) G_1(\underset{\sim}{x}, \underset{\sim}{x}'')\theta(z - h(x_\perp))\, d\underset{\sim}{x} \\ &\quad + \int \partial_m' G_2^o(\underset{\sim}{x}', \underset{\sim}{x}_s) n_m(x_\perp) [G_2(\underset{\sim}{x}_s, \underset{\sim}{x}'') - G_1(\underset{\sim}{x}_s, \underset{\sim}{x}'')]\, dx_\perp \end{aligned} \tag{3}$$

where the step function

$$\theta(z - h(x_\perp)) = \begin{cases} 1 & z > h(x_\perp) \\ 0 & z < h(x_\perp) \end{cases} \tag{4}$$

is used to explicitly indicate the discontinuous nature of the l.h.s. of (3), and the normal is $n_m(x_\perp) = \delta_{m3} - \partial_{m\perp} h(x_\perp)$ with δ_{m3} the Kronecker delta. In order to derive (3) we have used a continuity condition on the normal derivative of the Green's function

$$\partial_m G_2(\underset{\sim}{x}_s, \underset{\sim}{x}'') n_m(x_\perp) = \partial_m G_1(\underset{\sim}{x}_s, \underset{\sim}{x}'') n_m(x_\perp). \tag{5}$$

A further continuity condition is necessary on the Green's function and we express it generally as

$$G_2(\underset{\sim}{x}_s, \underset{\sim}{x}'') = a(x_\perp) G_1(\underset{\sim}{x}_s, \underset{\sim}{x}'') \tag{6}$$

The explicit form for a is derived from the flat surface limit of the theory in the Appendix and it is shown to be a constant. For the moment we keep it general.

Using (6) in (4), defining the "field" Green's function G_1^D and the "surface" Green's function G^S as

$$G_1^D(\underset{\sim}{x}, \underset{\sim}{x}'') = G_1(\underset{\sim}{x}, \underset{\sim}{x}'') \theta(z - h(x_\perp)) \tag{7}$$

$$G^S(\underset{\sim}{x}_s, \underset{\sim}{x}'') = 1/2\,[1 + a(x_\perp)]\, G_1(\underset{\sim}{x}_s, \underset{\sim}{x}'') \tag{8}$$

yields the result

$$G_1^D(\underset{\sim}{x}', \underset{\sim}{x}'') = G_2^o(\underset{\sim}{x}', \underset{\sim}{x}'') + (k_1^2 - k_2^2) \int G_2^o(\underset{\sim}{x}', \underset{\sim}{x}) G_1^D(\underset{\sim}{x}, \underset{\sim}{x}'') d\underset{\sim}{x} \tag{9}$$

$$-2 \int \partial'_m G_2^o(\underset{\sim}{x}', \underset{\sim}{x}_s) n_m(x_\perp) r(x_\perp) G^s(\underset{\sim}{x}_s, \underset{\sim}{x}'') dx_\perp$$

with

$$r(x_\perp) = [1 - a(x_\perp)] / [1 + a(x_\perp)] . \tag{10}$$

Letting $\underset{\sim}{x}' \to \underset{\sim}{x}'_s$ through positive z' values in (9) yields

$$G^s(\underset{\sim}{x}_s', \underset{\sim}{x}'') = G_2^o(\underset{\sim}{x}_s', \underset{\sim}{x}'')+(k_1^2-k_2^2)\int G_2^o(\underset{\sim}{x}_s', \underset{\sim}{x})G_1^D(\underset{\sim}{x}, \underset{\sim}{x}'')d\underset{\sim}{x} \qquad (11)$$

$$- \int P_m^{(2)}(\underset{\sim}{x}_s', \underset{\sim}{x}_s)n_m(x_\perp)r(x_\perp)G^s(\underset{\sim}{x}_s, \underset{\sim}{x}'')dx_\perp$$

where

$$P_m^{(2)}(\underset{\sim}{x}_s', \underset{\sim}{x}_s) = (2\pi)^{-3}\int d\underset{\sim}{k} \exp\left[i\underset{\sim}{k}\cdot(\underset{\sim}{x}_s'-\underset{\sim}{x}_s)\right] G_2^o(k)\ P_m^{(2)}(\underset{\sim}{k}) \qquad (12)$$

