A Noise Directionality Model including Variable Bathymetry

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Abstract

In this paper we investigate the noise directionality due to distributed surface sources in several environments with a uniformly sloping sea bed. Of particular interest are frequency dependence and the question of the relevance of multipaths to very high frequencies. We look at both uniform source distributions and swathes of sources lying parallel to the coastline. Some closed form solutions are presented, backed up by numerical solutions using the CANARY range and azimuth dependent noise model.

1. Introduction

Noise directionality is an important consideration for the design of high frequency sonar systems as well as low frequency arrays. At low frequencies one tends to convert the noise directionality into a correlation matrix for a given array in the given noise field. This can subsequently be used in beam forming or adaptive beam forming calculations. At very high frequencies it may be more appropriate simply to point the array physically in the desired direction. In both cases noise directionality is probably as important as absolute noise level.

There are several interesting problems in shallow and coastal waters across the frequency range. One is that of noise production mechanisms such as wind, wave and rain [1, 2]. Another is the effect of multipaths, in particular, bottom reflections from a sloping seabed. Yet another is the effect of uniform distributions of noise sources, such as wind, as opposed to discrete shipping sources. And finally there is the effect of more localised, but still distributed, sources such as waves where the water depth approaches zero.

In this paper we present some calculations of the effects of multipaths with uniform and non-uniform source distributions and a sloping sea bed. Since we are interested in directionality only our results are relative (ie relative to a unit source level per unit area of surface). At frequencies of tens or hundreds of kHz one would usually neglect multipaths because of high bottom losses, amongst other things. Here we investigate the validity of this assumption by looking at the frequency dependence of the directionality when sources may not be immediately overhead. At lower frequencies we already know that in deep water (the absence of a sea bed) we obtain Cron & Sherman's [3] result; in a range-independent environment we find a noise notch which is a simple Snell's law refraction phenomenon for surface sources; and in a range and azimuth dependent environment the noise notch is filled in by downslope multipaths [4].

2. Approach

Our calculations are based on a formulation developed by Harrison for range-independent [5] and for range-dependent environments [6] (in press). Analytical results are possible for a wedge-shaped environment with either uniform sources or swathes of sources parallel to the coastline [7]. We will also present some numerical results from the model CANARY (Coherence of Ambient Noise for Arrays) which uses a similar approach.

If we attempt to calculate from first principles, say, the array response A for a receiver with beam pattern $B(\phi,\theta_r)$ we obtain an integral over the entire sea surface whose integrand is the dipole source strength per unit area $q(\phi,r)\sin^2\theta_s$ times a propagation factor $P(\phi,\theta_r,r)$ times $B(\phi,\theta_r)$,

$$A = \iint q(\phi,r)sin^2\theta_s P(\phi,\theta_r,r)B(\phi,\theta_r)rdrd\phi \tag{1}$$

where ϕ is azimuth, θ_r, θ_s are ray elevation angle at the receiver and surface respectively, and r is range.

Harrison [5] shows that this simplifies as follows. If we imagine the receiver as a source, the ray spreading at the surface because of refraction and distance is what causes the usual weakening of intensity. By reciprocity, sources at the surface provide weakened contributions at the receiver. Simultaneously, however, the number of noise sources in an elementary area goes up (if they are locally uniformly distributed) by the same factor and exactly cancels. Thus the geometric spreading effect disappears even in an arbitrary 3D environment.

For each arrival angle at the receiver θ_r a ray potentially has had many surface hits and we have to add a noise contribution for each one. In a range-independent environment this is relatively simple, and because each ray cycle introduces one extra upper and lower turning point loss and a volume absorption loss we obtain a geometric series which can be solved analytically.

More generally in a range and azimuth dependent environment [6] we find

$$A = \iint q(\phi,r)D(\phi,\theta_r)B(\phi,\theta_r)d\phi cos\theta_r d\theta_r \tag{2}$$

Here D is the noise directionality that we seek; it is a noise power per unit solid angle. It can be separated into two terms U and S.

$$D(\phi,\theta_r) = U(\theta_r)S(\phi,\theta_r) \tag{3}$$

The function U is the residual attenuation due to bottom reflection (power reflection coefficient R_b) and volume absorption a between the last ray upper turning point and the receiver. For the upward ray the partial path length is s_p, and for the downward path it is (s_c-s_p), where s_c is the complete cycle path length. So for a path steep enough to hit the surface, U is given by

$$\begin{aligned} U(\theta_r) &= e^{-as_p} & ;\theta_r \geq 0 \\ &= R_b(\theta_b)e^{-a(s_c-s_p)} & ;\theta_r < 0 \end{aligned} \tag{4}$$

