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PARAMETRIC RELATIONS OF FM SONAR

by

H.W.K. KELLY

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INTRODUCTION

Frequency Modulated (FM) sonar usually refers to the special form of frequency modulation where the transmitted pulse is a gliding tone whose frequency varies linearly with time. This pulse is especially simple to process compared with, for example, random or pseudo-random noise pulses, yet it has advantages over simple pure tone pulses which approach those obtained from the more sophisticated transmission.

In simple pulse sonar the requirements of good range discrimination and high signal-to-noise ratios are incompatible. The better the range discrimination required the shorter the pulse that must be used. Since in sonar the peak intensity of the pulse is usually limited by cavitation, shortening the pulse reduces the total signal energy and hence worsens the signal-to-noise ratio achievable. With FM sonar this incompatibility can be overcome, since a long gliding tone pulse may be processed to give a range discrimination equivalent to a very much shorter pure tone pulse. The importance of good range discrimination in sonar is twofold, it enables target size (and, to some extent, target shape) to be estimated more accurately, and it reduces the reverberation background.

It is of interest to examine the parametric relations of FM sonar to see what are the inherent limitations to its performance and whether its potential advantages make it valuable in any particular sonar applications. The present report deals with the simple theory of FM sonar operations and shows how an optimum system may be designed. It also indicates the effects of doppler, the equipment and other factors which may be expected, in practice, to set a limit to the achievable performance.

NOMENCLATURE

Operating frequency	F
Frequency sweep width	B
Filter channel width	b
Range discrimination	r
Equivalent time discrimination	τ
Duration of FM pulse	T
Signal-to-noise gain of FM over short pulse	S
Coefficient of error terms proportional to B	K
Coefficient of error terms independent of B and T	γ
Coefficient of error terms proportional to B/T	ϵ
Slope of frequency sweep	β
Velocity of ship in direction of transmission	V
Velocity of target in direction of transmission	v
Velocity of sound in medium	C

THEORY FOR STATIONARY SOURCE AND TARGET

Consider a frequency modulated pulse of length T whose frequency at any instant, $t \leq T$, is $F - \beta t$. If this pulse is initiated at zero time towards a target at a distance, R, it will be reflected from the target and return to the source at time $\frac{2R}{C}$. The source frequency at this time, (assuming that the source tone continues to glide after the pulse transmission is finished) is $F - \frac{2R\beta}{C}$, so that at the instant the leading edge of the echo pulse is received there is a difference of frequency $\frac{2R\beta}{C}$ between the source and the echo.

Considering next the trailing edge of the transmitted pulse, which is transmitted at time T, and has a frequency $F - \beta T$ ($\sim F - B$), it will return to the source at time $T + \frac{2R}{C}$. The source frequency will then be $F - \beta\,(T + \frac{2R}{C})$. The difference in frequency between the source and the echo is still $\frac{2R\beta}{C}$.

If the source glide tone is then mixed with the incoming echo from a target at range R a steady note of duration T and frequency $\frac{2R\beta}{C}$ results. The echo from a target at range $R + r$ will similarly produce a steady note of duration T at a frequency $\frac{2\beta}{C}(R + r)$. The problem of resolving these two targets in range, therefore, is the separation of two pure tone pulses of length T and frequency difference $\frac{2r\beta}{C}$.

If the incoming signal is fed into a bank of narrow filters, each of width b, the two targets will be resolved if

$$b \leq \frac{2r\beta}{C}$$

The limit of range difference which can be resolved will then be set by the minimum value which can be given to b . This will depend on a number of factors. Since the echo is a pulse, its spectrum has a width of approximately $\frac{1}{T}$, and this sets a lower limit to b . Furthermore the echo pulse will not be perfectly constant in frequency. Three types of frequency variation may be recognized, each of which contributes to broadening the echo spectrum and hence to an increase in the value of b necessary to receive the majority of the echo energy.

