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A MECHANICAL MODEL FOR ESTIMATING THE MOTION OF LOW ALTITUDE SATELLITES WITH RESPECT TO THE EARTH'S SURFACE

by

G. R. LINDSEY and A. BERTI

1 AUGUST 1965

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APPROVED FOR DISTRIBUTION

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TABLE OF CONTENTS

A MECHANICAL MODEL FOR ESTIMATING THE MOTION OF LOW ALTITUDE SATELLITES WITH RESPECT TO THE EARTH'S SURFACE

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ABSTRACT

The principles and design of a mechanical model used for demonstrating or approximately estimating the motion of low altitude satellites with respect to the earth's surface is described.

INTRODUCTION

After the propulsion of the launching rocket has ceased, the motion of a satellite orbiting the earth is closely approximated by an ellipse, with one focus at the centre of the earth. For a mathematical description of the motion, it is most convenient to employ a coordinate system whose origin moves with the centre of the earth, but which does not rotate with the earth. Refinements can be introduced to allow for the effects of the oblateness of the earth (Ref. 1), atmosphere drag (Refs. 2 & 3) and perturbation caused by the sun and moon (Ref. 4).

In many cases it is important to relatethe motion of the satellite to certain points on the surface of the earth, rather than to its centre. Examples would be the flight of an astronaut making several circuits of the earth, the use of satellites to relay wireless messages commencing and finishing on the surface of the earth, or the employment of photographic satellites to observe the surface for meteorological or military purposes.

It is always possible to derive a mathematical solution to this problem. However, a good representation of the motion of low altitude satellites can be obtained with a mechanical model. Although less accurate than a mathematical solution, a mechanical model is more useful for obtaining a mental comprehension of the motion.

Such a model has been constructed to serve the dual purpose of giving instructional demonstrations and of making approximate calculations.

1. DYNAMICAL PRINCIPLES

The model was designed so that its stationary parts represent an inertial frame of reference. The earth is represented by a geographical globe, rotating about its axis in the west-to-east direction. Time is represented by the rotation of the globe, one revolution corresponding to a sidereal day (23 hr, 56 min, 4 sec of mean solar time) rather than a solar day, since the rotation is being referred to inertial space. The position of the satellite is represented by a small indicator moving on a circle. The plane of this circle can be adjusted to make any angle between 0° and 90° with respect to the plane of the earth's equator. This angle (i) represents the "inclination" of the satellite's orbit. A gear linkage with an adjustable ratio relates the rotation of the satellite indicator to that of the globe.

The indicator marks the "sub-satellite point", i.e. the point on the earth's surface directly beneath the satellite. Since the gear ratio does not change during one orbit, the motion of the indicator represents a constant angular velocity. In fact, unless the orbit of a real satellite is circular, the angular velocity varies throughout the orbit by an amount that depends on the eccentricity of the orbit. However, if the apogee (the point in the orbit farthest from the earth) is less than 2000 km above the earth, the eccentricity cannot be greater than 0.14.

Figure 1 illustrates three orbits, each with perigee at h = 274 km above the earth, a height at which the atmosphere density is low enough to permit a life of at least several months before the orbit collapses due to air drag. For the circular orbit (with apogee also 274 km above the earth) the period T (i.e. time to complete one orbit) is 90 min, the eccentricity is 0, and the angular velocity is constant. For the other two orbits (with apogee at heights of 967 and 1752 km) the periods are 97.2 and 105.4 min, and the eccentricities are 0.05 and 0.10.

Figure 2 shows the difference between η, the true anomaly (i.e. the angle swept out by the satellite) and M, the mean anomaly (i.e. the angle swept out by a body precessing at a constant angular velocity and the same T). $(\eta - M)$ is multiplied by T/2, and therefore represents the time difference between the two anomalies. The actual satellite runs ahead of the constant velocity body in the first 180° of the orbit, the two are coincident at perigee and apogee, and the actual satellite lags behind the other for $180^{\circ} < M < 360^{\circ}$.