With $G_2^o(k)$ the Fourier transform of $G_2^o(x)$ and

$$P_m^{(2)}(\underset{\sim}{k}) = 2i\left\{ k_{m\perp} + \delta_{m3}P\ \frac{k_2^2 - k_\perp^2}{k_z} \right\} . \qquad (13)$$

Here P in (13) stands for the Cauchy principle value. These functions were previously calculated when we discussed scattering from a random Neumann surface[1]. Equations (9) and (11) can be thought of as coupled surface-volume integral equations for the Green's function of the problem. Their utility is realised under Fourier transformation.

2. Fourier Transformation

Introducing Fourier transforms of the form

$$G_1^D(\underset{\sim}{x}', \underset{\sim}{x}'') = (2\pi)^{-6}\iint d\underset{\sim}{k}'d\underset{\sim}{k}''\exp\left\{ i(\underset{\sim}{k}'\cdot\underset{\sim}{x}' - \underset{\sim}{k}''\cdot\underset{\sim}{x}'')\right\} G_1^D(\underset{\sim}{k}', \underset{\sim}{k}'') \qquad (14)$$

and analogous functions for G^s and G_2^o in (9) and (11), then setting the resulting integrands to zero using a gauge condition argument previously discussed[1] yields two equations which can be combined. The most elegent way to define the result is to first define singularity free Green's functions Γ_1^D and Γ^s via

$$G_1^{D,S}(\underset{\sim}{k}',\underset{\sim}{k}'') = (2\pi)^3\,\delta(\underset{\sim}{k}'-\underset{\sim}{k}'')\ G_1^o(k')$$

$$+ (2\pi)^3 G_1^o(k')\ \Gamma_1^{D,S}(\underset{\sim}{k}', \underset{\sim}{k}'')\ G_1^o(k'') \qquad (15)$$

Note that we now have G_1^o rather than G_2^o occurring. This arises naturally from the algebra. Then we can derive an integral relation for Γ_1^D

$$\Gamma_1^D(\underset{\sim}{k}',\underset{\sim}{k}'') = B^o(\underset{\sim}{k}',\underset{\sim}{k}'') + \int B^o(\underset{\sim}{k}', \underset{\sim}{k})G_1^o(k)\,\Gamma^s(\underset{\sim}{k}, \underset{\sim}{k}'')d\underset{\sim}{k} \tag{16}$$

where

$$B^o(\underset{\sim}{k}',\underset{\sim}{k}'') = -2i(2\pi)^{-3}k_m' r_m(\underset{\sim}{k}' - \underset{\sim}{k}'') \tag{17}$$

and

$$r_m(\underset{\sim}{k}) = \int \exp(-i\underset{\sim}{k}\cdot\underset{\sim}{x}_s)n_m(x_\perp)r(x_\perp)dx_\perp, \tag{18}$$

and an integral equation for Γ^s

$$\Gamma^s(\underset{\sim}{k}',\underset{\sim}{k}'') = W(\underset{\sim}{k}', \underset{\sim}{k}'') + \int W(\underset{\sim}{k}', \underset{\sim}{k})G_1^o(k)\Gamma^s(\underset{\sim}{k}, \underset{\sim}{k}'')d\underset{\sim}{k} \tag{19}$$

where

$$W(\underset{\sim}{k}',\underset{\sim}{k}'') = V_m(\underset{\sim}{k}')r_m(\underset{\sim}{k}' - \underset{\sim}{k}'') \tag{20}$$

and

$$V_m(\underset{\sim}{k}) = -\frac{2i}{(2\pi)^3}G_2^o(k)\left\{(k_1^2-k_2^2)k_m+(k^2-k_1^2)\left[k_{m\perp} + \delta_{m3}P\ \frac{k_2^2-k_\perp^2}{k_z}\right]\right\} \tag{21}$$

