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SACLANT ASW
RESEARCH CENTER

THE REVERBERATION OF SOUND from SMOOTH OCEAN FLOORS

by

R. R. GOODMAN

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APPROVED FOR DISTRIBUTION:

John M. Ide
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Director

1. INTRODUCTION

Recently several authors have reported the results of measurements of sound reverberation from ocean floors. Urick[1] and Mackenzie[2] have both found that this type of reverberation, over a certain range of time, is empirically described by a Lambert's law scattering. Both of these experiments were performed over deep, very flat, ocean floors. The presence of a scattering mechanism which would produce a Lambert's law is somewhat surprising under these circumstances since it is usually associated with reflection from very rough surfaces.

Since the reverberation[+] clearly contains much information about the character of the ocean floor it is of great interest to understand the mechanisms which produce it. If reverberation is ever to be interpreted it is clear that a systematic study of its theory is necessary. This report is a contribution to such a study. The reverberations produced by a completely flat surface separating two homogeneous isotropic fluids is investigated. There is no effort to include the effect of rough surfaces which would be a logical extension of the work presented.

The following section contains a derivation of the reverberated sound

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[1] Superscripts refer to similarly numbered entries in the References at the end of the Report.

[+] The word "reverberation" is defined in this report to be all sound which is not associated with specular reflection arriving back at the source.

field produced by a pulsed source. In Sect. 3 the reverberation is considered for several pulse shapes and compared with experimental results. A discussion follows in Sect. 4.

2. THEORETICAL DERIVATION

The derivation of the reverberation field for a point source above a flat boundary separating two media is given in this section. The approach is similar to one given by Pekeris,[3] who first pointed out the existence of a "tail" on the specularly normally reflected signal. The general introductory theory presented here is reasonably complete but may be found in an expanded form in several reference books[4,5] as well as in numerous articles.

Consider first the CW case of a point source. The source is taken to be a distance above the flat boundary separating two semi-infinite fluids. The fluid containing the source has a velocity and density given by c and ρ respectively. These quantities in the other medium are symbolized by c' and ρ'.

The physical situation is illustrated in Fig. 1.

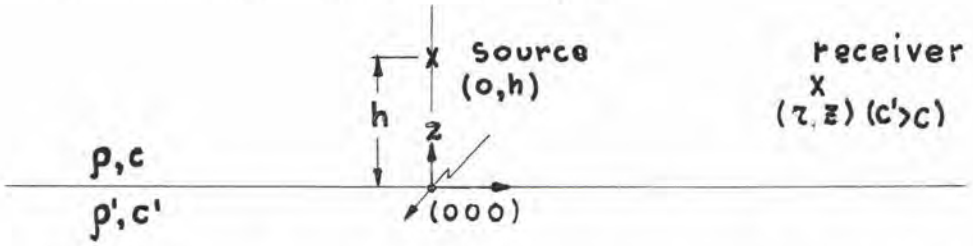

Fig. 1. Point Source and Receiver Above a Flat, Infinite Boundary Separating Two Semi-infinite Media.

The receiver is shown at an arbitrary position (r, z) and will later be considered to be at the source (0, h). The coordinate system is cylindrical with its origin at the surface below the source. The only case considered here is $C' > C$. The fields in both fluids must satisfy the Helmholtz equation given by

$$\left.\begin{aligned} \left(\nabla^2 + \frac{\omega^2}{c^2}\right)\varphi_1 &= 0 \quad ; \quad z \geqslant 0 \\ \left(\nabla^2 + \frac{\omega^2}{c'^2}\right)\varphi_2 &= 0 \quad ; \quad z \leqslant 0 \ , \end{aligned}\right\} \tag{1}$$

where φ_1 and φ_2 are the velocity potentials in the upper and lower half spaces respectively. A time dependence of the type $e^{i\omega t}$ has been assumed. The field in the upper medium φ_1 is the sum of the source field φ_s and the field φ_r due to the presence of the boundary. In cylindrical coordinates, which are the natural choice for this type problem, the most convenient representation of the point source of unit amplitude (for the velocity potential at unit distance) is

$$\varphi_s = e^{i\omega t} \int_0^\infty J_0(kr)\, e^{-\sqrt{k^2 - \frac{\omega^2}{c^2}}\,|z-h|} \frac{k\,dk}{\sqrt{k^2 - \frac{\omega^2}{c^2}}} \tag{2}$$