In practice it is impossible to make a perfectly linear frequency sweep. Any non-linearity will broaden the processed echo spectrum. For a given system the non-linearity will tend to be a constant proportion of the total frequency sweep so that the actual increase in bandwidth will be proportional to B . As will be discussed later, doppler effects which alter the slope of the returned echo, behave in the same way, so that these distortions may be grouped together to give a spectrum width increase, K B .

Other types of instrumental error may occur, such as the "wow" in tape recordings which may be used at any stage of the data processing. These errors will not depend on B or T . For the purpose of determining relationships between B and T , therefore, in order to specify the transmitted pulse, they may be regarded as constant, say γ .

Unless the target is precisely a point, not all parts of it will be at exactly the same range. Such range variations, within the limit of resolving power of the equipment will again show up as spectrum broadening. For a given target extent the frequency spread will be proportional to β , say $\epsilon\beta$. Variations in path length due to inhomogeneity in the medium, or reflection from the bottom will have the same effect.

Combining all these factors, it is seen that the filter channel necessary to take the echo signal is given by

$$b \geq \frac{1}{T} + KB + \gamma + \epsilon\beta$$

Since the required range resolution sets a maximum value for b , and the spectrum width of the echo sets a minimum, the compatible value of b may be determined by removing the signs of inequality and writing

$$b = \frac{2r}{C}\beta$$

$$b = \frac{1}{T} + KB + \gamma + \epsilon\beta$$

Replacing β by B/T and $\frac{2r}{C}$ by τ

$$b = \tau\frac{B}{T} = \frac{1}{T} + KB + \gamma + \epsilon\frac{B}{T}$$

Eliminating b , a relation is obtained between the values of B and T necessary to give a specified time resolution τ ($\sim$ range resolution r) :

$$\tau = \frac{1}{B} + KT + \gamma\frac{T}{B} + \epsilon \qquad \text{Eq. (1)}$$

Separating the variables this may be written:

$$B = \frac{1 + \gamma T}{(\tau - \epsilon) - KT} \qquad \text{Eq. (2)}$$

The criterion for a specified range resolution clearly does not determine B and T uniquely. A further relationship may be obtained by setting the requirement that in an optimum system the signal-to-background ratio in the receiving filter channel should be a maximum. Since the intensity of the echo is a constant which will be independent of B and T (it will generally be determined either by cavitation or transducer mechanical strength for a given target and given propagation loss) this ratio will be a maximum when the background in the channel of width b is a minimum.

Two cases must be distinguished; when the background is predominantly noise and when it is predominantly reverberation. In the former case it will generally be either ambient sea noise, or ship noise. Each of these may be regarded as white noise over the relatively narrow frequency bands concerned.

If the background is predominantly reverberation its intensity will depend on the energy in the transmitted pulse, the reflectivity of the medium and its boundaries, and the volume of the medium, or surface of the boundaries, which can simultaneously reflect sound into the receiving channel. The pulse energy will affect signal and reverberation equally, and so will not alter the signal-to-background ratio. The reflectivity is a function of the ambient conditions, and is not, therefore, a disposable design parameter. The insonified volume contributing to the reverberation will depend on the range, the equipment directivity and the effective pulse length (or range resolution). Only the last of these factors can affect a comparison between FM and pure tone pulse sonars.

Since the reverberation background is proportional to the range resolution, the FM pulse will be more effective in reducing this background than a pure tone pulse of the same length. In fact from the reverberation point of view the FM pulse and the short pulse with the same resolution are equivalent. The system does not, therefore, allow the signal-to-reverberation ratio to be varied independently of the range resolution, and this must be borne in mind when choosing τ.

