THE CONTINUOUS GRADIENT RAY TRACING SYSTEM (CONGRATS)

by

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INTRODUCTION

CONGRATS, an acronym for the Continuous Gradient Ray Tracing System, is an integrated collection of ray tracing programs designed to model acoustic propagation and reverberation. The fundamental programs of the series, CONGRATS I, construct ray diagrams and generate eigenrays, that is, rays that join a given source to a given target [Ref. 1]. The most distinguishing feature of CONGRATS I is that the velocity of sound in the ocean is represented by a function of depth whose first derivative is continuous, and still permits one to integrate the resulting ray tracing equations in closed form.

CONGRATS II processes the eigenray information that was generated by CONGRATS I, and displays it in such useful forms as total propagation loss as a function of range, and pulse shape as a function of time [Ref. 2]. The various multipath arrivals can be summed using random phase or coherent phase addition.

The most recent contribution to the ray tracing series, CONGRATS III, is the main topic of discussion [Ref. 3]. Bottom, surface, and volume reverberation are computed as a function of time for a given set of environmental and sonar parameters. The total reverberation level is assumed to be the sum of the three components. Among the notable features of CONGRATS III is the large number of multipath arrivals that can be considered in the reverberation computation.

REVERBERATION THEORY

Let us review reverberation theory. Consider an acoustic signal which originates at a point source at a reference time equal to zero and is transmitted through the ocean [Fig. 1]. A portion of the signal is scattered back toward the source as the signal encounters scatterers on the ocean bottom or surface or within the ocean volume. When the rescattering of the scattered sound is neglected, a closed ray path from a source to a scatterer and back to the source can be constructed from an incident ray (a ray from the source at point 'a' to the scatterer at point 'b') and a backscattered ray (a ray from the scatterer back to the source). Let the incident ray enter the water at time t_o and have travel time t_1 and let the backscattered ray have travel time t_2. Then the closed ray path will have round trip travel time

$$t = t_1 + t_2 \quad . \qquad \text{[Eq. 1]}$$

It will return to the source at time

$$T = t_o + t \quad , \qquad \text{[Eq. 2]}$$

and have an intensity

$$I = I_s \eta_d \eta_d' \eta_w \eta_w' k \qquad \text{[Eq. 3]}$$

where

- I_s is the reference intensity 1 yd from the source,
- η_d is the transmitting response of the sonar,
- η_d' is the receiving response of the sonar,
- η_w is the propagation loss factor of the incident ray,
- η_w' is the propagation loss factor of the backscattered ray, and
- k is the backscattering coefficient expressing the ratio of reflected intensity to incident intensity per scatter.

If the acoustic signal has pulse length τ, it follows that

$$0 \leq t_o \leq \tau \quad , \qquad \text{[Eq. 4]}$$

and the closed ray paths contributing to the reverberation intensity at time T are those with travel time satisfying

$$T - \tau \leq t \leq T \quad . \qquad [\text{Eq. } 5]$$

The corresponding scatterers will be contained in a region R. Let this region be partitioned into numerous subregions ΔR_i in each of which $\eta_d \; \eta_d' \; \eta_w \; \eta_w'$ and k are representative values, and let there be N scatterers per unit region. Then if the reverberation intensity I_{rev} is the sum of the intensities of the individual contributors

$$I_{rev} = I_s \sum_R \eta_d \, \eta_d' \, \eta_w \, \eta_w' \; kN\Delta R_i \qquad [\text{Eq. } 6]$$

In the limit as ΔR_i approaches zero,

$$I_{rev} = I_s \int_R \eta_d \, \eta_d' \, \eta_w \, \eta_w' \; m dR \quad , \qquad [\text{Eq. } 7]$$

where

$$m = kN \qquad [\text{Eq. } 8]$$

is the backscattering strength.