It is evident from Fig. 2 that for eccentricities less than $e = 0.1$ the use of a constant velocity indicator will never produce errors larger than about 3 min of time. The data on Fig. 1 are calculated from the formulae given by Krafft Ehricke (Ref. 5).

$$r_p = \text{perigee distance} = a\ (1 - e)\ \text{km},$$

$$r_A = \text{apogee distance} = a\ (1 + e)\ \text{km},$$

$$T = 1.659 \times 10^{-4}\ a^{3/2}\ \text{min},$$

$$R = \text{earth's equatorial radius} = 6378\ \text{km},$$

where a is the semi-major axis of the ellipse.

For Fig. 2 the necessary additional formulae are

$$M = E - e \sin E$$

$$t = MT/2\pi\ \text{min}$$

$$\tan \eta/2 = \sqrt{(1 + e)/(1 - e)}\ \tan E/2$$

where E is the eccentric anomaly.

In the model, the plane of rotation of the satellite orbit remains fixed. In reality, the fact that the earth is not truly spherical causes the orbital plane of a low altitude satellite to precess (Ref. 1). For an eastbound satellite, the orbital plane rotates westward at a rate of

$$\Delta \Omega = 9.97 \quad (R/R+h) \cos i \ \text{deg/day.}$$

This rate can be as high as several deg/day, so that appreciable errors could be caused over a period of several days if the precession was ignored. One way to correct the indications of the model is to modify the time scale so that one rotation of the globe represents an elapsed time of

$$360/(360 + \Delta \Omega) \text{ of a sidereal day}$$

2. MECHANICAL DESIGN

To achieve a satisfactory mechanical design it proved necessary to meet the following requirements:

(1) The globe and the satellite should be moved by the same control, which should be actuated by either hand or motor.

(2) The ratio between the rotation velocity of the satellite and the globe should be continuously variable, for satellite periods between a minimum of 80 and a maximum of 130 min. This determined the velocity ratio of the gears and the dimensions of the changing gear.

(3) There should be no lag between the motions of the globe and the satellite.

(4) The inclination of the plane of the satellite orbit had to be adjustable through an arc of 90^{o}.

(5) The direction of rotation of the satellite had to be reversible (in order to simulate retrograde as well as direct orbits).

In the following, brief description of the device, mention is made of those parts for which precise construction was necessary.

The entire mechanism is mounted in a casing of light alloy contained in a wooden box. (See Figs. 3 & 4). Housings for ball bearings for the various transmission shafts are mounted in the metal casing, together with holes for shaft exits, handles for lifting the casing, and the graduated arch around the globe.

The mechanism (Fig. 5) is set in motion by a handle (1) attached to a horizontal main shaft (2) on the right of the casing. This shaft can be turned either clockwise or counterclockwise.

On the main shaft there are the changing gear (3) for passing from hand to motor control, a gear for rotation of a secondary shaft (4) above the main shaft, a cog (5) for driving a revolution counter, and a gear to rotate the shafts (6) and (7). These latter turn in opposite directions, and each is reeved and carries a coupler at each end which can be connected to the satellite transmission.

Changing between manual and motor drive is effected by a lever (8) protruding through the container. There is a neutral, intermediate position. A chain of reducing gears is placed between the small, high-speed electric motor (9) and the main shaft.

The speed of rotation of the globe is determined by a continuously variable drive, whose ratio is controlled by a handle (10) mounted on the left wall of the casing. An indicating arm shows the position of the fast element of the drive. The globe is fixed to the vertical shaft (11), which is rotated by the disc (12) that forms the slow element of the variable drive. The assembly is spring-mounted to maintain alignment and contact.

A revolution counter (13) records the number of rotations of the globe, thus producing the measure of time. The other revolution counter — geared to the main shaft — records the number of rotations of the satellite. Both counters can be reset to zero by levers.