When the background is white noise its intensity will depend on the channel width, b, in which it is being received. To keep b to a minimum means that B/T must be kept to a minimum, and this, therefore, gives a further relation between B and T for determining the optimum system. Inserting the value for B from Equation (2) results in

$$\frac{B}{T} = \frac{\frac{1}{T} + \gamma}{(\tau - \epsilon) - KT}\text{, which is to be a minimum.}$$

Differentiating, equating to zero and solving for T gives

$$1 + \gamma T_o = \sqrt{1 + \frac{\gamma}{K}(\tau - \epsilon)} \qquad \text{Eq. (3)}$$

Substituting this optimum value of T in the equation for B and simplifying provides

$$B_o = \frac{1}{KT_o} \quad \text{i.e.} \quad B_o T_o = \frac{1}{K} \qquad \text{Eq. (4)}$$

Since B_o and T_o have now been determined uniquely the other parameters of the optimum system may be readily determined. The filter bandwidth, b_o, will be

$$b_o = \frac{\tau B_o}{T_o} \qquad \text{or}$$

$$b_o = \frac{\tau}{KT_o^{\,2}} \qquad \text{or}$$

$$b_o = K \tau B_o^{\,2} \qquad \text{Eq. (5)}$$

Since the system has been optimized for signal-to-noise ratio it is important to have a measure of the gain that has been achieved. A convenient yardstick is the signal-to-noise ratio that would be obtained under the same conditions using the same signal intensity for a short pulse with the same range resolving power. Clearly this pulse would have a length τ, and would need a channel width $1/\tau$ to receive it. The signal-to-noise gain, S, is therefore the ratio of this channel width to the narrower channel width, b_o, in which the FM signal can be received.

$$S = \frac{1}{b_o \tau}$$

hence

$$S = \frac{1}{\tau^2} \; \frac{T_o}{B_o} \qquad \text{or}$$

$$S = \frac{1}{K \tau^2 \; B_o^{\,2}} \qquad \text{or}$$

$$S = \frac{KT_o^{\,2}}{\tau^2} \qquad \text{Eq. (6)}$$

DISCUSSION OF PARAMETERS

The equations which have been derived above enable the relative importance of the various error terms to be assessed. The coefficient, γ, is considered first since this is the only purely instrumental error. Squaring Equation (3) gives

$$1 + 2\gamma T_o + \gamma^2 T_o^2 = 1 + \frac{\gamma}{K}(\tau - \epsilon)$$

$$\text{i.e.}\quad 2 T_o \left(1 + \frac{\gamma}{2} T_o\right) = \frac{1}{K}(\tau - \epsilon) \qquad \text{Eq. (7)}$$

The fact that γ is not zero will reduce the optimum pulse length by a factor of approximately $(1 + \frac{\gamma}{2} T_o)$. Since S depends on T_o^2, the loss in signal-to-noise will be $(1 + \frac{\gamma}{2} T_o)^2$. Hence a value of (γT_o) of 2 would give about a 6 db loss, and, if the loss is to be less than 1 db, T_o must be less than 0.25. Since T_o is likely to be about 1 second, γ should be less than 0.25 cps.

The coefficient ϵ can be seen to be a time which is subtracted from τ. Thus to obtain an actual time discrimination, τ, it is necessary to design for a smaller time discrimination $\tau - \epsilon$. As is intuitively obvious this immediately shows that it is not possible to obtain a time discrimination better than ϵ. Equation (7) shows that the effect of ϵ is to reduce the optimum value of T_o by the factor $1 - \frac{\epsilon}{\tau}$. Once again this will reduce the signal-to-noise ratio by the square of this factor, so that for a 1 db loss we have $\frac{\epsilon}{\tau} = 0.11$. The loss will be 6 db for $\frac{\epsilon}{\tau} = 0.5$.

Since ϵ is likely to be due to inhomogeneities in the medium, or unevenness of the surface or bottom, the losses so introduced cannot be overcome, and will set a limit to the degree of range or time discrimination for which it is possible to aim. Some preliminary sea experiments have indicated values of ϵ in the range 2 to 5 milliseconds for propagation paths a few miles long, so that designing for values of τ less than, say, 10 milliseconds is likely to be unproductive.

It should be noted, in passing, that though non-zero values of ϵ will reduce the signal-to-noise ratio they will not necessarily affect the performance of the FM system relative to the short pulse system, since the latter may be expected to be similarly degraded by errors of this nature. Data are not yet available on which, if either, system is superior in this respect.