NUMERICAL INTEGRATION OF THE REVERBERATION INTEGRALS

When the scatterers are confined to the ocean volume, the corresponding reverberation is called volume reverberation and Eq. 7 becomes

$$I_{rev} = I_s \int_{\Delta\Phi} \int_{A(\Phi)} \eta_d \, \eta_d' \, \eta_w \, \eta_w' \; mrdAd\Phi \quad , \qquad [\text{Eq. } 9]$$

where

- r is the horizontal range,
- $\Delta\Phi$ is the change in azimuthal angle, and
- $\Delta(\Phi)$ is the intersection of the insonified region R with the half-plane Φ = constant.

In order to accomplish this integration numerically, the ocean is partitioned by vertical half-planes $\Phi = \Phi_k$ and each vertical plane is further partitioned by a range-depth grid. The eigenrays to each point in the grid are computed and combined according to the summation

$$I_{rev} = I_s \sum_k \sum_{i,j} (\eta_d \eta'_d \eta_w \eta'_w m) r_i A_{ij} \Delta\Phi_k \qquad \text{[Eq. 10]}$$

where A_{ij} is an area insonified around the (r_i, z_j)-th grid point.

A similar analysis results in the boundary reverberation summation

$$I_{rev} = I_s \sum_k \sum_i (\eta_d \eta'_d \eta_w \eta'_w m) \frac{r_i \Delta r_i}{\cos \theta_{bot}} \Delta\Phi_k \qquad \text{[Eq. 11]}$$

At present, analytical bottom and surface backscattering equations developed by Mackenzie [Ref. 4]

$$10 \log_{10} \mu = -27 + 10 \log (|\sin \theta_1 \sin \theta_2|) \qquad \text{[Eq. 12]}$$

and Chapman-Harris [Ref. 5],

$$10 \log_{10} \mu = 3.3 \beta \log \frac{\theta}{30} - 42.4 \log_{10} \beta + 2.6 \qquad \text{[Eq. 13]}$$

where

$$\beta = 158 (vf^{1/3})^{-0.58} \quad , \qquad \text{[Eq. 14]}$$

v is the wind speed, and f is the frequency, respectively have been implemented in the computer program. The volume backscattering strength is found by interpolating in a table of strength versus depth. Figure 2 illustrates a typical backscattering strength-depth curve. The actual unit of strength is more complicated than simply decibels and should be associated with a unit of volume, in this case, cubic yards.

Independent array response models supply the transmitting and receiving responses of the sonar in the form of tables of loss versus inclination angle at a particular azimuthal angle. Then by looping through the reverberation program numerous times, one is able to consider three dimensional beam pattern effects.

The propagation losses and travel times of the eigenrays, which are required in the reverberation calculation, are computed in CONGRATS I using the continuous gradient ray tracing technique to be described shortly.

A CONTINUOUS GRADIENT RAY TRACING TECHNIQUE

The basic assumption of CONGRATS I is that the velocity of sound in the ocean can be adequately approximated by a function of depth only, say $V(z)$. Let us also confine our attention to ray segments that do not intersect ocean boundaries. Then Snell's law

$$V(z) = C_v \frac{dr}{ds} \qquad \text{[Eq. 15]}$$

uniquely determines the coordinates (r,z) of a point on the ray segment as a function of initial position (r_a, z_a), initial direction $\left(\frac{dr}{ds}\Big|_a, \frac{dz}{ds}\Big|_a\right)$, and arc length s. The vertex velocity C_v is constant along the segment and can be expressed in terms of the initial conditions through Snell's law. Travel time t is related to s and V by

$$\frac{ds}{dt} = V(z) \qquad \text{[Eq. 16]}$$

It is well known that a ray passing from an initial depth z_a to a greater depth z_b will undergo the change in range

$$\Delta r = \int_{z_a}^{z_b} \frac{V\,dz}{\sqrt{C_v^2 - V^2}} \qquad \text{[Eq. 17]}$$

and travel time

$$\Delta t = \int_{z_a}^{z_b} \frac{C_v\,dz}{V\sqrt{C_v^2 - V^2}} \qquad \text{[Eq. 18]}$$

providing that C_v is always greater than V. When C_v equals V, the ray is horizontal and is said to vertex. It can also be shown that these integrals are convergent unless the velocity gradient $V'(z)$ vanishes at a vertexing depth.

