A Novel Method to Enlarge the Scanning Region of a Focused Beamformer

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Abstract

Often focused beamforming systems use the Fresnel approximation to compute the required delays. Unfortunately, such an approximation has a validity region that is very narrow around the broadside direction. To investigate wider areas, in this paper, a novel approximation composed of the weighted terms of the conventional Fresnel approximation is presented. The attractive results (in terms of mean square error and enlargement of the validity region) obtained by applying the proposed approximation to a planar array are presented.

1. Introduction

Beamforming is a linear technique aimed at processing array signals in order to enhance incoming signals from a selected steering direction and to abate incoming signals from any other direction [1]. Thanks to its flexibility, beamforming can be successfully employed in many application fields (e.g., sonar, radar, medical imaging, non-destructive testing, etc.) with different objectives. In any case, if the array works under far-field conditions, the delays required by the beamforming operation can be easily computed in an exact way, whereas, if the array works under near-field conditions, the focalization of beamforming is required to take into account the curvature of waves [1]. In the latter case, due to the presence of a square root operation, a fast computation of the exact delays is often prohibitive and an approximate version is preferred: generally, the Fresnel approximation (obtained by the expansion of the time-independent free-space Green's function [2][3]) is adopted [1]. Moreover, the Fresnel approximation makes it possible to apply the Fast Fourier Transform (FFT) in the implementation of beamforming even when focalization is necessary [4], thus resulting in a great computational profit.

These issues are particularly important in three-dimensional (3D) sonar imaging systems, where: the focalization is required, the enormous number of different delays to be used disallows an off-line computation of them, and the FFT implementation of the beamforming is highly demanded. In this case, the Fresnel approximation allows one to compute on-line the delays and to apply the FFT implementation, thus achieving a 3D real-time imaging [4]. Despite its simplicity and advantages, the Fresnel approximation has a well defined region of validity [2] that forces potential steering directions to be contained inside a narrow scanning region around the broadside direction. This constraint is heavy in applications like acoustic imaging that require a wide region of view, for both medical and underwater investigations. To avoid this drawback, some imaging techniques that do not need the Fresnel approximation have been devised [5][6] but, unfortunately, they increase the computational load and/or the system complexity.

This paper presents a novel approximation (not too different from the Fresnel one) based on the minimization of the mean square error, so resulting in an acceptable precision inside wide scanning regions. In more detail, the same terms as used for the Fresnel expansion are weighted by coefficients whose values are fixed, given the array geometry, by a least-squares procedure on the basis of the desired scanning region. Thus, the new approximation keeps the low computational load and the low system complexity that characterize the Fresnel expansion and maintains the opportunity to be implemented in an FFT focused beamformer, too.

This paper is organized as follows. Section 2 presents a background concerning the delay approximation and the related validity region. In Section 3, the weighting of the Fresnel approximation is presented and its application to a planar array is described in Section 4. Finally, results are discussed and some conclusions are drawn in Section 5.

2. Background on Delay Approximation

A beam signal (generated by a conventional delay and sum beamformer [1]) steered in the direction of the unit vector (versor) **u** is defined as:

$$b(\mathbf{u},t) = \sum_{i=1}^{M} w(i) \cdot x_i\left(t - \tau(\mathbf{u},i)\right) \tag{1}$$

where $x_i(t)$ is the temporal signal received by the i-th sensor, $\tau(\mathbf{u}, i)$ and $w(i)$ are the delay and the weight applied to such a signal, respectively, and M is the number of array elements.

Under the far-field hypothesis, the exact delay $\tau(\mathbf{u}, i)$ can be written as:

$$\tau_{ff}(\mathbf{u},i) = \frac{\left(\mathbf{u}\,\mathbf{v}_i^+\right)}{c} \tag{2}$$

where $\mathbf{v}_i = [x_i, y_i, z_i]$ is the position vector of the i-th sensor, $^+$ indicates the transposition operator (both **u** and $\mathbf{v}_i$ are row vectors), and c is the carrier speed. This delay formulation is compatible with the FFT implementation of beamforming.

If focalization is necessary, as the far-field hypothesis does not hold any more, then the exact delay is:

$$\tau_{ex}(\mathbf{u},i) = \frac{R - \sqrt{R^2 + \|\mathbf{v}_i\|^2 - 2R\,\mathbf{u}\,\mathbf{v}_i^+}}{c} \tag{3}$$

where R is the focalization distance in the steering direction **u** and $\|\cdot\|^2$ is the Euclidean norm. These delays are heavy to compute on line (mainly due to the presence of a square root) and inhibit the implementation of focused beamforming by the FFT.