Assuming that γ is small, it is possible, as shown previously, to write for T_o,

$$T_o = \frac{1}{2K} \frac{\tau - \epsilon}{1 + \frac{\gamma}{2} T_o}$$

Substituting in the equation for S gives

$$S = \frac{K}{\tau^2} \frac{1}{4K^2} \frac{(\tau - \epsilon)^2}{(1 + \frac{\gamma}{2} T_o)^2}$$

$$= \frac{1}{4K} \left(\frac{1 - \frac{\epsilon}{\tau}}{1 + \frac{\gamma}{2} T_o} \right)^2 \qquad \text{Eq. (8)}$$

Apart from the relatively small correction terms already discussed, S is determined solely by K. The value of K should clearly be kept as small as possible. Doppler is one of the main factors affecting K, and, as will be discussed in a later part of this report, it is probably impracticable to make K much less than 0.005. From this $S = \frac{1}{4K} = 50$, i.e. a 17 db improvement in signal-to-noise over the corresponding short pulse system.

Equation (7) states the relation $K B_o T_o = 1$, and it can be seen that the system gain is, as would be expected, primarily determined by the product $B_o T_o$. Considering the above restriction on the value of K it can be seen that this product will not be able to exceed 200 in a practical case.

It is important to know how critical are the values of B_o and T_o in affecting S. If for simplicity the error coefficients, γ and ϵ are ignored, Equation (1) may be written

$$\tau = \frac{1}{B} + KT \qquad \text{Eq. (9)}$$

This relation must be satisfied by all systems which have the specified discrimination τ . In particular it is satisfied by B_o and T_o , so that, since $B_o T_o = \frac{1}{K}$,

$$\frac{1}{B_o} = K T_o = \frac{\tau}{2} \qquad \text{Eq. (10)}$$

Substituting in Equation (9) values of τ and K derived from Equation (10) gives

$$\frac{B_o}{B} + \frac{T}{T_o} = 2 \qquad \text{Eq. (11)}$$

for all systems giving the required discrimination. Since all the terms must be positive this immediately sets limits to the permissible values of B and T

$$B \geq B_o/2$$

$$T \leq 2 T_o$$

Outside these limits, the required discrimination cannot be obtained at all, since the echo will have too wide a spectrum. Even within this range, however, there is a considerable sacrifice in signal-to-noise, if there is much departure from the optimum values.