In practice, V is known only at discrete data points (z_i, V_i), and one must evaluate Eqs. 17 and 18 numerically. Unfortunately, the number of data points is usually insufficient to expect standard numerical integration formulae to give accurate results. An alternative approach is to first fit the data with an interpolating function, and then either integrate Eqs. 17 and 18 in closed form if possible, or generate additional data points to be used in conjunction with standard numerical integration formulae. However, it is desirable to approximate the velocity-depth profile with a function that has a continuous derivative, for discontinuities in the velocity gradient often cause erroneous values of geometrical spreading loss [Ref. 6].

Before describing the particular velocity representation that is used in the CONGRATS programs, it is convenient to clarify the notation. The function $V(z)$ denotes the velocity of sound in the ocean and is defined from the ocean surface to the ocean bottom. V_i is the value of $V(z)$ evaluated at the depth z_i [Fig. 3].

The function $V_i(z)$, on the other hand, is only defined in the interval $z_i \leq z \leq z_{i+1}$. Primes of functions denote differentiation with respect to depth, while G_i is the value of $V'(z)$ evaluated at z_i. In general G_i is not known but can be estimated from the given data V_i using numerical differentiation. The quantities v_0, g_0, g_1 and g_2 are parameters. They are constant in each horizontal layer but may differ from layer to layer. Then a continuous gradient velocity approximation can be constructed as follows:

For each pair of adjacent data points i and i+1, find a four parameter analytic function $V_i(z)$ such that its derivative $V_i'(z)$ is continuous in the closed interval $z_i \leq z \leq z_{i+1}$ and satisfies the boundary conditions

$$V_i(z_i) = V_i \quad , \qquad \text{[Eq. 19]}$$

$$V_i(z_{i+1}) = V_{i+1} \quad , \qquad \text{[Eq. 20]}$$

$$V_i'(z_i) = G_i \quad , \qquad \text{[Eq. 21]}$$

and

$$V_i'(z_{i+1}) = G_{i+1} \quad . \qquad \text{[Eq. 22]}$$

Finally, set $V(z) = V_i(z)$ in the half-closed interval $z_i \leq z < z_{i+1}$.

It can be shown that parameters v_0, g_0 g_1 and g_2 can be chosen so that the function

$$V_i(z) = \left[v_0 + \Delta z \frac{g_0 + \Delta z\, g_1}{(1 + \Delta z\, g_2)^2} \right]^{-\frac{1}{2}} \quad . \qquad \text{[Eq. 23]}$$

where

$$\Delta z = z - z_i \qquad \text{[Eq. 24]}$$

satisfies the above conditions [Ref. 7]. It can also be shown that the corresponding range integral Eq. 17 and time integral Eq. 18 can be evaluated in closed form in terms of elementary transcendental functions.

There are some difficulties involved with this curve fitting method, most of which are due to computer truncation errors. All can be removed by making appropriate modifications. The method has been used extensively in CONGRATS I, and every velocity profile that was considered could be satisfactorily fitted.

A useful option in the program allows one to relax certain boundary conditions. The resulting representation still has a continuous gradient but is not forced to go through all of the data points. Instead, the condition

$$\max |V(z_i) - V_i| \leq \epsilon \qquad \text{[Eq. 25]}$$

must be satisfied, where $\varepsilon \geq 0$ is a velocity tolerance to be supplied by the user. This option tends to reduce the number of functions in the form of Eq. 23 required to fit the given profile. Hence fewer integrals need to be evaluated, and a substantial saving in computer execution time is realized.