For these reasons, one tries to approximate the exact delay by a formulation that allows an easy on line computation and the FFT implementation. This target is generally achieved thanks to the Fresnel expansion, which results in the following approximation [1] to the exact delay:

$$\tau_{Fr}(\mathbf{u},i) = \frac{\left(\mathbf{u}\,\mathbf{v}_i^+\right)}{c} - \frac{\|\mathbf{v}_i\|^2}{2Rc} \approx \tau_{ex}(\mathbf{u},i) \tag{4}$$

The first term of the addition depends on both $\mathbf{v}_i$ and **u**, and represents the distance-independent delay equal to that defined in (2). The second term takes into account the wave curvature, depends on $\mathbf{v}_i$ and does not depend on **u**. Thanks to the latter fact, one can show that the FFT implementation of focused beamforming is feasible [4].

According to Ziomek [2], there are three necessary conditions that define the validity region of the Fresnel approximation. The first imposes a small steering sector, the second condition stipulates the minimum focalization distance, and the third establishes the boundary between near-field and far-field regions:

$$72° \le \phi \le 108° \tag{5}$$

$$R > 1.356V \tag{6}$$

$$R < \pi V^2/\lambda \tag{7}$$

where ϕ is the angle between **u** and $\mathbf{v}_i$, V is the maximum value of $\|\mathbf{v}_i\|$, and λ is the wavelength of the carrier. These restrictive conditions (in particular, the one that limits the angular extension) are often not satisfied in imaging systems, with potential low performances in the lateral regions of images.

3. Weighting the Fresnel Approximation

In order to relax the constraint of (5), a possible solution is to weight the terms of the Fresnel approximation by two constants, k_1 and k_2, computed on the basis of the desired steering region and of a fixed focalization distance. The importance of keeping unchanged the two terms of the Fresnel approximation lies in their computational simplicity and in the possibility of implementing focused beamforming by the FFT. One can write the novel delay approximation as follows:

$$\tau_{ls}(\mathbf{u},i) = k_1 \frac{\left(\mathbf{u}\,\mathbf{v}_i^+\right)}{c} + k_2 \frac{\|\mathbf{v}_i\|^2}{Rc} \tag{8}$$

and try to fix the values of the two constants by minimizing the sum of the square differences between the delays provided by the approximation and the exact delays. Square errors can be measured over a two-dimensional grid containing all the possible pairs ($\mathbf{u}$, i). Denoting by $e(\mathbf{u}, i)$ the error between the approximate and exact delays, and by $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_N$ the steering directions of interest, one can write an overdetermined system of equations by using a matrix formulation:

$$\begin{bmatrix} \tau_{ex}(\mathbf{u}_1,1) \\ \mathrm{M} \\ \tau_{ex}(\mathbf{u}_1,M) \\ \tau_{ex}(\mathbf{u}_2,1) \\ \mathrm{M} \\ \tau_{ex}(\mathbf{u}_2,M) \\ \mathrm{M} \\ \mathrm{M} \\ \mathrm{M} \\ \tau_{ex}(\mathbf{u}_N,M) \end{bmatrix} = \begin{bmatrix} (\mathbf{u}_1\,\mathbf{v}_1^+)/c & \|\mathbf{v}_1\|^2/(Rc) \\ \mathrm{M} & \mathrm{M} \\ (\mathbf{u}_1\,\mathbf{v}_M^+)/c & \|\mathbf{v}_M\|^2/(Rc) \\ (\mathbf{u}_2\,\mathbf{v}_1^+)/c & \|\mathbf{v}_1\|^2/(Rc) \\ \mathrm{M} & \mathrm{M} \\ (\mathbf{u}_2\,\mathbf{v}_M^+)/c & \|\mathbf{v}_M\|^2/(Rc) \\ \mathrm{M} & \mathrm{M} \\ \mathrm{M} & \mathrm{M} \\ \mathrm{M} & \mathrm{M} \\ (\mathbf{u}_N\,\mathbf{v}_M^+)/c & \|\mathbf{v}_M\|^2/(Rc) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} + \begin{bmatrix} e(\mathbf{u}_1,1) \\ \mathrm{M} \\ e(\mathbf{u}_1,M) \\ e(\mathbf{u}_2,1) \\ \mathrm{M} \\ e(\mathbf{u}_2,M) \\ \mathrm{M} \\ \mathrm{M} \\ \mathrm{M} \\ e(\mathbf{u}_N,M) \end{bmatrix} \qquad (9)$$

The system in (9) can be written in a shortened form as:

$$\mathbf{d} = \mathbf{A}\mathbf{k} + \mathbf{e} \qquad (10)$$

where $\mathbf{d}$ is the column vector (MN by 1) of the exact delays, $\mathbf{k}$ is the column vector (2 by 1) of the unknowns, $\mathbf{e}$ is the column vector (MN by 1) of the errors, and $\mathbf{A}$ is the matrix (MN by 2) containing the two terms of the approximation.