Note that if we set $k_2^2 - k_\perp^2 = k_z^2$ in the principle value part of (21) (called the "on-shell" or "0" condition) then

$$V_m(\underset{\sim}{k}')\Big|_o = k_m' \tag{22}$$

so that

$$\Gamma^s(\underset{\sim}{k}', \underset{\sim}{k}'')\Big|_o = \Gamma_1^D(\underset{\sim}{k}', \underset{\sim}{k}'') \tag{23}$$

Hence we have an integral equation for Γ^s and an algebraic procedure for calculating Γ_1^D from it. The latter is intimately related to scattering as we point out in the next section. Our results here reduce to the analogous results for the Neumann problem[1] when $r(x_\perp) = 1$ and $k_1 = k_2$.

3. Reduction

To point out the relation between Γ_1^D and the scattering problem we note that we can write the outgoing scattered field ψ^o in terms of the incident field ψ^i by using the Fourier transform of the Γ_1^D part of (15). The result is

$$\psi^o(\underset{\sim}{x}') = (2\pi)^3 \iint G_1^o(\underset{\sim}{x}', \underset{\sim}{x}_1)\, \Gamma_1^D(\underset{\sim}{x}_1, \underset{\sim}{x}_2)\, \psi^i(\underset{\sim}{x}_2)\, d\underset{\sim}{x}_1\, d\underset{\sim}{x}_2 \tag{24}$$

Each of these fields can be further decomposed into plane wave fields ϕ^o and ϕ^i via

$$\psi^o(\underset{\sim}{x}) = \int \exp(i\underset{\sim}{k}\cdot\underset{\sim}{x})\phi^o(k_\perp)\, dk_\perp \qquad k_z = (k_1^2 - k_\perp^2)^{\frac{1}{2}} = K_1 \tag{25}$$

$$\psi^i(\underset{\sim}{x}) = \int \exp(i\underset{\sim}{k}\cdot\underset{\sim}{x})\phi^i(k_\perp)\, dk_\perp \qquad k_z = -K_1 \tag{26}$$

and the plane wave fields related via

$$\phi^{o+}(k_\perp) = \int T(k_\perp, k_\perp')\phi^{i+}(k_\perp')\, dk_\perp' \tag{27}$$

where

$$T(k_\perp, k_\perp') = \left\{(\pi i/k_z)\, \Gamma_1^{D+}(\underset{\sim}{k}, \underset{\sim}{k}')\right\}\Bigg|_{\substack{k_z = K_1 \\ k_z' = -K_1'}} \tag{28}$$

The various conditions on the z-components of the wave numbers describe asymptotic conditions necessary to ensure that we have appropriate incoming and outgoing waves. It is interesting and useful to note that, using these conditions, we can again algebraically relate Γ_1^D and Γ^s

$$\Gamma_1^{D+}(\underset{\sim}{k}, \underset{\sim}{k}') = \Gamma^s(\underset{\sim}{k}, \underset{\sim}{k}') \quad \text{if } k_z = K_1 \text{ , } k_z' = -K_1' \tag{29}$$

Similarly complex conjugate fields can be defined and the scattered intensity $I(k_\perp)$ given by

$$I(k_\perp)\delta(k_\perp - k_\perp') = \phi^{o+}(k_\perp)\left\{\phi^{o+}(k_\perp')\right\}^*$$

$$= (\pi^2/K_1^2)\int \Gamma^{s+}(k_\perp, K_1;\ k_\perp'', -K_1'')\,\phi^{i+}(k_\perp'')dk_\perp''.$$

$$\int \left\{\Gamma^{s+}(k_\perp', K_1';\ k_\perp''', -K_1''')\,\phi^{i+}(k_\perp''')\right\}^* dk_\perp''' \qquad (30)$$

For single plane wave incidence

$$\phi^i(k_\perp) = \delta(k_\perp - k_\perp^i) \qquad (31)$$

(30) considerably simplifies. Note that we've used the functions Γ^s in the calculations via (29).