The motion of the satellite is transmitted from the main shaft (2) through a set of rigid shafts (13) and gears (14). At first, a flexible cable was tried, but there was too much elastic play in the motion. The satellite did not begin to move until the globe had turned through an appreciable angle, and it continued to rotate for a short distance after the globe had stopped. Although very convenient for adjusting the inclination of the orbit, the flexible cable had to be abandoned.

The chief difficulty with the rigid shaft linkage lay in the need to be able to change the plane of rotation of the satellite through 90°. The problem was solved by using sections (13) connected by gears (14) able to accommodate any position of the end section (15) within an arc of over 90°. To obtain another 90° of freedom, the linkage could be connected to one of the couplings at the other end of the case. At either end, the coupling turning in the opposite sense could be used, thus changing from a direct to a retrograde orbit.

The satellite is represented by a small sphere attached to a curved arm that just clears the surface of the globe. The arm is mounted in a bearing (16), and is turned by the last section (15) of the rigid linkage. The bearing (16) is mounted in a support that slides along the scaled sector (17) and can be locked to the sector at a point that determines the inclination of the orbit.

The box containing the whole mechanism has been hinged to a table, and can be inclined at an angle of $23^{\circ}30'$ to the horizontal to simulate the inclination of the earth's axis to the plane of the ecliptic.

Good quality materials were used in the construction to avoid the need for maintenance and repair.

3. APPLICATIONS

The flight of an astronaut orbiting the earth for a few days or less can be very well simulated. If the period and the inclination of the orbit are known, and set on the model, it is then necessary to make an adjustment that will place the satellite above a given point on the globe at which the time of transit was known. This could be the launch point, the re-entry point, or any intermediate point. Time can be moved forward or backward from this known moment by turning the handle in either direction. The time of passage above various points on the earth can be estimated, and thus the possibility of observing the capsule or of receiving messages from it. Conversely, the opportunities for the space vehicle to observe the earth can be studied.

The opportunity for observing a satellite such as "Echo" with the naked eye can be studied, provided that the time of a recent transit is known, and whether it was headed on a northbound or southbound passage.

The paths of intercontinental rockets can be shown.

Orbits with high eccentricity — or those that can be changed in mid-flight by acceleration rockets — can not be demonstrated by this device. The precision is insufficient to follow an orbit for many days. In this case it is necessary to make occasional correction settings.

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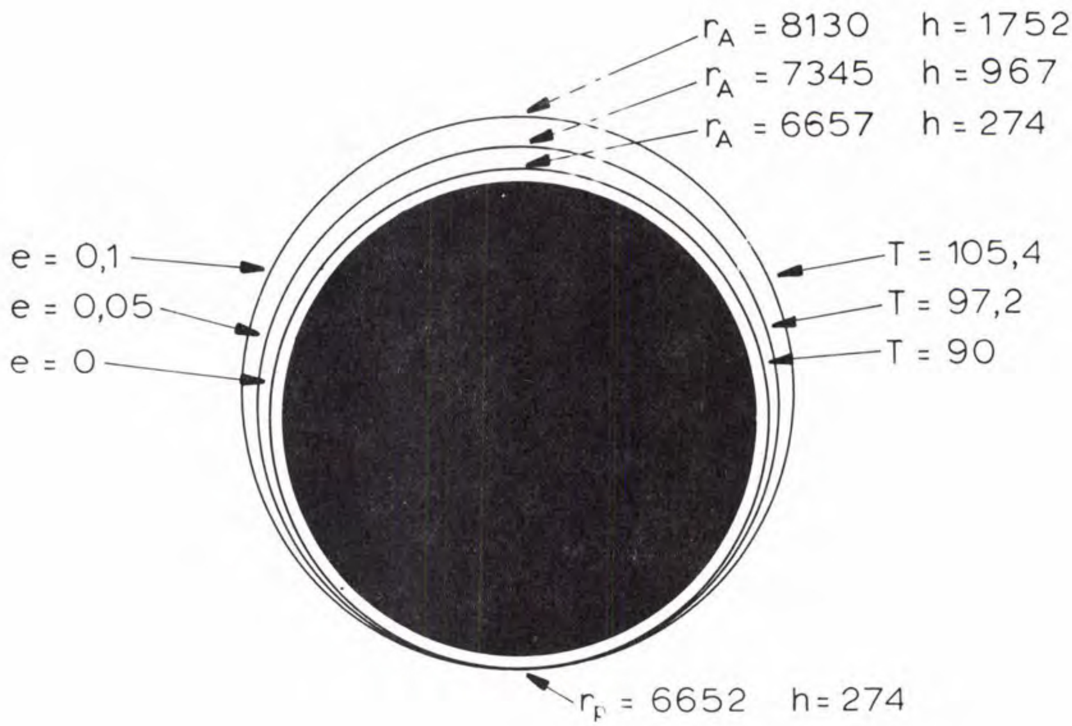