Central to this paper is the function S which represents the contribution arriving along one ray from multiple dipole sources. It is S that forms a geometric series in a range-independent environment with a uniform source distribution, but elsewhere it is clearly more complicated.

$$S(\phi,\theta_r) = \sum_{n=0}^{N} q(\phi,r)sin(\theta_s)_n.exp(-\sum_{j=1}^{n} L_j) \tag{5}$$

with

$$e^{-L_j} = \Re_j = R_s((\theta_s)_j)R_b((\theta_b)_j)exp(-a(s_c)_j) \tag{6}$$

Here the surface and bottom reflection coefficients R_s and R_b (evaluated at appropriate angles) are treated as symmetrical functions of angle.

2.1 **Analytical results**

The position dependence of the source strength q (ϕ,r) can either be assumed to be a constant (uniform distribution) or can be converted to angle dependence given the bathymetry and a ray invariant [8].

In a wedge-shaped ocean the arrival angle θ_r corresponds to a *fixed* ray angle at a given distance from the apex because the water depth at this distance is fixed. Therefore analytical solutions for the noise directionality $D(\phi,\theta_r)$ are also possible with swathes of sources parallel to the coastline because (5) can still be solved [7].

Harrison (in press) [6] derives solutions for uniform sources in a wedge combined with isovelocity or range-independent downward refraction, sound channel or surface duct. The refraction cases explicitly show the classical filling in of the noise notch by upslope sources.

In the isovelocity case we take the bottom to be a tilted plane of (low) gradient ε_0, so that, adopting a N×2D approach the effective slope at a particular azimuth is given by $\varepsilon(\phi) = \varepsilon_0 \cos\phi$. After each bottom bounce the ray angle is incremented or decremented by 2ε so that θ and j are linearly related. Joint boundary loss is taken to be αsinθ per bounce where α is a constant up to a critical angle θ_c. We can then approximate the sums in (5) as integrals and solve them. Looking downslope from the receiver we have

$$S_{down}(\phi, \theta_r) = \frac{1}{\alpha}(1 - e^{-(\alpha/2|\varepsilon|)(1-\cos\theta_r)}) \tag{7}$$

whereas going upslope from the receiver we have

$$S_{up}(\phi, \theta_r) = \frac{1}{\alpha}(1 - e^{-(\alpha/2|\varepsilon|)(\cos\theta_r - \cos\theta_c)}) \tag{8}$$

where θ_c is a critical angle. It is now clear that neither S_{up} nor S_{down} can be greater than the range-independent equivalent which is 1/α. They can only equal it for low slopes or large losses.

Intensity contours for normalised directionality (αD=αSU) are shown in cartesian (ϕ,θ_r) projection in Fig 1. Parameters are: Bottom slope $\varepsilon_0 = 0.01$, bottom loss $\alpha = 0.23$ (ie 1 dB per radian), critical angle $\theta_c = 30°$. The intensity for downward angles steeper than θ_c at the receiver is obviously zero as indicated by the black area. The highest intensities (white) are seen slightly up-slope of across-slope. Upslope the weakest returns are for steep ray angles; downslope the weakest returns are in the horizontal. The up/down asymmetry is entirely due to bottom loss $R_b = \exp(-\alpha \sin|\theta_r|)$ in U (see (4)) since absorption has been set to zero. At upward elevation angles greater than critical the formula reverts to Cron and Sherman's [3] leaving $D = \sin\theta_s$. For comparison Fig 2 shows the equivalent plot from [6] for a range - independent surface duct in a wedge.

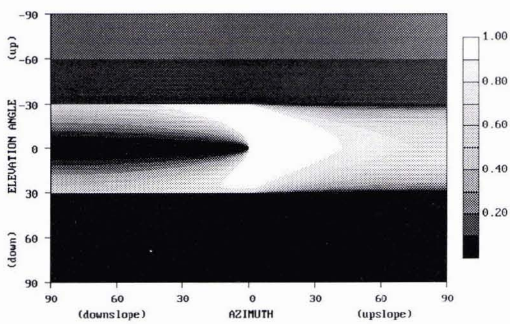

Fig 1 *Analytical noise directionality for an isovelocity wedge using (7) and (8). Intensity is linear.*

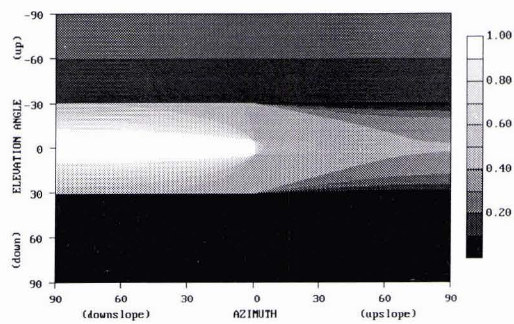

Fig 2 *Analytical noise directionality for a surface duct (formulae in [6]). Intensity is linear.*

2.2 Numerical results

Behaviour of boundary losses and volume loss is clearly crucial. At higher frequencies we need to take better account of the detailed variation of these quantities, so now we turn to numerical methods. In the model CANARY we still have to solve for $S(\phi,\theta_r)$, (5), and again we take a N×2D approach. The gist of this numerical method is given in [9] and [10] (in press).