4. Random Surface

Up to now we have been describing a deterministic problem. In this section we write down the integral equations for the first two moments of Γ^s without going into detail about their derivation. Details can be found in the references. We consider the surface to be Gaussian distributed. Note that the only place $h(x_\perp)$ occurs explicitly in (19) is in the r_m integral which is part of the function W. Integration of r_m by parts reduces the problem to taking the ensemble average of an exponential function. If we were to write an iterative solution of (19), then take the ensemble of the result term by term and resum the result it is necessary to consider ensemble averages of products of the r_m. This can be simplified using the characteristic function

$$\left\langle \prod_{j=1}^{n} \exp\left[-ik_z^{(j)} h(x_\perp^{(j)})\right] \right\rangle$$

$$= \exp\left\{ -\tfrac{1}{2}\gamma(0) \sum_{j=1}^{n}\left[k_z^{(j)}\right] - \sum_{\substack{i,j=1 \\ i<j}}^{n} \gamma(x_\perp^{(i)} - x_\perp^{(j)} k_z^{(i)} k_z^{(j)}) \right\} \qquad (33)$$

where

$$\gamma(x_{\perp}^{(i)} - x_{\perp}^{(j)}) = \langle h(x_{\perp}^{(i)})\, h(x_{\perp}^{(j)}) \rangle \tag{34}$$

is the two-point correlation function. Equation (33) follows from Gaussian statistics. In addition the products of r_m are cluster decomposed using methods previously discussed[1,2].

Using these properties we can write the integral equation for the first moment of Γ^s, the Dyson equation, as

$$\langle \Gamma^{s+}(\underset{\sim}{k}', \underset{\sim}{k}'') \rangle = M(\underset{\sim}{k}', \underset{\sim}{k}'') + \int M(\underset{\sim}{k}', \underset{\sim}{k})\, G_1^{o+}(k) \langle \Gamma^{s+}(\underset{\sim}{k}, \underset{\sim}{k}'') \rangle \, d\underset{\sim}{k} \tag{35}$$

where the function M is called the mass operator in analogy with random volume scattering theory[3]. Although three-dimensional, the integral equation (35) appears simple. This is deceiving since M is an infinite series of successively more complicated terms involving multiple integrals. It cannot be summed, although each term can be formally written down quite easily using diagram techniques.[1,3] Solutions for this first moment describe coherent scattering.

The integral equation for the second moment is a Bethe-Salpeter type equation and is

$$\begin{aligned}
&\langle \Gamma^{s+}(\underset{\sim}{k}, \underset{\sim}{k}_a)\, \Gamma^{s-}(\underset{\sim}{k}', \underset{\sim}{k}_a') \rangle \\
&\quad = \langle \Gamma^{s+}(\underset{\sim}{k}, \underset{\sim}{k}_a) \rangle \langle \Gamma^{s-}(\underset{\sim}{k}', \underset{\sim}{k}_a') \rangle + L(\underset{\sim}{k}, \underset{\sim}{k}_a;\ \underset{\sim}{k}', \underset{\sim}{k}_a') \\
&\quad + \int L(\underset{\sim}{k}, \underset{\sim}{k}'';\ \underset{\sim}{k}', \underset{\sim}{k}_a') G_1^{o+}(k'')\ \Gamma^{s+}(\underset{\sim}{k}'', \underset{\sim}{k}_a)\ d\underset{\sim}{k}'' \\
&\quad + \int L(\underset{\sim}{k}, \underset{\sim}{k}_a;\ \underset{\sim}{k}', \underset{\sim}{k}'')\, G_1^{o-}(k'')\ \Gamma^{s-}(\underset{\sim}{k}'', \underset{\sim}{k}_a')\ d\underset{\sim}{k}'' \\
&\quad + \iint L(\underset{\sim}{k}, \underset{\sim}{k}'';\ \underset{\sim}{k}', \underset{\sim}{k}''') G_1^{o+}(k'') G_1^{o-}(k''') \cdot \\
&\qquad \cdot \langle \Gamma^{s+}(\underset{\sim}{k}', \underset{\sim}{k}_a)\, \Gamma^{s-}(\underset{\sim}{k}''', \underset{\sim}{k}_a') \rangle \, d\underset{\sim}{k}''\, d\underset{\sim}{k}'''
\end{aligned} \tag{36}$$

Where the intensity operator L is again an infinite series of terms also most easily written down using diagram techniques. The solution of (36) yields the incoherent intensity. Writing down higher order moment equations is also possible.