(See Ref. 4, p. 126.) It is easily shown that the differential equation (1), when written in cylindrical coordinates and separated, gives rise to

the solutions

$$\left.\begin{aligned} \varphi_1(k,r,z) &= A(k)\, J_0(kr)\, F_1(z,k) \\ \varphi_2(k,r,z) &= B(k)\, J_0(kr)\, F_2(z,k), \end{aligned}\right\} \qquad (3)$$

where

$$\left.\begin{aligned} \frac{d^2F_1}{dz^2} + \left(\frac{\omega^2}{c^2} - k^2\right) F_1 &= 0 \\ \frac{d^2F_2}{dz^2} + \left(\frac{\omega^2}{c^2} - k^2\right) F_2 &= 0 \,. \end{aligned}\right\} \qquad (4)$$

The solutions of Eq.(4) are well known to be

$$\left.\begin{aligned} F_1 &= a_1 e^{-\sqrt{k^2-\frac{\omega^2}{c^2}}\, z} \,; \quad z \geq h \\ F_1' &= a_2 e^{-\sqrt{k^2-\frac{\omega^2}{c^2}}\, z} + a_3 e^{+\sqrt{k^2-\frac{\omega^2}{c^2}}\, z} \,; \quad 0 \leq z \leq h \\ F_2 &= a_4 e^{-\sqrt{k^2-\frac{\omega^2}{c'^2}}\, z} \,; \quad z \leq 0 \,. \end{aligned}\right\} \qquad (5)$$

The radiation condition has been used for the first and last of the solutions of Eq.(5). The root $\sqrt{k^2-\frac{\omega^2}{c^2}}$ must always be taken such that $\mathrm{Re}\sqrt{k^2-\frac{\omega^2}{c^2}} \geq 0$ and $\mathrm{Re}\sqrt{k^2-\frac{\omega^2}{c'^2}} \geq 0$ in order to be consistent with the solutions chosen. Everywhere, except at the source, the pressure and displacement must be continuous.

Thus at $z = 0$

$$\left.\begin{aligned} \rho \frac{\partial \varphi_s}{\partial t} + \rho \frac{\partial \varphi_r}{\partial t} &= \rho' \frac{\partial \varphi_2}{\partial t} \\ \frac{\partial \varphi_s}{\partial z} + \frac{\partial \varphi_r}{\partial z} &= \frac{\partial \varphi_2}{\partial z} \end{aligned}\right\} \tag{6}$$

and at $z = h$

$$\left.\begin{aligned} F_1 &= F_1' \\ \frac{dF_1}{dz} &= \frac{dF_1'}{dz} \end{aligned}\right\} \tag{7}$$

Using Eqs. (2), (3), (5), (6), and (7), it is easy to show that the solutions may be written as

$$\left.\begin{aligned} \varphi_r &= e^{i\omega t} \int_0^\infty \frac{k\,dk\, J_0(kr)}{\sqrt{k^2 - \frac{\omega^2}{c^2}}} \left\{ \frac{\sqrt{k^2 - \frac{\omega^2}{c^2}} - \frac{\rho}{\rho'}\sqrt{k^2 - \frac{\omega^2}{c'^2}}}{\sqrt{k^2 - \frac{\omega^2}{c^2}} + \frac{\rho}{\rho'}\sqrt{k^2 - \frac{\omega^2}{c'^2}}} \right\} e^{-\sqrt{k^2 - \frac{\omega^2}{c^2}}\,(z+h)} \\ \varphi_2 &= \frac{2\rho}{\rho'} e^{i\omega t} \int_0^\infty \frac{k\,dk\, J_0(kr)}{\sqrt{k^2 - \frac{\omega^2}{c^2}}} \left\{ \frac{\sqrt{k^2 - \frac{\omega^2}{c'^2}}}{\sqrt{k^2 - \frac{\omega^2}{c^2}} + \frac{\rho}{\rho'}\sqrt{k^2 - \frac{\omega^2}{c'^2}}} \right\} e^{-\left\{\sqrt{k^2 - \frac{\omega^2}{c'^2}}\,z + \sqrt{k^2 - \frac{\omega^2}{c^2}}\,h\right\}} \end{aligned}\right\} \tag{8}$$