Because each term in the first sum of (5) represents the contribution from the nth surface hit multiplied by the cumulative losses up to that point it is possible to calculate both sums efficiently in a single loop. We simply trace the ray (at ϕ,θ_r) backwards, simultaneously multiplying the current cumulative loss factor by the RHS of (6) for the latest cycle, and adding q sin θ_s times the current cumulative loss factor as in (5) for each surface hit.

Note that a coarse spread in ray angles will suffice since we do not need to bother with geometric spreading. Some comparisons between CANARY and the analytical cases of [6] are given in [11].

In the following examples we have used Thorp [12] for volume loss a (dB/km)

$$a = f^2\left[3.01\ 10^{-4} + \frac{0.109}{(1+f^2)} + \frac{43.7}{(4100+f^2)}\right] \tag{9}$$

with frequency f in kHz; Marsh, Schulkin and Kneale [13] for surface loss

$$R_s(\theta_s) = exp\left[-5.57\ 10^{-4}\, f^{3/2} w^4 sin\theta_s\right] \tag{10}$$

with wind speed w in m/s. To represent bottom loss frequency dependence we have invented a formula loosely based on a graph of Marsh's shown in Urick's book [14]. This gives the linear rise to a plateau in angle combined with an increase in frequency.

$$R_b(\theta_b) = exp\left[-1.35(1 + log_{10}f + (log_{10}f)^2)tanh(2.86\theta_b)\right] \tag{11}$$

For frequencies below 1kHz we drop the $(log_{10}f)^2$ term.

2.2.1 *Frequency dependence in a wedge with uniform source distribution*

Figures 3-5 illustrate the effect of the increasing losses on the directionality at frequencies of 0.25, 1, 4kHz in an isovelocity wedge of slope 0.01 assuming a wind speed of 5m/s. Under these conditions we can safely ignore multipaths above 4kHz.

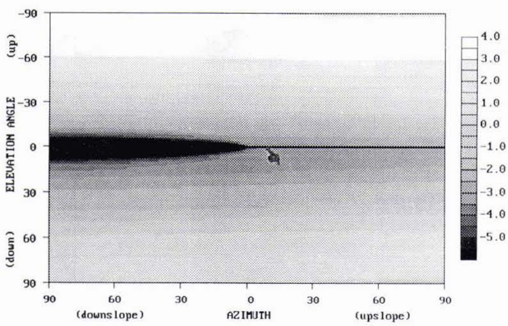

Fig 3 *Numerical noise directionality (dB) for isovelocity with uniform sources at 250Hz*

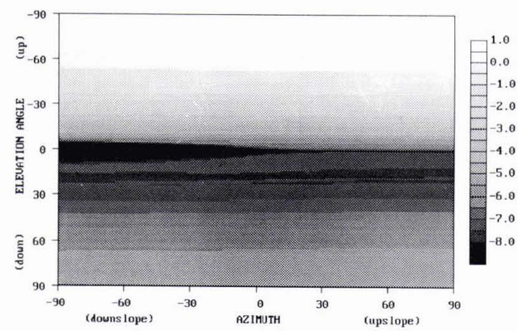

Fig 4 *Numerical noise directionality (dB) for isovelocity with uniform sources at 1kHz*

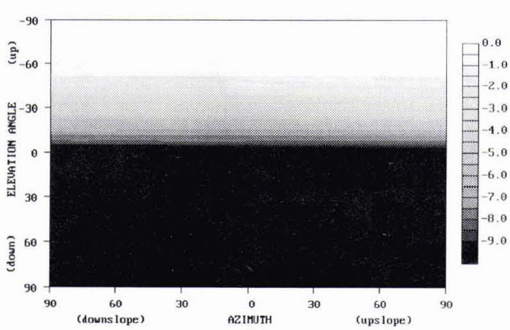

Fig 5 *Numerical noise directionality (dB) for isovelocity with uniform sources at 4kHz*