CONTINUOUS VERSUS CONSTANT GRADIENT FIT TO AN EPSTEIN PROFILE

A second option in CONGRATS I allows one to substitute the well known constant gradient technique for the continuous gradient technique. The velocity-depth profile is then approximated by straight line segments instead of by functions in the form of Eq. 23, and one can integrate the resulting ray tracing equations more easily. In fact, because of the relative simplicity of the constant gradient technique, it is the most commonly used ray tracing method. The reason we prefer the more complicated continuous gradient technique will become clear after the following example.

Figure 4 shows an Epstein velocity-depth profile which was fitted with five CONGRATS velocity functions. There were fifty original data points as indicated by plusses. The CONGRATS velocity break-points are indicated by circles and by plusses within circles. A 0.1 m/s velocity tolerance was used. That is, the maximum error in the curve fit was less than 0.1 m/s. Since the continuous gradient technique would perform only one-tenth the number of integrations required by the constant gradient technique, and a single integration uses five times more computer execution time, CONGRATS appears to be twice as fast, at least in this particular example.

When the source is placed on the channel axis at 76 m, there is a high degree of focussing as shown in Fig. 5. The CONGRATS ray diagram (on top) and the constant gradient ray diagram (on the bottom) are similar. Note, however, that the constant gradient solution is not focussed as sharply.

The corresponding propagation loss curves for a 60 m target depth are given in Fig. 6. Anomalies in the computed value of geometrical spreading loss which were caused by discontinuities in the velocity gradient, are clearly visible in the constant gradient solution. There are no such anomalies in the CONGRATS solution.

EIGENRAYS

The most important function of CONGRATS I with respect to the reverberation program CONGRATS III is to determine acoustic eigenrays, that is, those rays that join a given source (r_s, z_s) to a given target (r_t, z_t). One method of determining eigenrays which was tried and later discarded involves an iterative scheme:

a. Set $i = 1$ and choose an initial vertex velocity $C_v^{(1)}$.

b. Trace the ray with vertex velocity $C_v^{(i)}$ to the target depth z_t and denote the corresponding range by $r^{(i)}$.

c. If $|r_t - r^{(i)}|$ is sufficiently small, convergence has occurred. If $|r_t - r^{(i)}|$ is not sufficiently small, set

$$C_v^{(i+1)} = C_v^{(i)} + \frac{r_t - r^{(i)}}{\left.\frac{\partial r^{(i)}}{\partial C_v}\right|_{z = z_t}} \qquad \text{[Eq. 26]}$$

increment i, and return to step b.

This iterative scheme has two drawbacks. First of all, convergence is slow near caustics (points at which $\left.\frac{\partial r}{\partial C_v}\right|_{z = z_t}$ vanishes) unless a convergence acceleration technique is used. Secondly, the method is inefficient when many eigenrays are to be determined. For example, assume that there are 1000 targets, that each target has 5 eigenrays, and that each eigenray requires 3 iterations for convergence to occur. Then the total number of rays to be traced (15 000) becomes excessive.

An alternative approach involves a preselected set of rays. When two adjacent rays bound a target, an interpolation is performed to determine the eigenray. If the derivatives $\left.\frac{\partial r}{\partial C_v}\right|_{z = z_t}$ are known, one can use a higher order interpolation routine. Otherwise a linear interpolation can be used instead.

The interpolation technique was incorporated into CONGRATS I and was found to be as accurate as the convergence method. One of the outputs of the program is geometrical spreading loss N_{sp}, where

$$N_{sp} = 60 + 10 \log_{10} \left| \frac{C_v \sin \theta_s \sin \theta_t}{\cos^2 \theta_s} r_t \frac{\partial r_t}{\partial C_v} \right|_{z = z_t} \qquad \text{[Eq. 27]}$$

is in decibels, and θ_s and θ_t are the inclination angles at the source and target, respectively. The number 60 appearing in Eq. 27 is present because the unit of range is in kiloyards. The total propagation loss N along a ray is given by

$$N = N_{sp} + N_a + N_s + N_b \quad , \qquad \text{[Eq. 28]}$$

where N_a is the attentuation loss, N_s is the loss incurred at surface reflections, and N_b is the loss incurred at bottom reflections.