Since $$S_o = \left(\frac{1}{\tau^2}\right)\left(\frac{T_o}{B_o}\right)$$

and $$S = \left(\frac{1}{\tau^2}\right)\left(\frac{T}{B}\right)$$

the loss in signal-to-noise ratio on departing from the optimum is

$$S/S_o = \left(\frac{T}{B}\right)\left(\frac{B_o}{T_o}\right)$$

$$= \left(\frac{T}{T_o}\right)\left(\frac{B_o}{B}\right)$$

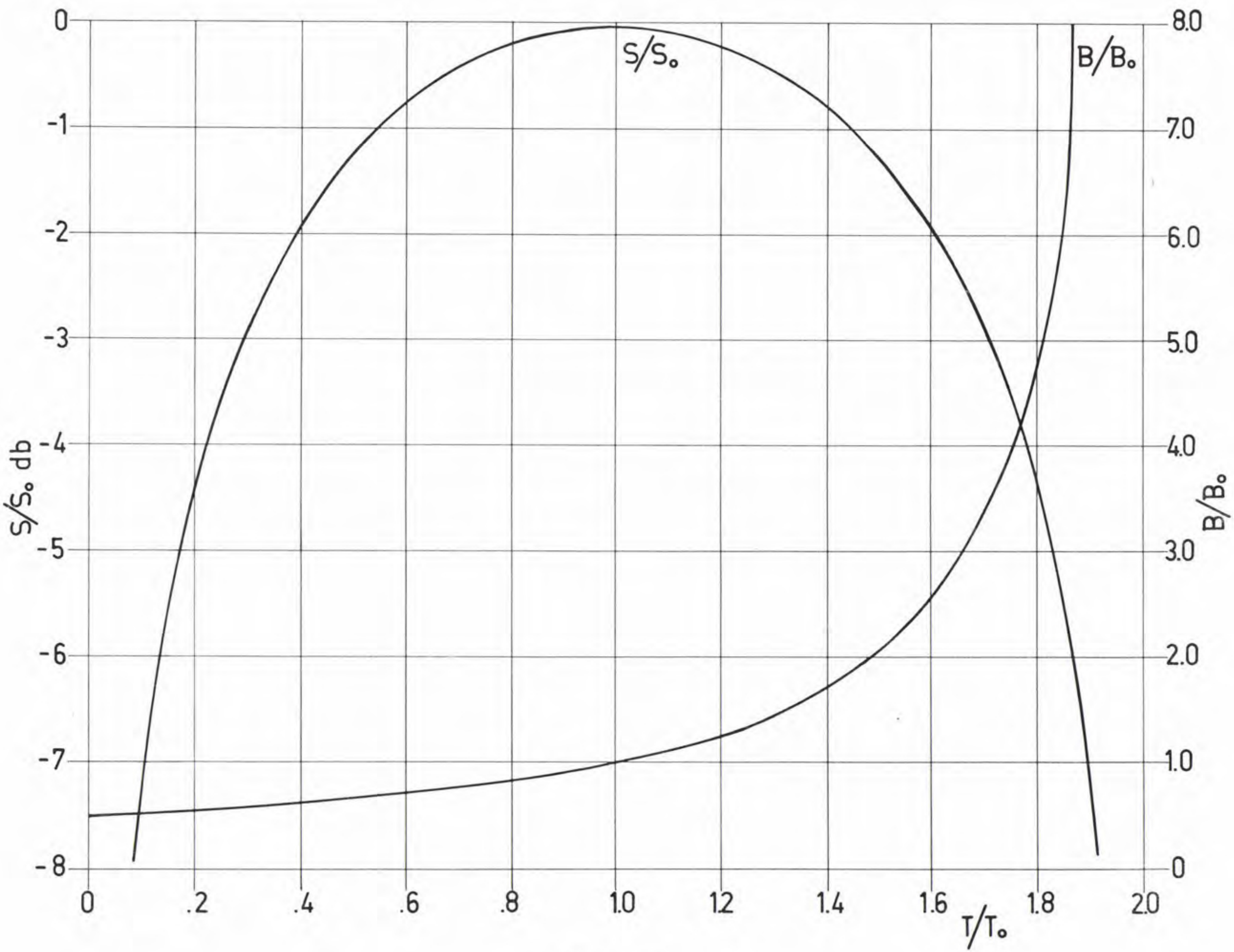
$$= \left(\frac{T}{T_o}\right)\left(2 - \frac{T}{T_o}\right) \qquad \text{Eq. (12)}$$

Figure 1 shows the variation in this ratio as T varies from its minimum value, zero, to its maximum consistent with τ , $2\,T_o$. It will be seen that the permissible range for a signal-to-noise ratio within 3 db of optimum is only

$$0.3 \leq T/T_o \leq 1.7, \quad 0.6 \leq B/B_o \leq 3.3$$

while for 1 db the parameters are determined within the limits

$$0.55 \leq T/T_o \leq 1.45, \quad 0.7 \leq B/B_o \leq 1.80$$



S/S₀
B/B₀
S/S₀ db
B/B₀
T/T₀
0
-1
-2
-3
-4
-5
-6
-7
-8
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0

Figure 1

EFFECT OF DOPPLER

The analysis so far considered assumes no relative motion between the source and the target. This will not, in general, be the case, and the effect of relative motion must now be considered. If the source has a velocity, V, and the target a velocity, v, both in the direction of transmission, i.e. from source to target, the earlier analysis may be repeated to determine the form of the received signal and its relation to the transmission.

The transmitted signal at time t is, as before, $F - \beta t$, in the time interval $0 \leq t \leq T$. Since the source is moving, this signal will enter the water with a frequency $(F - \beta t)\left(\frac{C}{C-V}\right)$.