The interest in this report is concentrated on the field at the position of the source. At this position φ_{r} has the form

$$\varphi_{r} = e^{i\omega t} \int_{0}^{\infty} \frac{k\,dk}{\sqrt{k^2 - \frac{\omega^2}{c^2}}} \left\{ \frac{\sqrt{k^2 - \frac{\omega^2}{c^2}} - b\sqrt{k^2 - \frac{\omega^2}{c^2}\alpha^2}}{\sqrt{k^2 - \frac{\omega^2}{c^2}} + b\sqrt{k^2 - \frac{\omega^2}{c^2}\alpha^2}} \right\} e^{-2h\sqrt{k^2 - \frac{\omega^2}{c^2}}} \qquad (9)$$

where

$$\alpha^2 = c^2/c'^2$$

$$b = \rho/\rho'$$

Before a field due to a pulse is formulated, a more convenient form φ_{r} $(0, h)$ is desired. Such a form is obtained by making the transformation

$$i\omega\zeta = \sqrt{k^2 - \omega^2/c^2} \qquad (10)$$

The path of integration has two separate paths in the ζ plane: one for $\omega > 0$ and one for $\omega < 0$, as shown in Fig. 2.

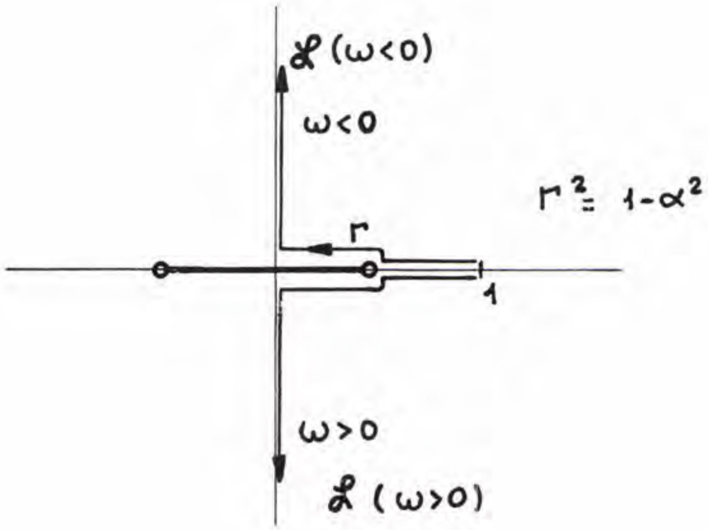

Fig. 2. Contour Paths in the ζ Plane

The paths are determined by the condition $\mathcal{R}e\sqrt{k^2-\frac{\omega^2}{c^2}} \geq 0$ and $\mathcal{R}e\sqrt{k^2-\frac{\omega^2}{c'^2}} \geq 0$. The branch line is also determined by these conditions to lie along the real axis between the branch points $\zeta = \pm\Gamma$. The equation for φ_r in terms of the integral over ζ is

$$\varphi_r = \frac{i\omega}{c} e^{i\omega t} \int_{\mathcal{L}(\omega)} d\zeta \left\{ \frac{\zeta - b\sqrt{\zeta^2-\Gamma^2}}{\zeta + b\sqrt{\zeta^2-\Gamma^2}} \right\} e^{-2hi\omega\zeta} , \qquad (11)$$

where

$$\Gamma^2 = 1 - \alpha^2 .$$

Both paths of $\mathcal{L}(\omega)$ may be replaced by one along the real axis from 1 to ∞ provided the integrand vanishes along an arc of infinite radius. This requirement is met by changing the integrand with the addition and subtraction of a term