If several eigenrays arrive at the same target, the effective propagation loss N_{eff} is given by

$$N_{eff} = -10 \log_{10} \sum_j 10^{-\frac{N^{(j)}}{10}} \qquad \text{[Eq. 29]}$$

(random phase addition) or by

$$N_{eff} = -20 \log_{10} \left| \sum_j 10^{-\frac{N^{(j)}}{20}} e^{i[2\pi f t^{(j)} + \Phi^{(j)}]} \right| , \qquad \text{[Eq.}$$

where $i = \sqrt{-1}$, f is the frequency in hertz, $t^{(j)}$ is the travel time in seconds, $\Phi^{(j)}$ is the phase change in radians of the j^{th} eigenray (coherent addition). Phase changes are caused by interactions of the ray with the ocean surface and bottom, and they also occur when a ray passes through a caustic curve. In the last instance, a phase change of π radians is added although the generally accepted value is $-\pi/2$ radians. The erroneous phase shift of π radians reduces the high intensities predicted by ordinary ray theory. It is used as an artifice to make reasonable predictions until one of the more sophisticated theories treating caustics can be added to the computer program.

EXAMPLES

Figures 7, 8 and 9 illustrate some of the more widely used programs in the CONGRATS series. They pertain to the Mediterranean Sea in the summer. Some of the results have been compared to measured data taken during the summer of 1970 by the Ocean Sciences Division of the Naval Underwater Systems Center. Figure 7 is a plot of sound speed as a function of depth. The warm surface temperatures cause a sharp negative gradient near the surface, which causes the energy from a near surface source to be initially directed downward as shown in Fig. 8.

The ray plot shows both bottom bounce and convergence zone rays. The convergence zone is defined by the caustic line intersecting the surface at a range of about 44 kyds. For the set of data, the predicted zones and measured zones have agreed in range to approximately 200 yards. This is a relative error of about one-half of one percent.

Figure 9 displays the level at a point as a function of time. Since the intensity at a point changes whenever a sound pulse arrives and since the travel times associated with each arrival are generally different, the curves are composed of numerous step functions. These curves were obtained by adding beam pattern information to each arrival, and then adding the resultant signals in random phase. For the relatively large pulse length of one-half second the intensity builds up quickly, remains constant until the first arrival ceases to contribute, and then decays. On the other hand, if the pulse length is sufficiently small such as 10 ms, then the dominant individual arrivals become more apparent. The computer program can also add the signals coherently if the user so desires.

In order to plot propagation loss as a function of range, a single point from the pulse shape plot is chosen as a resultant level. When random phase addition is used, the maximum level is usually chosen.

CONGRATS VALIDATION

For certain simple propagation situations, for example constant or linear sound speed profiles, the ray integrals, the spreading loss formulae, and the reverberation integrals have been evaluated analytically and compared to the results produced by the computer programs. The analytic answers and computer answers have been in excellent agreement. For the numerical integration scheme described for the reverberation calculation, the range-depth grid over which the integral is taken must be sufficiently fine. At any rate, the close agreement indicates that the mathematical equations used to model propagation and reverberation are being evaluated correctly in the computer for the circumstances examined. We are now trying to determine how well the program predicts propagation and reverberation levels measured at sea.

The propagation loss for a 25-foot target depth is shown in Fig. 10. The strengths of the various arrivals were summed using random phase addition, as shown by the dashed line, and coherent phase addition, as shown by the solid line. The predictions generally agree well with the measured data. Here the peak level is about 80 dB.

Figure 11 is similar to Fig. 10 except the target depth is now at 503 ft. Note that the maximum level has dropped about 10 dB which is also in agreement with measured data.

Figure 12 shows reverberation level as a function of time, as predicted by the CONGRATS programs. Again the predictions were good.