Fig. 1 – THREE LOW–ALTITUDE ORBITS

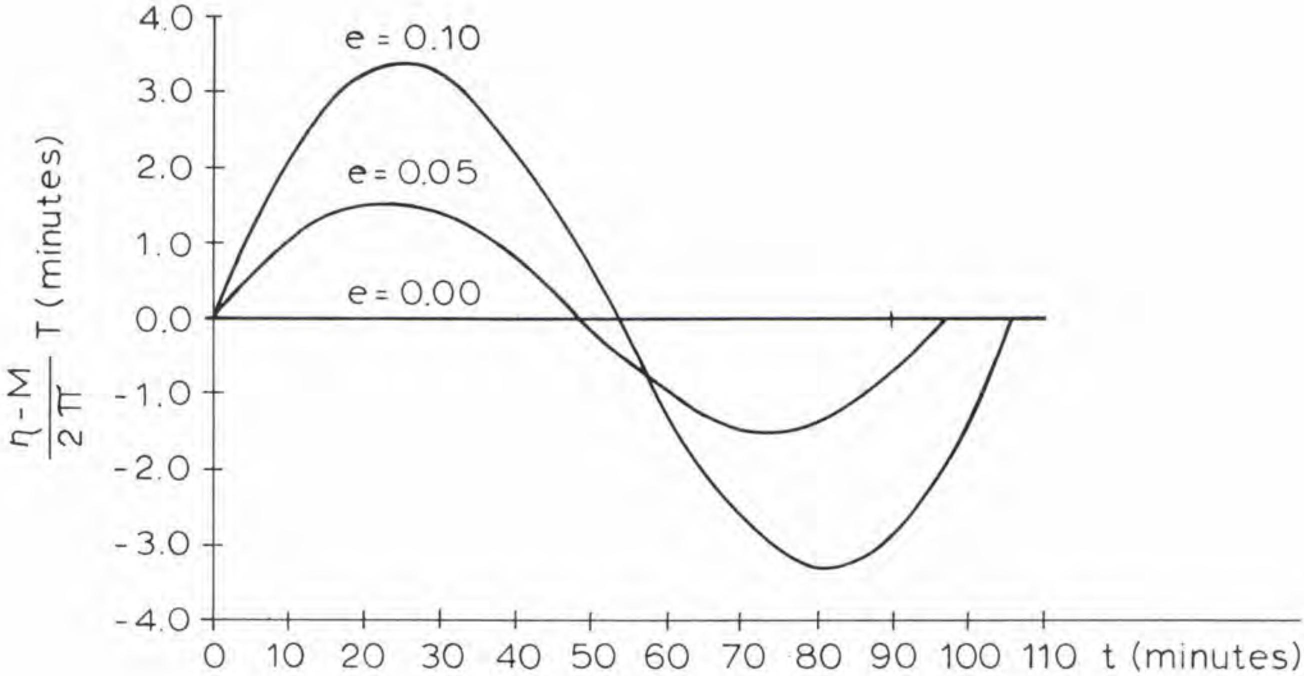

Fig. 2 – DIFFERENCE IN TIME BETWENN THE