We have presented the above as examples of what can be done using this general method of approach. The problem can be considered formally in a very straightforward way. But the general cases of (35) and (36) are too complicated to be solved as yet. What is available however is a simple example of (35) which can be solved. If we approximate M as the first term in its series expansion it is possible to write an approximation to the coherent specular intensity for plane wave incidence (at angle θ_i) as

$$I_C(k_\perp) = R^2(\rho, N, \theta_i) \left| V^+(Ki, -Ki) \right|^2 \delta(k_\perp - ki_\perp) \tag{37}$$

where

$$\begin{aligned} \rho &= \rho_1/\rho_2 \\ N &= k_1/k_2 \\ ki &= k_1 \sin\theta_i \\ Ki &= (k_1^2 - k_\perp^{i2})^{\frac{1}{2}} \end{aligned} \tag{38}$$

and where R is the plane wave reflection coefficient (see the Appendix)

$$R(\rho, N, \theta_i) = \frac{(1 - N^2\sin^2\theta_i)^{\frac{1}{2}} - \rho\cos\theta_i}{(1 - N^2\sin^2\theta_i)^{\frac{1}{2}} + \rho\cos\theta_i} \tag{39}$$

Here V^+ satisfies a one-dimensional integral equation of the form

$$T^+(\xi', \xi'') = C_0(\xi' - \xi'') \tag{40}$$

$$+ \frac{R(\rho, N, \theta_i)}{\pi i} \int_{-\infty}^{\infty} \frac{C_0(\xi' - \xi)}{\xi^2 - 1 - i\epsilon} P\left(\frac{1}{\xi}\right) T^+(\xi, \xi'')\, d\xi$$

where $$C_0(\xi) = \exp(-\tfrac{1}{2}\xi^2\Sigma^2) \tag{41}$$

and $$\Sigma = k_1\sigma \cos\theta_i \tag{42}$$

with σ is the rms height of the surface, Σ the Rayleigh roughness parameter, and the definition

$$V^+(K^i\xi, -K^i\xi') = T^+(\xi, \xi') \tag{43}$$

If we further approximate V^+ by the C_0 term in (40) we get the Kirchhoff result. The full equation (41) can be interpreted as a lowest order (in the mass operator) multiple Kirchhoff expansion. Numerical solutions of (41) for the Neumann surface ($R = 1$) have been presented[4] and are shown in Fig. 2. For $\Sigma < 1$ the result agrees with the Kirchhoff approximation but for $\Sigma > 1$ the multiple scattering yields more coherent specular intensity than that expected from the Kirchhoff result. This effect has been experimentally observed in diverse scattering problems and explained using various theoretical models.[5] A comparison of our results with others will be discussed elsewhere.[6]

5. Summary

We have presented rather briefly an outline of our method of approach to scattering from a random interface. It is based on using only a single free-space Green's function in deriving the coordinate-space integral equations, the use of Fourier transform methods, and cluster decomposition methods similar to those used in statistical mechanics. Once the general method is understood it is straightforward to write down moment equations as in Sec. 4. A program to investigate the general properties of these equations as well as numerical solutions of specific examples is under way. Early results indicate the necessity of considering multiple scattering in problems of this type.