In addition to the in-house validation effort, a joint programme between NUSC and SACLANTCEN here at La Spezia has been developed. Two joint reports will be published, one presenting propagation loss measurements and predictions; the other comparing reverberation levels. A preliminary investigation has shown very good agreement in the propagation loss area, and good agreement in reverberation.

SUMMARY

To summarize, CONGRATS I is the fundamental ray tracing program of the CONGRATS series. It uses a continuous gradient ray tracing technique in order to reduce the problem of false caustics, a problem which often occurs when discontinuities in the velocity gradient are introduced. Eigenrays are found by interpolation rather than by iteration in order to reduce the running time of the program. Options allow the user to reduce the number of horizontal layers into which the ocean is divided, to substitute the constant gradient ray tracing technique for the continuous one, to print and plot ray data, and to use various attentuation, surface loss, and bottom loss models. Although the examples discussed pertained to oceans in which the boundaries are assumed to be horizontal, one can also input a linear segmented surface and bottom.

The propagation loss curves and pulse shape curves were supplied by CONGRATS II. There are several other programs in this series, but most are of a specific nature and would not be of general interest. The user may choose between random phase and coherent phase additions to obtain resultant intensities.

CONGRATS III computes bottom, surface, and volume reverberation as a function of time. Future plans include the addition of a program to compute echo-to-reverberation level as a function of time, and attempts to increase the efficiency of the existing reverberation program.

CONGRATS IV when completed, will allow the sound speed to vary with range as well as depth.

The existing programs have been implemented on UNIVAC 1108 computers in New London, in California, in Rome, and at other installations. It has run on General Electric and Burrough's computers also. There is an effort presently underway to program a modified version of CONGRATS on a shipboard computer. Thus CONGRATS has become a useful tool in the field of underwater acoustics.

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DISCUSSION

Typical times to run the CONGRATS system were given as one to two minutes for a range of 100 kyd and a flat bottom, and 10 to 15 minutes for a volume reverberation calculation, both on a UNIVAC 1108.