ADVANCE OF THE SATELLITE AND THAT OF A POINT MOVING WITH CONSTANT SPEED

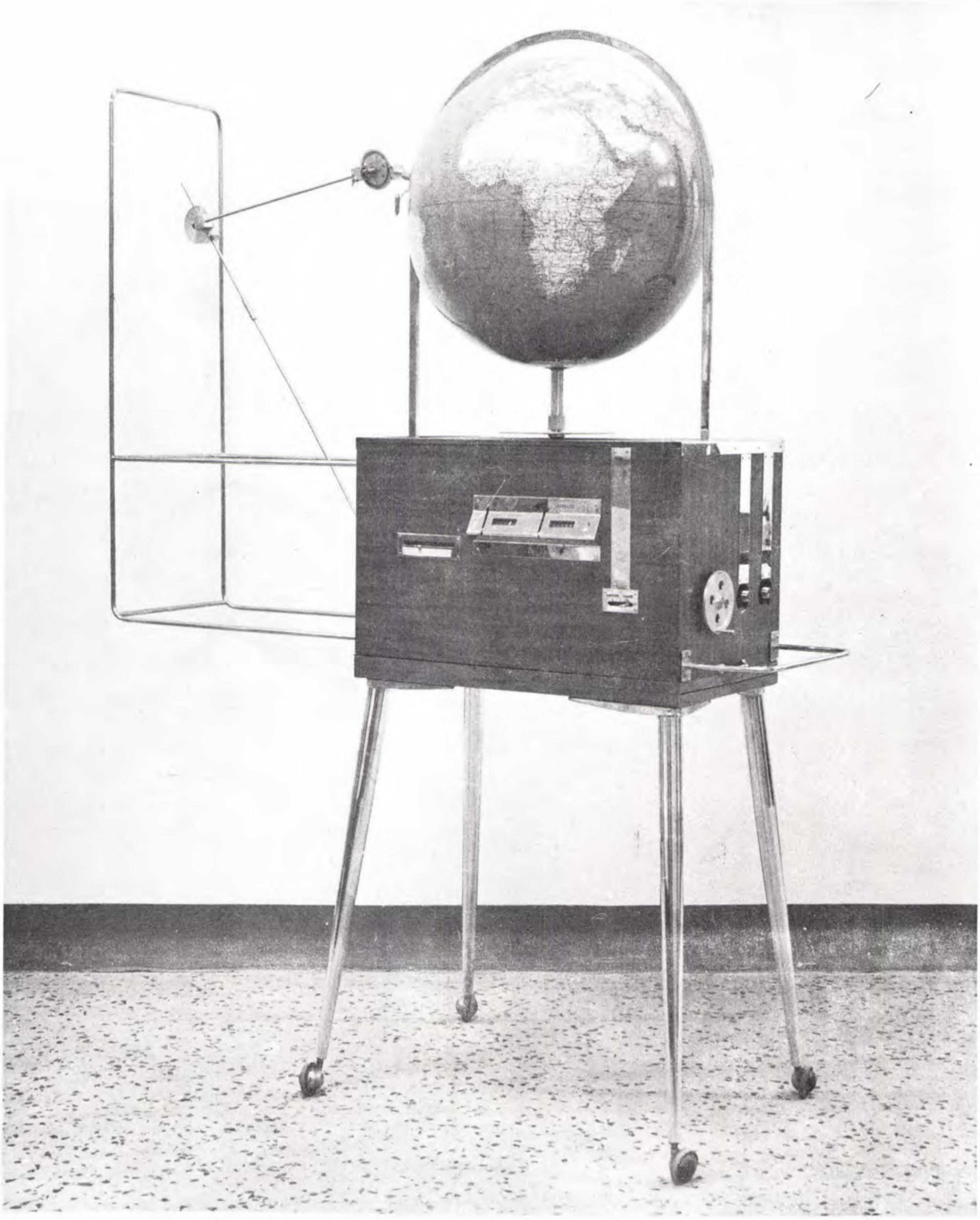