By using a least-squares inverse [7], $\mathbf{A}^{ls}$ (2 by MN), of the matrix $\mathbf{A}$, one can compute a system solution $\mathbf{k}^*$ that minimizes the mean square error:

$$\min_{\mathbf{k}}\left\{\|\mathbf{e}\|^2\right\} \qquad (11)$$

as follows:

$$\mathbf{k}^* = \left(\mathbf{A}^{ls}\right)\mathbf{d} = \left[\left(\mathbf{A}^+\mathbf{A}\right)^{-1}\mathbf{A}^+\right]\mathbf{d} \qquad (12)$$

Once the solution, $\mathbf{k}^*$, has been computed off-line, one can begin the focused beamforming operation under the guarantee that the approximate delays τ_{ls} are optimum in the least-squares sense. One can verify that, if one fixes a steering region perfectly overlapped with the validity region of the Fresnel approximation, the differences between the least-squares and Fresnel approximations are negligible, i.e., $k_1 \approx 1$ and $k_2 \approx -0.5$.

4. Results for a Planar Array

The effectiveness of the proposed method has been assessed in the following cases: equispaced linear array (as described in [8]) and equispaced square array. In the latter case, the steering faculty is extended to the 3D space, as shown in Fig. 1. Therefore, supposing the array to be placed on the plane $z = 0$ and according to Fig. 1, the steering versor $\mathbf{u}_{\alpha\beta}$ can be defined as a function of the angles α and β; the vector $\mathbf{v}_i$ and the least-squares delay τ_{ls} can be written as follows:

$$\mathbf{u}_{\alpha\beta} = [\sin\alpha, \sin\beta, \cos\alpha\cos\beta] \qquad (13)$$

$$\mathbf{v}_i = [x_i, y_i, 0] \qquad (14)$$

$$\tau_{ls}(\mathbf{u}_{\alpha\beta}, i) = k_1 \frac{(x_i\sin\alpha + y_i\sin\beta)}{c} + k_2\frac{x_i^2 + y_i^2}{Rc} \qquad (15)$$

where i is an integer included between 1 and M. One can define an angle θ as the angle between the steering versor and the z axis (see Fig. 1), as a consequence, the relation with α and β is: $\theta = \arccos[\cos(\alpha)\cos(\beta)]$.

To test the accuracy of the delay approximation, one can use the total mean square error (i.e., $MSE = \|\mathbf{e}\|^2/MN$) plotted versus the focalization distance, the MSE as a function of the steering direction (computed at a fixed focalization distance):

$$MSE(\alpha,\beta) = \frac{1}{M}\sum_{i=1}^{M} \epsilon(\mathbf{u}_{\alpha\beta}, i)^2 \quad , \tag{16}$$

and the *MSE* as a function of the array element position (computed at a fixed focalization distance):

$$MSE(x_i, y_i) = \frac{1}{N}\sum_{l=1}^{N} e(\mathbf{u}_l, i)^2 \tag{17}$$

where $\mathbf{u}_l$ is one of the N steering directions.

As an example, one can consider an array composed of 11×11 3λ-spaced elements, working at 500 kHz, with a sound speed c = 1500 m/s (i.e., M = 121 and λ = 3 mm). Such an array can be employed in a 3D imaging systems working under wide-band conditions to avoid grating lobes [9]. A collection of 25×25 steering directions with an angular spacing of 2.8° for both α and β has been considered for the computation of (k_1, k_2). From these 625 steering directions, the pairs (α, β) for which the related θ is higher than 30° has been disregarded; as a consequence, the final number of considered steering directions is N = 357. Finally, a range domain 0.25 m ≤ R ≤ 5 m has been required. The angular extension of this scanning region is larger than that of the validity region of the Fresnel approximation (i.e., from (5) one can derive the constraint: |θ| < 18°), whereas the range extension is similar (i.e., the Fresnel approximation is valid for 0.09 m < R < 4.24 m). Figure 2 shows the computed values of k_1 (panel a) and k_2 (panel b) versus the focalization distance R, and the total *MSE* (panel c), measured in μs^2, for both the least-squares approximation τ_{ls}, the Fresnel approximation τ_{Fr}, and the far-field delay τ_{ff}. Moreover, Fig. 3 compares the *MSE*(α,β) of the least-squares approximation with that of the Fresnel approximation, after fixing β = 0°; Fig. 4 shows the same comparison for the *MSE*(x_i, y_i), after fixing y_i = 0.045 m. Both errors were computed after fixing R = 0.5 m and were measured in μs^2.