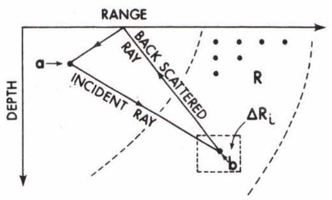


FIG. 1

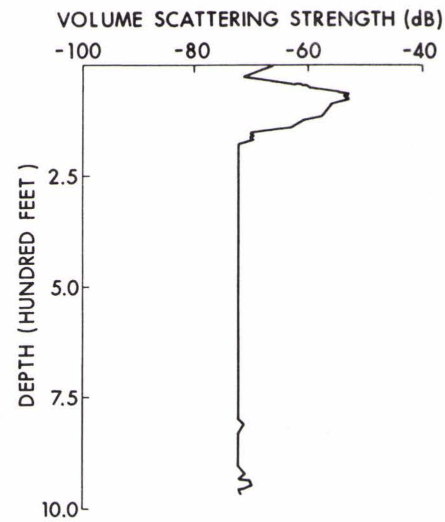


FIG. 2

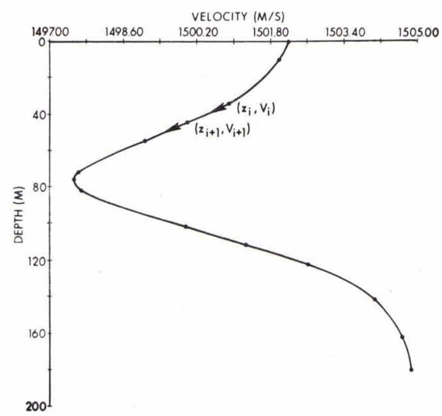


FIG. 3

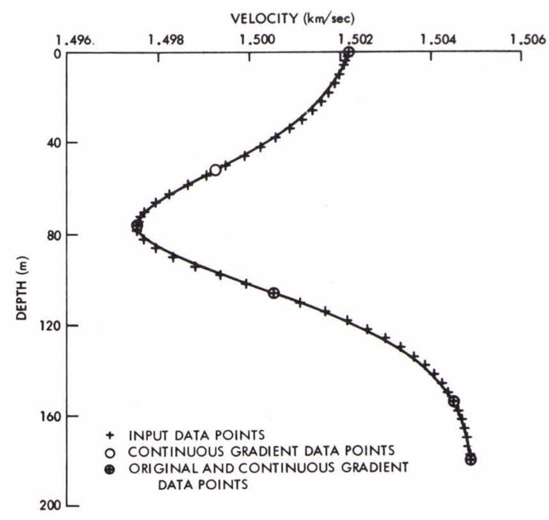
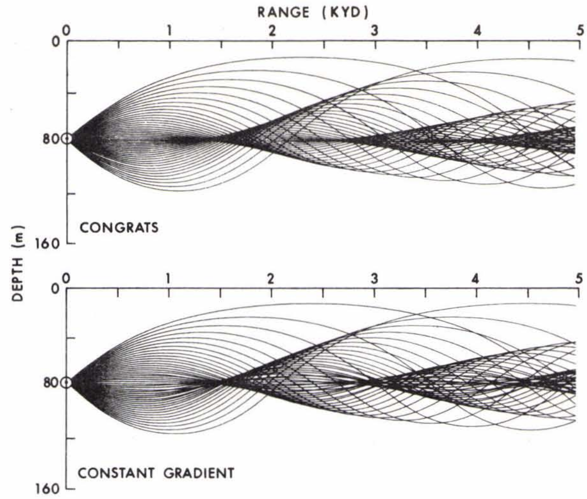


FIG. 4



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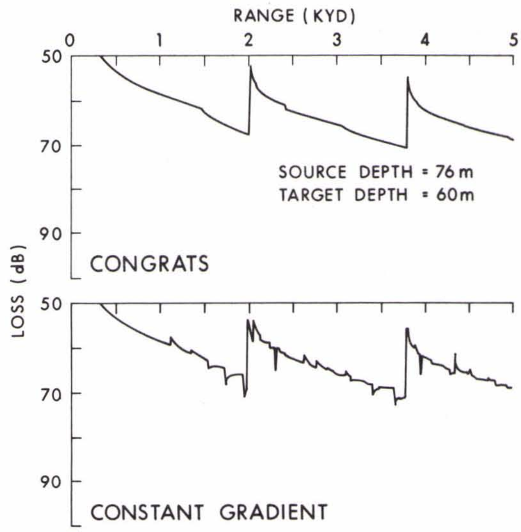

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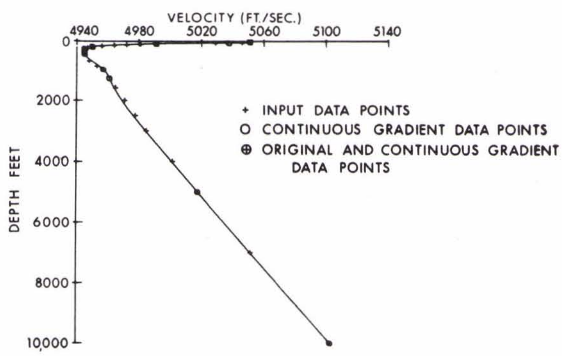