Fig. 3 – PHOTOGRAPH OF MODEL

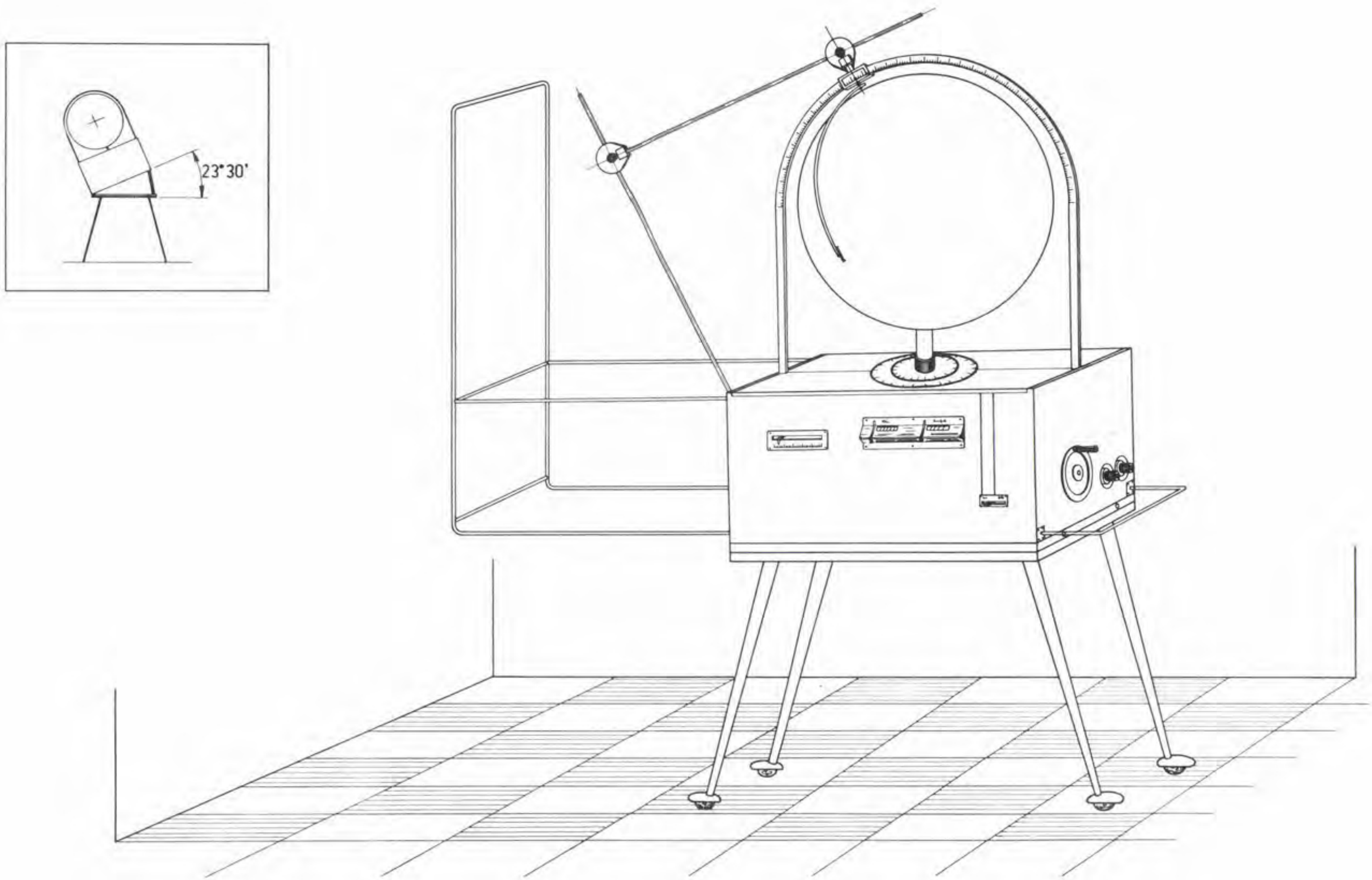

Fig. 4 — DIAGRAM OF MODEL