$$\left\{ \frac{1-b}{1+b} \right\} e^{-2h i \omega \zeta}$$

giving for φ_n the final form

$$\varphi_n = e^{i\omega t} \left\{ \frac{(1-b)}{(1+b)} \frac{e^{-\frac{2hi\omega}{c}}}{2h} + \frac{i\omega}{c} \int_1^\infty dx \left\{ \frac{x - b\sqrt{x^2 - \Gamma^2}}{x + b\sqrt{x^2 - \Gamma^2}} - \frac{1-b}{1+b} \right\} e^{-\frac{2hi\omega x}{c}} \right\} \qquad (12)$$

This agrees with the equation derived by Pekeris,[3+] for signals in shallow water. The convenience of this form will be demonstrated in the following extension to pulses of arbitrary shape.

The field for arbitrary pulse shapes is a straightforward extension of the CW case. For the time dependent velocity potential $\Phi_s(t)$

[+] The Eq. (A-34) in Pekeris is printed incorrectly. Its correct form agrees with Eq. (12) above.

of the source whose Fourier transform is

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \; \Phi_s(t)\, e^{-i\omega t}, \tag{13}$$

the field $\Phi_r(0, h, t)$ is given by

$$\Phi_r(0,h,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\omega \, e^{i\omega t} g(\omega)\, \varphi_r(\omega, h, 0). \tag{14}$$

The substitution of Eq. (12) into (14) gives

$$\Phi_r(0,h,t) = \frac{1}{2h}\left(\frac{1-b}{1+b}\right) \Phi_s\left(t - \frac{2h}{c}\right) + \frac{1}{\sqrt{2\pi}\, c} \int_{-\infty}^{+\infty} g(\omega)\, i\omega\, e^{i\omega t} d\omega \int_{1}^{\infty} dx \left\{\frac{x - b\sqrt{x^2-\Gamma^2}}{x + b\sqrt{x^2-\Gamma^2}} - \frac{1-b}{1+b}\right\} e^{-\frac{2hi\omega}{c}x}, \tag{15}$$

which may be written as

$$\Phi_r(0,h,t) = \frac{1}{2h}\left(\frac{1-b}{1+b}\right) \Phi_s\left(t - \frac{2h}{c}\right) + \frac{1}{\sqrt{2\pi}\, c} \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} d\omega \, g(\omega)\, e^{i\omega t} \int_{1}^{\infty} dx \left\{\frac{x - b\sqrt{x^2-\Gamma^2}}{x + b\sqrt{x^2-\Gamma^2}} - \frac{1-b}{1+b}\right\} e^{-\frac{2hi\omega}{c}x}. \tag{16}$$

Integration over ω gives

$$\Phi_r(0,h,t) = \frac{1}{2h}\left(\frac{1-b}{1+b}\right)\Phi_s\left(t-\frac{2h}{c}\right) + \frac{1}{c}\frac{\partial}{\partial t}\int_1^\infty dx\, \Phi_s\left(t-\frac{2h}{c}x\right)\left\{\frac{x-b\sqrt{x^2-\Gamma^2}}{x+b\sqrt{x^2-\Gamma^2}} - \frac{1-b}{1+b}\right\}. \quad (17)$$

Since

$$\frac{\partial}{\partial t}\Phi_s\left(t-\frac{2h}{c}x\right) = -\frac{c}{2h}\frac{\partial}{\partial x}\Phi_s\left(t-\frac{2h}{c}x\right), \quad (18)$$

the integral in Φ_r may be integrated by parts, giving

$$\Phi_r = \frac{1}{2h}\left(\frac{1-b\alpha}{1+b\alpha}\right)\Phi_s\left(t-\frac{2h}{c}\right) - \frac{b\Gamma^2}{h}\int_1^\infty \frac{dx\;\Phi_s\left(t-\frac{2h}{c}x\right)}{\sqrt{x^2-\Gamma^2}\left(x+b\sqrt{x^2-\Gamma^2}\right)^2}. \quad (19)$$

The first term in Eq. (19) is clearly associated with the direct reflection of the pulse. It has the form of an image. The last term of Eq. (19) is the "reverberation" term which will be studied in the remainder of this report. For convenience it will be given the symbol $\Phi_R(t)$.