Appendix - Flat Surface Limit

We have already pointed out that the formalism reduces to the Neumann case when $r(x_\perp) = 1$, but have not shown how to derive $r(x_\perp)$, or equivalently $a(x_\perp)$, which we used in the boundary condition (6). We do this here. For a flat surface, $h(x_\perp) = 0$, and

$$r_m(\underset{\sim}{k}) = \delta_{m3}\,\tilde{r}\,(k_\perp) \tag{A.1}$$

where $$\tilde{r}(k_\perp) = \int dx_\perp \exp(-ik_\perp \cdot x_\perp) r(x_\perp) \tag{A.2}$$

and thus $$W(\underset{\sim}{k}', \underset{\sim}{k}'') = V_3(\underset{\sim}{k}')\,\tilde{r}\,(k_\perp' - k_\perp''). \tag{A.3}$$

Substituting (A.3) into (19) it is easy to show that

$$\Gamma^s(\underset{\sim}{k}', \underset{\sim}{k}'') = V_3(\underset{\sim}{k}')\,\tilde{r}\,(k_\perp' - k_\perp''). \tag{A.4}$$

Similarly (16) yields

$$\Gamma_1^D(\underset{\sim}{k}', \underset{\sim}{k}'') = -2i(2\pi)^{-3} k_z'\,\tilde{r}\,(k_\perp' - k_\perp'') \tag{A.5}$$

for a flat surface. Substituting (A.5) into (15), Fourier transforming the result, carrying out the integrals and defining

$$\tilde{r}(k_\perp' - k_\perp'') = (2\pi)^2 \delta(k_\perp' - k_\perp'') R(k_\perp') \tag{A.6}$$

where $R(k_\perp)$ is the same plane wave reflection coefficient defined in (39), yields the result

$$G_1^D(\underset{\sim}{x}', \underset{\sim}{x}'') = (2\pi)^{-2} \int \exp\left\{ik_\perp' \cdot (x_\perp' - x_\perp'')\right\} G_1(z', z'')\,dk' \tag{A.7}$$

$$G_1(z', z'') = \frac{\exp(iK_1' z'')}{-2iK_1'} \left[\exp(-iK_1' z') + R(k_\perp)\exp(iK_1' z')\right] \tag{A.8}$$

which is the one-dimensional Green's function for the flat interface. Similarly, results can be derived for the transmitted field. If we set $k_\perp'' = 0$ in (A.6) we can solve for $\tilde{r}(k_\perp)$, $r(x_\perp)$ by Fourier inversion using (A.2), and hence $a(x_\perp)$ to yield

$$a(x_\perp) = \rho_1/\rho_2 \qquad (A.9)$$

just the ratio of densities of the two media. Thus starting with a general form for a we are led to a coordinate independent result via the flat surface limit. Several equations in the paper are thus simplified.

Footnotes

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5. See, for example; C. S. Clay, H. Medwin and W. M. Wright - J Acoust Soc Am 53, 1677 (1973), M. L. Boyd and R. L. Deavenport - J Acoust Soc Am 53, 791 (1973), H. Medwin, R. A. Helbig and J. D. Hagy - J Acoust Soc Am 54, 99(1973), P. J. Lynch and R. J. Wagner - J Math Phys 11, 3032 (1971) and W. A. Kuperman, "Attenuation From Boundary Scattering In Shallow Waters", in Sound Propagation in Shallow Water II. Unclassified Papers, SACLANTCEN Conf Proc No 14 (15 Nov 1974)
6. J. A. DeSanto - "Multiple Scattering From a Random Interface", Invited Paper for Special Underwater Acoustics Session on Scattering From Rough Surfaces, Acoustical Society of America, San Francisco meeting, 4-7 November 1975