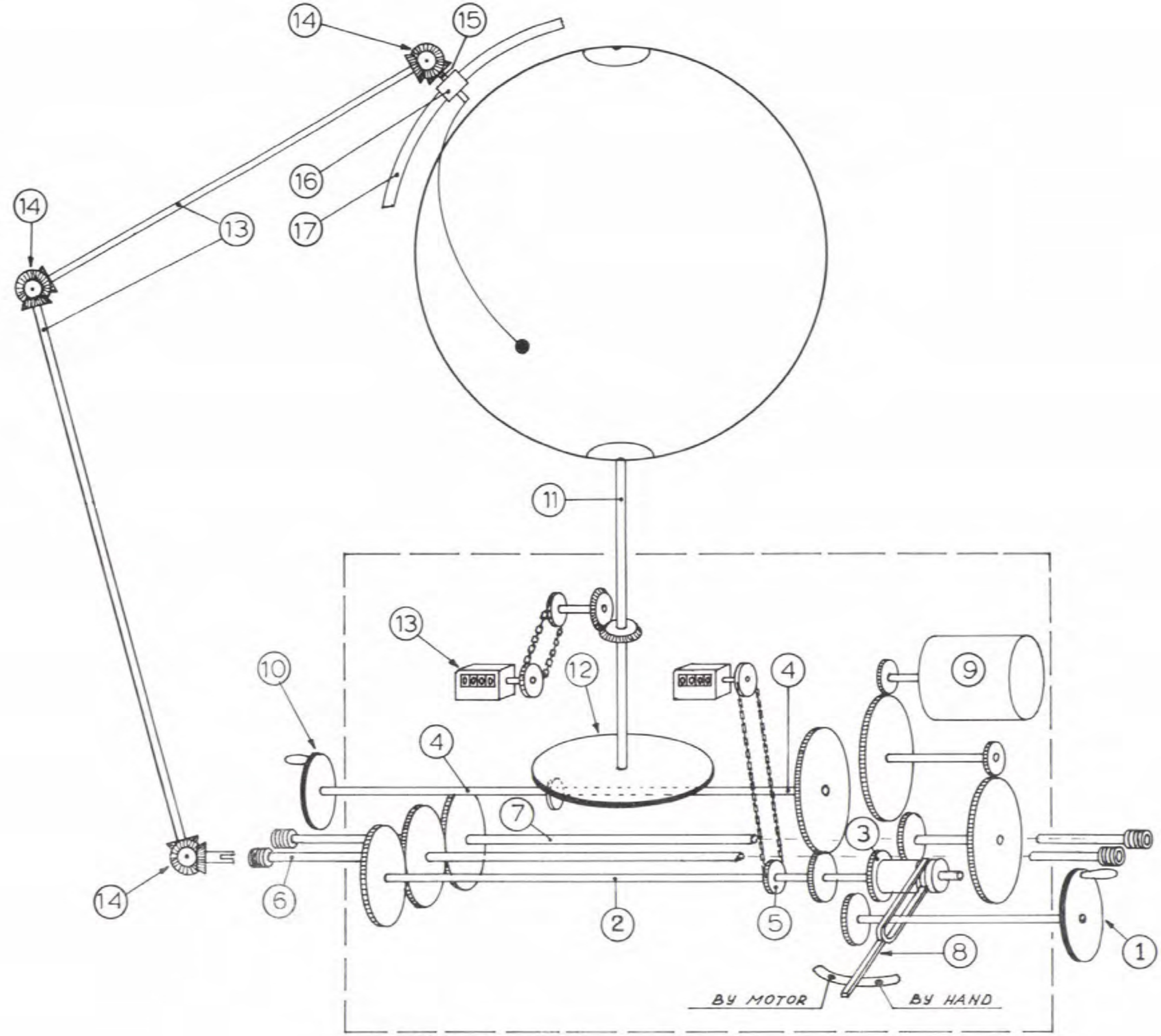

Fig. 5 – SCHEMATIC DIAGRAM OF MECHANISM

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