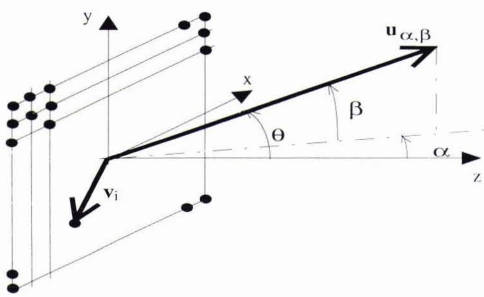

Figure 1: Geometry of a square array and related notation.

5. Discussion and Conclusions

One can notice that, for every delay approximation, the total *MSE* decreases as the focalization distance increases. Within the near-field region, the proposed approximation has a total *MSE* lower than that of the Fresnel approximation. Moreover, the weights k_1 and k_2 tend to assume constant values for a focalization distance larger than few meters, so excluding the strict necessity for computing them for each R (in particular, k_1 tends to be equal to 1, as in the Fresnel approximation).

Concerning the *MSE* versus the steering direction and the array element, one can notice that the proposed approximation has the lowest error for whatever array element, whereas the situation is more complex for the error as a function of the steering angles. In the latter case (see Fig. 3), as β was fixed equal to 0° one can notice that θ = α and that the error of the Fresnel approximation increases with |α| and its value at the border of the validity region (i.e., *MSE*(18°, 0°)) is about 0.006 μs^2 (at R = 0.5 m). Despite the error of the least-squares approximation is not null for α = 0°, its value does not exceed 0.006 μs^2 over the domain |α| < 26°. Therefore, as within the validity region of the Fresnel approximation the loss of image quality is negligible, one can deduce that, generally, moving from the Fresnel approximation to the least-squares approximation, the safe scanning region can be enlarged from |θ| < 18° to |θ| < 26°, with a gain factor of about 1.45.

By repeating the same reasoning on the errors computed for $R \neq$ 0.5 m, the same conclusion (about the potential enlargement of the scanning region) will be reached. Moreover, if a change of the desired scanning sector is performed, one can verify that the least-squares approximation provides a sort of compromise between the approximation precision and the extension of the desired scanning region.