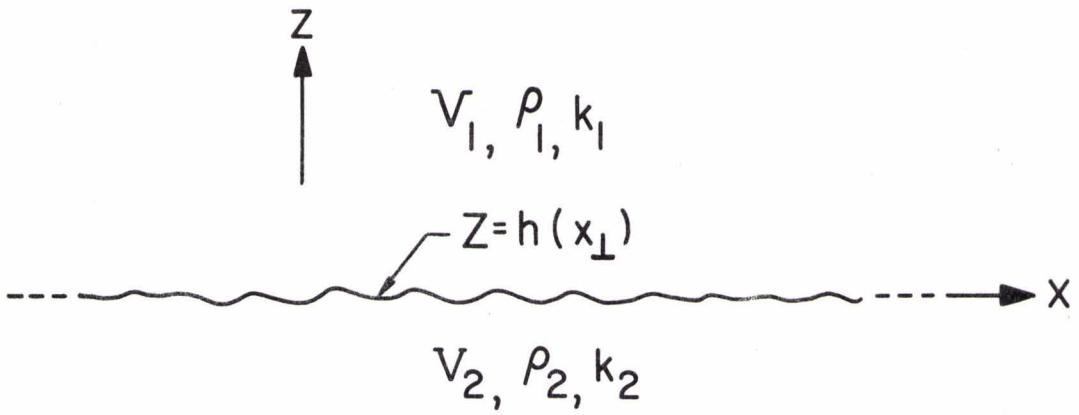

FIG. 1 SECTION OF A RANDOM INTERFACE $z = h(x_1)$ SEPARATING TWO SEMI-INFINITE FLUIDS (REGIONS V_1 AND V_2) HAVING DIFFERENT DENSITIES ρ AND WAVENUMBERS k

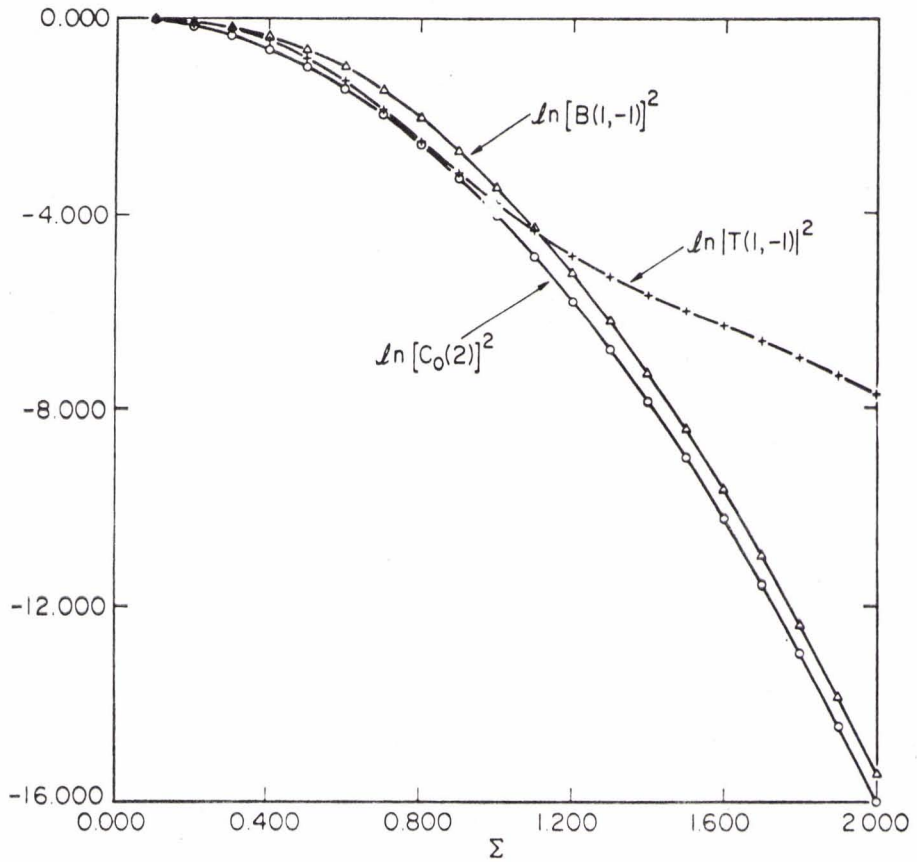

FIG. 2 COHERENT SPECULAR SCATTERING FROM A RANDOM NEUMANN SURFACE (Ref. 4) PLOTTED VERSUS $\sum = k_1 \sigma \cos \theta_i$, THE RAYLEIGH ROUGHNESS PARAMETER. $C_o(2) = \exp(-2 \sum^2)$ IS THE KIRCHHOFF RESULT, B(1, -1) ANOTHER SINGLE SCATTER RESULT INVOLVING COMBINATIONS OF EXPONENTIALS AND T(1, -1) THE MULTIPLE SCATTERING SOLUTION OF Eq. (41) HERE FOR $R = 1$