There are some interesting limits easily studied. If the sound velocities are equal ($c=c'$) in both media the term Φ_R vanishes for arbitrary ρ/ρ'. It also vanishes for $b \to \infty$. This last limit demonstrates

that this term cannot account for any reverberation at the air-sea interface leaving only the possibility of surface roughness as a source of reverberation.

In the following section some specific pulse shapes will be considered.

3. INVESTIGATION OF SPECIFIC PULSE SHAPES AND COMPARISON WITH EXPERIMENT

It has now been shown that reverberation occurs even in the presence of a boundary without roughness. It now remains to be seen whether or not this form of reverberation can account for the observed results. Mackenzie has reported results for CW pulses, and Urick has measured reverberation for explosive sources. In this section the magnitude of Φ_R will be compared with their results. First, Φ_R must be calculated for the specific pulse shapes. Those calculated here are:

(1) the δ pulse

(2) the CW pulse

(3) the exponential pulse.

It must be remembered that these are the pressure pulse shapes (p_s), not the velocity potentials. Since

$$p = \rho \frac{\partial \varphi}{\partial t} \tag{20}$$

the pressure reverberation term ($p_R(t)$) is given as

$$p_R = \rho \frac{\partial \Phi_e}{\partial t} = -\frac{b \rho \Gamma^2}{h} \int_1^\infty dx \, \frac{\partial \Phi_s}{\partial t} \Big/ \sqrt{x^2 - \Gamma^2}\left(x + b\sqrt{x^2-\Gamma^2}\right)^2$$

$$= -\frac{b\Gamma^2}{h}\int_1^\infty dx \, \frac{p_s(t - \frac{2h}{c}x)}{\sqrt{x^2-\Gamma^2}\left(x + b\sqrt{x^2-\Gamma^2}\right)^2} \quad . \qquad (21)$$

The results therefore may be calculated directly for the pressure Since every type of pulse considered will vanish before some time (say $t = 0$), p_R may be written as

$$p_R = -\frac{b\Gamma^2}{h}\int_1^{\frac{tc}{2h}} dx \, \frac{p_s(t - \frac{2h}{c}x)}{\sqrt{x^2-\Gamma^2}\left(x + b\sqrt{x^2-\Gamma^2}\right)^2} \quad . \qquad (22)$$

There is some convenience in using a dimensionless time unit $\frac{ct}{2h}$. The symbol τ will be used for such units of time. Then Eq. (22) becomes

$$p_R(\tau) = -\frac{b\Gamma^2}{h}\int_1^{\tau} dx \, \frac{p_s\left((\tau - x)\frac{2h}{c}\right)}{\sqrt{x^2-\Gamma^2}\left(x + b\sqrt{x^2-\Gamma^2}\right)^2} \quad . \qquad (23)$$

(1) The δ pulse

If the source emits a pulse $\delta(t)$ then,

since $\delta(t) = \frac{c}{2h}\delta(\tau)$, P_R becomes

$$P_R = -\frac{b\Gamma^2 c}{2h^2\sqrt{\tau^2-\Gamma^2}\left(\tau+b\sqrt{\tau^2-\Gamma^2}\right)^2} \quad (24)$$

(2) The explosive pulse

Consider the signal

$$\left.\begin{aligned} p_s(t) &= p_0 e^{-\alpha t}, \quad t \geq 0 \\ &= 0 \quad\quad\;\; , \quad t < 0 . \end{aligned}\right\} \quad (25)$$

Equation (22), upon substitution of Eq. (25), becomes

$$P_R = -\frac{b\Gamma^2 p_0}{h}\int_1^{\tau} dx \, \frac{e^{-\frac{2\alpha h}{c}(\tau - x)}}{\sqrt{x^2-\Gamma^2}\left(x+b\sqrt{x^2-\Gamma^2}\right)^2} \quad (26)$$