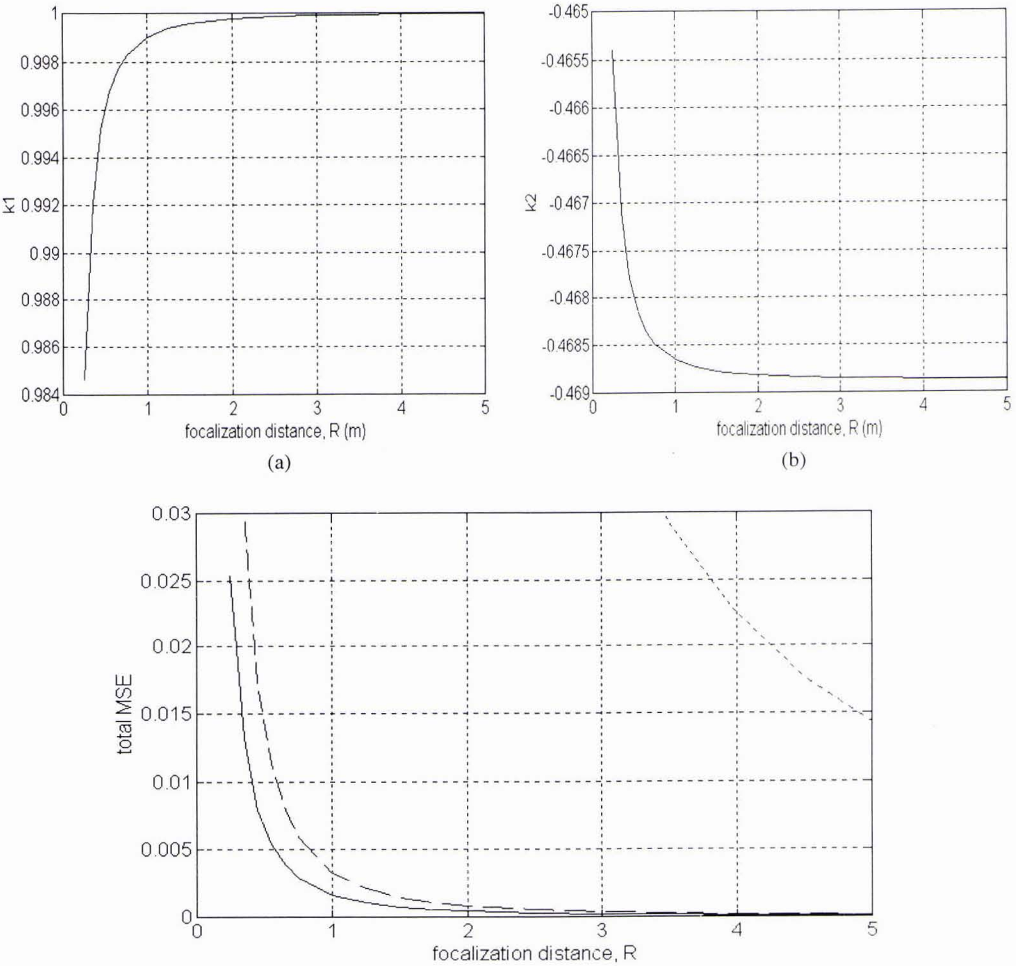

Figure 2: Behaviours of the weights k_1 (a) and k_2 (b) of the least-squares approximation versus the focalization distance R in m. (c) Total MSE in μs^2 versus R for the least-squares approximation (solid line), the Fresnel approximation (dashed line), and the far-field hypothesis (dotted line).

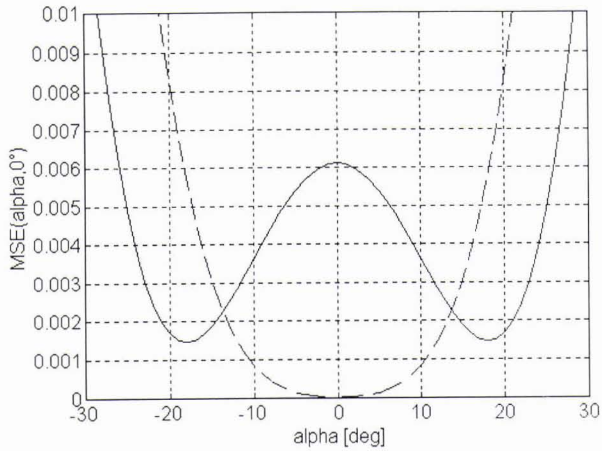

Figure 3: Behaviours of $MSE(\alpha, \beta)$ versus α, after fixing $\beta = 0°$, computed at $R = 0.5$ m and measured in μs^2, for the least-squares approximation (solid line) and the Fresnel approximation (dashed line).

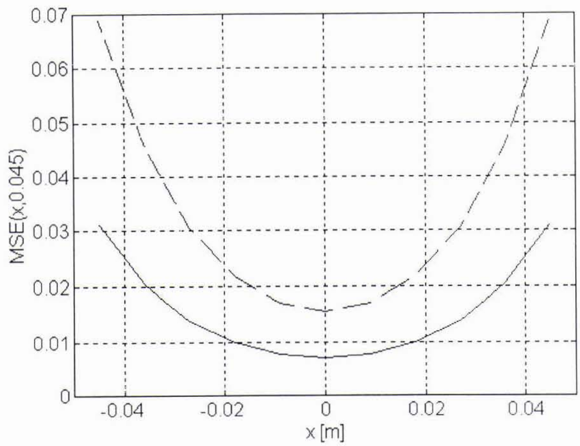

Figure 4: Behaviours of $MSE(x_i, y_i)$ versus x_i, after fixing $y_i = 0.045$ m, computed at $R = 0.5$ m and measured in μs^2, for the least-squares approximation (solid line) and the Fresnel approximation (dashed line).

Finally, concerning the FFT implementation of focused beamforming, one can verify that for a large set of focalization distances, the value of k_1 can be set equal to 1, without a notable loss of precision. Then, the equations that link the spatial frequencies of the FFT to the steering angles can hold their conventional form [4][10]. Instead, if $k_1 \neq 1$, such equations should be slightly updated by an adequate scaling factor.

In conclusion, an approximation for the delays required by focused beamforming has been proposed that minimizes the *MSE* computed over the scanning region of interest. The minimization has been obtained by weighting the terms of the conventional Fresnel approximation through a least-squares solution of an overdetermined equation system. In general, one can observe that, if the desired scanning region is not too extended, the least-squares approximation yields an acceptable precision for many practical operations. At the same time, thanks to the similarity of the proposed approximation to the Fresnel one, the computational load and the system complexity do not notably increase, and the opportunity of implementing focused beamforming via the FFT can be easily kept. The application of such an approximation to a planar array has been analysed and the advantages have been described.

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