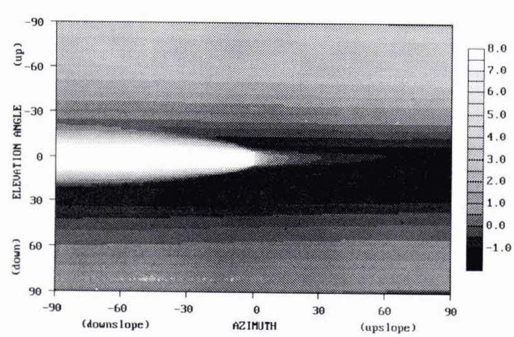

Fig 6 *Numerical noise directionality (dB) for surface duct with uniform sources at 250Hz*

If there is upward refraction with the receiver relatively near the surface then there is a better chance of multipaths without much bottom interaction. In Figs 6-8 (and Figs 3-5) we have the receiver at depth 10m in 100m of water. Here there is potential for strong multipath arrivals at low angles provided wind speeds are low.

Although all figures for uniform source distribution display directionality the direct upward path dominates at high frequencies and there is no azimuth dependence. There is a residual first bottom reflection in the downward direction. As frequency lowers there is a near horizontal contribution from out to sea in the surface duct but a corresponding low in the isovelocity case.

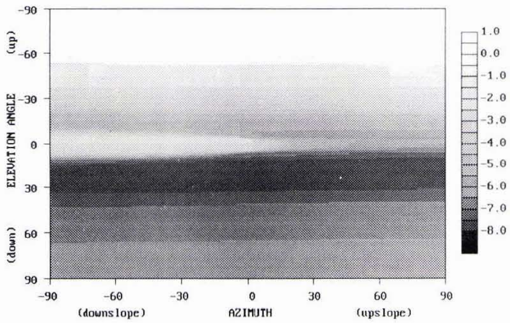

Fig 7 *Numerical noise directionality (dB) for surface duct with uniform sources at 1kHz*

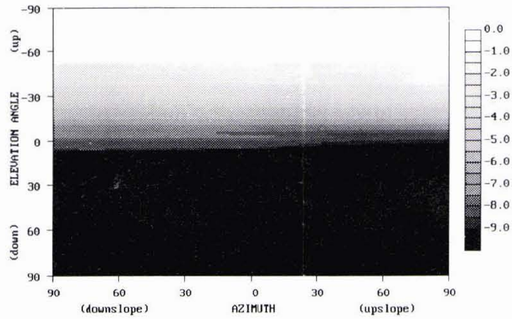

Fig 8 *Numerical noise directionality (dB) for surface duct with uniform sources at 4kHz*

2.2.2 Receiver depth dependence

In an upward or downward refracting duct there is a transition from surface/bottom reflected to waterborne paths some way up the slope depending on final arrival angle and receiver depth. This may have a noticeable effect on the noise directionality.

In a downward refracting duct there would be a 'noise notch' (an angle range of low noise) if the bottom were flat. The slope fills this on the upslope side, and the width of the notch on the downslope side depends on the receiver depth since it affects the velocity contrast for the limiting ray. The right hand side of Fig 9 shows the filled noise notch at 250Hz for a receiver at mid-depth. Doubling the velocity contrast in the sound speed profile or deepening the receiver opens up the noise notch as in Fig 10. Fig 11 shows (by changing the plotting contrast) that the effect is still there at 1kHz and the bottom paths dominate these angles although the noise is weak compared with that for overhead.

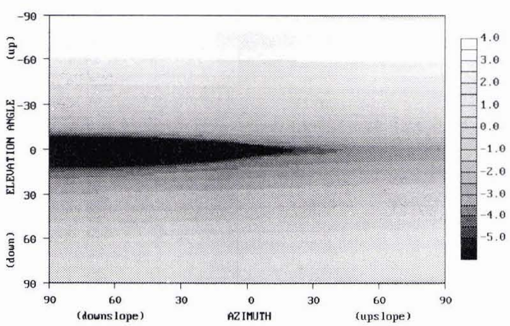

Fig 9 *Weak noise notch for mid-depth receiver at 250Hz for downward refraction with uniform sources*

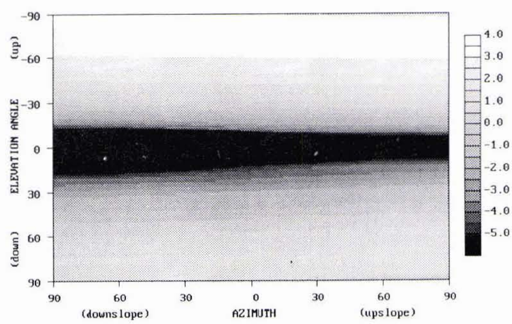

Fig 10 *Strong noise notch for mid-depth receiver at 250Hz for downward refraction with uniform sources*