Since pulses will be considered which have very fast decay rates compared to times $\frac{2h}{c}$ the term $\frac{2h\alpha}{c}$ is very much larger than one, therefore allowing

Eq.(26) to be approximated accurately by

$$p_R \cong - \frac{c\,b\,\Gamma^2 p_0}{2h^2\alpha\sqrt{\tau^2-\Gamma^2}\left\{\tau+b\sqrt{\tau^2-\Gamma^2}\right\}^2}\,, \qquad (27)$$

where

$$\tau - 1 \gg \frac{c}{2h\alpha}\,. \qquad (28)$$

(3) The CW pulse

Suppose

$$\left.\begin{aligned} p_s(t) &= 0\,, \quad t \leq 0 \\ &= p_0 \sin\omega t\,, \quad 0 \leq t \leq T \\ &= 0\,, \quad t \geq T \end{aligned}\right\} \qquad (29)$$

where

$$\frac{\omega}{2\pi} T = N \qquad \text{(N is an integer);} \qquad (30)$$

i.e., the pulse contains an integer number of cycles.

Eq. (23) becomes

$$p_R(\tau) = -\frac{b\Gamma^2 p_o}{h} \int_{\tau-\frac{cT}{2h}}^{\tau} dx \; \frac{\sin \frac{2h}{c}\omega(\tau - x)}{\sqrt{x^2-\Gamma^2}\,\left(x + b\sqrt{x^2-\Gamma^2}\right)^2} \tag{31}$$

for $\tau > 1 + \frac{cT}{2h}$. Letting $\Delta = \frac{cT}{2h}$ and substituting $\tau - x = \xi$, P_R may be written as

$$P_R(\tau) = -\frac{b\Gamma^2 p_o}{h} \int_0^{\Delta} d\xi \, \frac{\sin \frac{2h\omega}{c}\xi}{\sqrt{(\tau-\xi)^2-\Gamma^2}\left\{(\tau-\xi) + b\sqrt{(\tau-\xi)^2-\Gamma^2}\right\}^2} \tag{32}$$

For high frequencies in relatively deep water $\frac{2h\omega}{c}$ is very much greater than one; thus, the sin term in Eq. (32) oscillates very rapidly compared to changes in denominator. A good approximation to the integral can be made for such cases and its derivation is given in Appendix A. The final result is that

$$P_R \cong \frac{\pi b c^2 \Gamma^2 p_o}{h^3 \omega^2} \sum_{m=0}^{N-1} \frac{3(\tau - m\phi)\left(\tau - m\phi + b\sqrt{(\tau-m\phi)^2-\Gamma^2}\right) - 2\Gamma^2}{\left\{(\tau-m\phi)^2-\Gamma^2\right\}^{3/2}\left(\tau - m\phi + b\sqrt{(\tau-m\phi)^2-\Gamma^2}\right)^3} \tag{A-7}$$

which is very useful for numerical purposes along with the asymptotic limit,

$$P_R \xrightarrow{\tau >> \Delta} \frac{3 b c^2 \Gamma^2 p_0 T \left\{ 3\tau\left(\tau + b\sqrt{\tau^2 - \Gamma^2}\right) - 2\Gamma^2 \right\}}{2 h^3 \omega \left(\tau^2 - \Gamma^2\right)^{3/2} \left(\tau + b\sqrt{\tau^2 - \Gamma^2}\right)^3} . \quad \text{(A-8)}$$

It is now possible to use these equations to compare the effect of this type of reverberation with observed values.

For the purpose of very rough comparison with experimental observations these formulas will now be used for numerical evaluation. The pulsed and explosive signals were used by Mackenzie and Urick respectively and seem to offer the best opportunity for comparison with theory in that they present data of a resonably complete nature. These results are not examined in detail in this report for reasons which will be evident.