FIG. 7

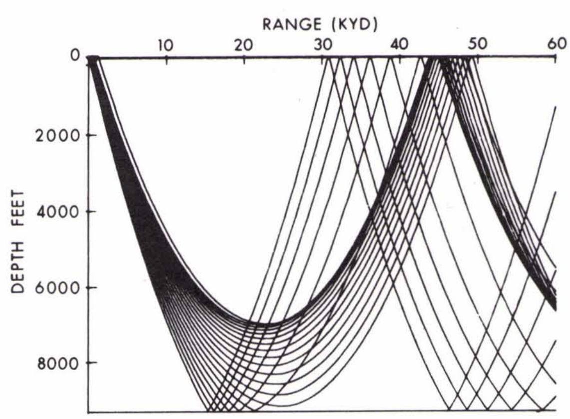

FIG. 8

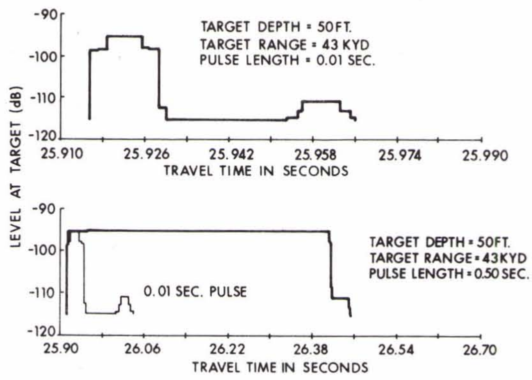

FIG. 9

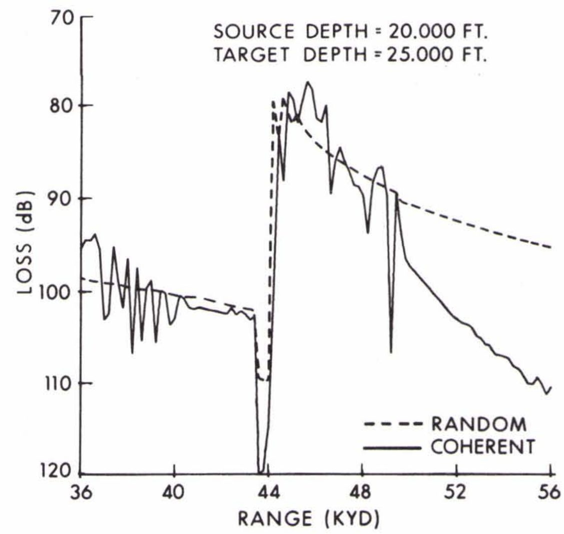

FIG. 10

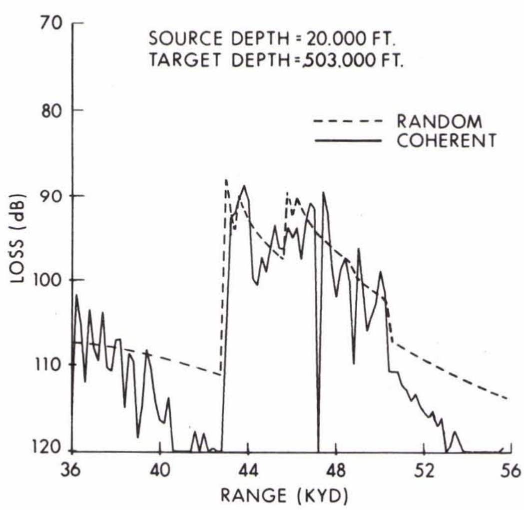

FIG. 11

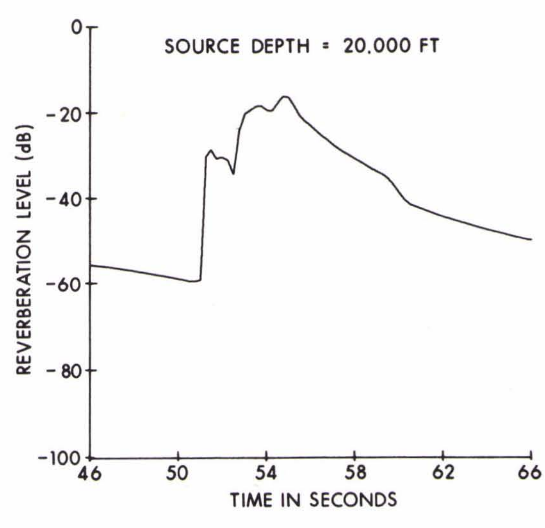

FIG. 12