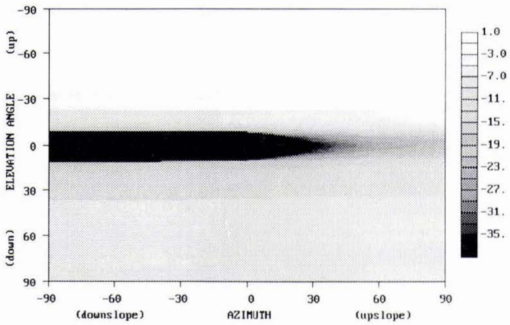

Fig 11 *Strong noise notch for mid-depth receiver at 1kHz for downward refraction with uniform sources (highlighting the lower contour levels)*

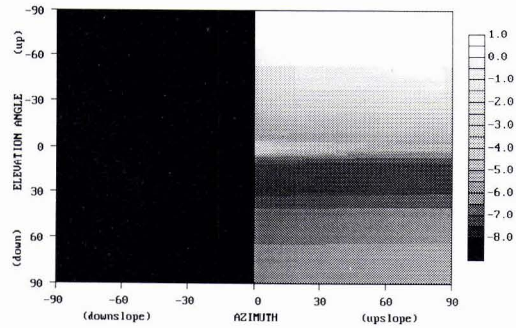

Fig 12 *Numerical noise directionality (dB) for surface duct at 1kHz with a swathe of sources; δ=1*

2.2.3 Dependence on source distribution

If we are considering wind noise then a uniform distribution seems a reasonable assumption. On the other hand wave noise sources could be assumed to be overhead or to be distributed along a distant beach. We now make the assumption that sources extend in a swathe parallel to the shore out to a fraction δ of the distance from the shore to the receiver. Note that once there are no noise sources (of this type) overhead multipaths to distant sources definitely could be important even at high frequencies because there are no other paths.

Figs 12-14 show the noise directionality at 1kHz for δ=1/3, 2/3, 1 for the upward refracting case. The most obvious effect is that as the swathe becomes narrower the angle range of the noise becomes more and more confined to a narrow range near the horizontal although the transition happens quite near δ=1. Once the swathe edge reaches the receiver position the directionality is close to that of a uniform distribution (see Fig 7) although of course, there is only noise from azimuths between upslope and across slope. Interestingly there is little azimuth dependence in Figs 13 and 14.

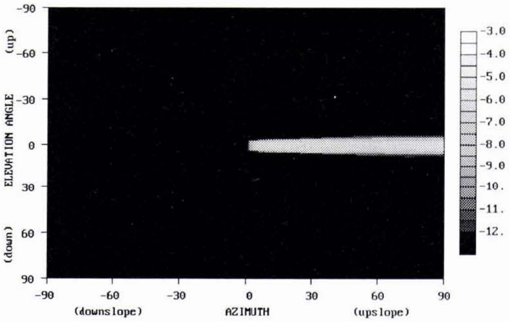

Fig 13 *Numerical noise directionality (dB) for surface duct at 1kHz with a swathe of sources; δ=2/3*

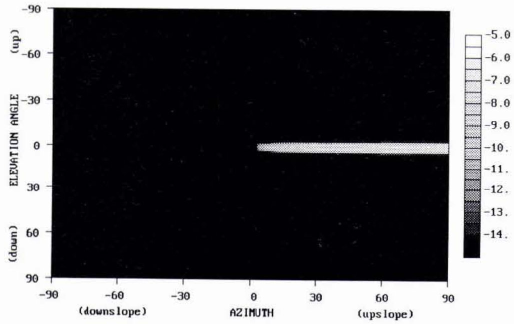

Fig 14 *Numerical noise directionality (dB) for surface duct at 1kHz with a swathe of sources; δ=1/3*

3. Conclusions

By treating surface noise sources as an extended sheet it is possible to derive closed form analytical and numerical results for various conditions in range-dependent environments. Full derivations of the analytical cases are given in [6], and examples are shown here in Figs 1 and 2.

Noise directionality was investigated numerically as a function of frequency, receiver depth and source swathe width for swathes parallel to the coastline. Naturally as frequency rises boundary losses become higher, and multipaths become less significant. If there is a maximum in the sound speed profile above the receiver and with a higher sound speed there will be a vertical noise notch in the downslope direction. Upslope the same angle range will be filled, even at high frequencies, with multipath arrivals. Similarly, if the noise sources only exist near the coastline then even at high frequencies there are significant multipaths, and angles are confined to those near the horizontal.

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