It is now of interest to see if P_R can account for the reverberation observed by Urick and Mackenzie. For comparison, ratios of velocity and density will be taken to be

$$\frac{c}{c'} = 0.9$$
$$\frac{\rho}{\rho'} = 2/3$$
$$C = 1500 \text{ meters/sec}$$

Reasonable variations in these will not lead to great changes in the magnitude of P_R . For comparison with Mackenzie, who normalized his

results, let

$$h = 2100 \text{ fathoms } (3840\text{m})$$
$$T = 1/2\text{sec}$$
$$\omega = 1060\pi \quad (530 \text{ cps})$$
$$P_0 = 1 .$$

Consider the value just before the second specular reflection arrives. For this point Eq. (A-8) suffices and gives

$$P_R = 3.95 \times 10^{-10} \text{ dynes}/\text{cm}^2 ,$$

or, in db the level of reverberation would be

$$L_R = 20 \log P_R = -198 \text{ db} .$$

This may be seen to be about 85 db too low to account for Mackenzie in Fig. 5. of his paper. At 10 sec his value is at noise level - 125db. It is quite clear that the difference is enormous and therefore the origin of his reverberation must be different than that proposed here. More will be said about this difference later.

With Urick's results comparison is more difficult since he works with db levels of various frequency bands. However, simple order of magnitude results are obtained using Eq. (27) with

$$h = 4400\text{m}$$
$$\alpha = 5 \times 10^5 \text{ sec}^{-1} \text{ (from Fig. 5. Ref. 1)}$$
$$P_0 = 6 \times 10^8 \text{ dynes} / \text{cm}^2$$

This gives

$$|p_R| = 2.76 \times 10^{-4} \text{ dynes/cm}^2$$

or

$$L_R = -71 \text{ db}.$$

This appears to be about the same magnitude below Urick's data (a few db above zero, estimating from Fig. 4 of his report) as the p_R was below Mackenzie's values. Thus, Urick's results also may not be explained by the theory developed here.

4. DISCUSSION AND SUMMARY

It has been shown that the simple theory of a point source above a plane fluid-fluid boundary contains a form of reverberation. It is not, however, large enough to explain the results obtained in two separate experiments. There are two obvious ways to extend the theory which would increase the calculated reverberation.

First, a roughness could be assumed at the boundary. It would account for an increase in the reverberated field but almost certainly not enough to account for Urick's and Mackenzie's results, at least for the roughness observed in the deep flat regions of the ocean floor. The roughness over the regions of the experiments mentioned above are not known but it is doubtful that it is large enough to account for the magnitude of the reverberation observed.

Second, the theory could be extended to include the case of an increasing velocity as a function of depth in the lower medium. Since the lower

medium probably does have this characteristic in the sea due to compaction this is a logical extension. It clearly brings more energy back up to the surface, but its magnitude as a function of velocity gradient is not known. The writer intends to make this extension in the near future.

It should be mentioned that in the theory above no account has been taken of the air-sea surface. Both Mackenzie and Urick made their experiments with both the source and the receiver near the surface (about 15 m). Although both state that backscattering from the bottom accounts for reverberation, neither includes the effect of nonspecular reflection from the air-sea surface. This reflection could clearly account for a large amount or reverberation for higher sea states. Mackenzie performed his experiments in sea state 3; Urick did not mention the sea state. This important contribution cannot be neglected for sources and receivers near the surface. For the purpose of studying reverberation from the sea floor it would be better to work at a depth sufficient to separate surface effects. Such an experiment, to the writer's knowledge, has never been reported in the literature although it would give a much clearer indication of the nature of the ocean floor.

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Appendix A

APPROXIMATE EVALUATION OF p_R FOR CW PULSED SIGNALS

For pulsed CW signals certain approximations allow the reverberation term p_R to be expressed in a form convenient to numerical evaluation. In this appendix, an approximation is obtained which replaces the integral by a sum over a finite but generally large number of terms.

Equation (32) is

$$p_R(\tau) = -\frac{b\Gamma^2 p_0}{h}\int_0^{\Delta} d\xi \, \frac{\sin\frac{2h\omega}{c}\xi}{\sqrt{(\tau-\xi)^2-\Gamma^2}\left\{\tau-\xi+b\sqrt{(\tau-\xi)^2-\Gamma^2}\right\}^2} \qquad \text{(A-1)}$$

where $\frac{2h\omega}{c} \gg 1$ will be assumed. The term $\sin\frac{2h\omega}{c}\xi$ is rapidly oscillating compared to any other term. By changing the scale of the variable of integration to

$$\frac{2h\omega}{c}\xi = \theta \qquad \text{(A-2)}$$

then Eq. (A-1) becomes

$$p_R(\tau) = -\frac{bc\Gamma^2 p_0}{2h^2\omega}\int_0^{2\pi N} d\theta \, \frac{\sin\theta}{\sqrt{\left(\tau-\frac{c\theta}{2h\omega}\right)^2-\Gamma^2}\left\{\tau-\frac{c\theta}{2h\omega}+b\sqrt{\left(\tau-\frac{c\theta}{2h\omega}\right)^2-\Gamma^2}\right\}^2} \qquad \text{(A-3)}$$

The integral may be expressed as the sum of integrals, each over a range π , giving

$$p_R(\tau) = -\frac{b c \Gamma^2 p_0}{2h^2\omega} \sum_{m=0}^{2N-1} \int_{m\pi}^{(m+1)\pi} d\theta \frac{\sin\theta}{\sqrt{\left(\tau-\frac{c\theta}{2h\omega}\right)^2-\Gamma^2}\left\{\left(\tau-\frac{c\theta}{2h\omega}\right)+b\sqrt{\left(\tau-\frac{c\theta}{2h\omega}\right)^2-\Gamma^2}\right\}^2} \quad \text{(A-4)}$$

Since the denominator does not change significantly over the range of integration Eq. (A-4) may be approximated by

$$p_R(\tau) \cong -\frac{b c \Gamma^2 p_0}{h^2\omega} \sum_{m=0}^{2N-1} \frac{(-1)^m}{\sqrt{\left(\tau-\frac{cm\pi}{2h\omega}\right)^2-\Gamma^2}\left\{\left(\tau-\frac{cm\pi}{2h\omega}\right)+b\sqrt{\left(\tau-\frac{cm\pi}{2h\omega}\right)^2-\Gamma^2}\right\}^2} \quad \text{(A-5)}$$

The summation may be written in a form

$$p_R(\tau) \cong -\frac{b c \Gamma^2 p_0}{h^2\omega} \sum_{m=0}^{N-1} \left\{ \frac{1}{\sqrt{(\tau-m\phi)^2-\Gamma^2}\left(\tau-m\phi+b\sqrt{(\tau-m\phi)^2-\Gamma^2}\right)^2} - \frac{1}{\sqrt{(\tau-(m+\frac{1}{2})\phi)^2-\Gamma^2}\left(\tau-(m+\frac{1}{2})\phi+b\sqrt{(\tau-(m+\frac{1}{2})\phi)^2-\Gamma^2}\right)^2} \right\}. \quad \text{(A-6)}$$

where $\phi = \frac{\pi c}{h\omega}$ which is very much smaller than one. The two terms in the summation differ by only a small amount, allowing them to be combined, to a good approximation, and written as

$$P_R(\tau) \cong \frac{\pi b c^2 \Gamma^2 p_0}{h^3 \omega^2} \sum_{m=0}^{N-1} \frac{3(\tau - m\phi)\left(\tau - m\phi + b\sqrt{(\tau - m\phi)^2 - \Gamma^2}\right) - 2\Gamma^2}{\left\{(\tau - m\phi)^2 - \Gamma^2\right\}^{3/2}\left(\tau - m\phi + b\sqrt{(\tau - m\phi)^2 - \Gamma^2}\right)^3} \qquad \text{(A-7)}$$

For times that are large compared to the signal lengths

$$P_R(\tau) \xrightarrow{\tau >> \Delta} \frac{b c^2 \Gamma^2 p_0 T \left\{3\tau\left(\tau + b\sqrt{\tau^2 - \Gamma^2}\right) - 2\Gamma^2\right\}}{2h^3\omega \; (\tau^2 - \Gamma^2)^{3/2}\left(\tau + b\sqrt{\tau^2 - \Gamma^2}\right)^3} \qquad \text{(A-8)}$$

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