At the instant, t, of transmission the distance between the source and the target, initially at range R, is $R + (v - V)t$. The relative velocity of the sound towards the target is $C - v$, so that the signal will reach the target at time $t + \left(\frac{R + (v - V)t}{C - v}\right)$. The target movement will alter the frequency of the signal on reflection, so that the echo will return with a frequency $(F - \beta t)\left(\frac{C}{C-V}\right)\left(\frac{C-v}{C+v}\right)$. Since the source is moving this echo will be received back at the source with a frequency

$$(F - \beta t)\left(\frac{C + V}{C - V}\right)\left(\frac{C - v}{C + v}\right) \qquad \text{Eq. (13)}$$

At the time the echo is reflected, $t + \left(\frac{R + (v-V)t}{C - v}\right)$, the distance between the target and the source will be

$$R + (v - V)\left[t + \frac{R + (v-V)t}{C - v}\right]$$

and as the relative velocity of the sound on the return path is $C + V$, the time for the return journey will be

$$\frac{1}{C+V}\left\{R + (v-V)\left[t + \frac{R + (v-V)t}{C - v}\right]\right\}$$

this reduces to

$$\frac{1}{C+V} \left\{ R + (v-V)\,t \right\} \left\{ 1 + \frac{v-V}{C-v} \right\}$$

$$= \frac{1}{C-v} \left\{ R + (v-V)\,t \right\} \left(\frac{C-V}{C+V} \right)$$

Adding this to the time of the outward journey, the time at which this element of the echo is received is

$$t + \left(\frac{R + (v - V)\,t}{C - v} \right) \left[1 + \frac{C - V}{C + V} \right] \qquad \text{Eq. (14)}$$

The signal being generated at this instant is

$$F - \beta \left[t + \frac{R + (v - V)\,t}{C - v} \left\{ 1 + \frac{C - V}{C + V} \right\} \right]$$

The difference between the generated and received signals is, therefore, to the first order in $\frac{v}{C}$ and $\frac{V}{C}$ only (since these ratios will be less than 1% in practice)

$$(F - \beta\, t) \, (1 + \frac{2V}{C}) \, (1 - \frac{2v}{C})$$

$$- (F - \beta\, t) + \left(\frac{2\,R\,\beta}{C} \right) \, (1 + \frac{v}{C}) \, (1 - \frac{V}{C})$$

$$+ 2 \, \beta\, t \, \left(\frac{v - V}{C} \right) \, (1 + \frac{v}{C}) \, (1 - \frac{V}{C})$$

$$= -2\,(F - \beta\, t) \left(\frac{v - V}{C} \right) + 2 \left(\frac{R\,\beta}{C} \right) \left(1 + \frac{v - V}{C} \right) + 2 \, \beta\, t \left(\frac{v - V}{C} \right)$$

$$= \frac{2\,R\,\beta}{C} + 2 \left(\frac{v - V}{C} \right) \left(\frac{R\,\beta}{C} - F \right) + 4 \, \beta\, t \left(\frac{v - V}{C} \right) \qquad \text{Eq. (15)}$$

From the above equations, by setting $t = 0$ or T as appropriate we can determine the time of arrival and duration, and the frequency and frequency slope of the difference between the echo and the generated signal.

Again neglecting orders of $\frac{v}{C}$ and $\frac{V}{C}$ above the first, the travel time for the leading edge of the echo will be, from Equation (14)

$$\frac{2R}{C} \left(1 + \frac{v - V}{C}\right)$$

Consequently, as in conventional sonar, the range determined by echo arrival time, in ignorance of the relative rate between the source and target, will not be R, the range at zero time, but the range at the time half way between the transmission time and the echo time.

The initial frequency difference between echo and generated signal will be

$$\frac{2 R \beta}{C} + 2 \left(\frac{v - V}{C}\right)\left(\frac{R \beta}{C} - F\right)$$

Comparing this with the stationary case it is seen that the range determined by frequency will also differ from the range at the time the transmission was initiated owing to the relative velocity. It can be seen, however, that for any selected range this difference can be made zero by a suitable choice of F and β . This is one consequence of using a downward gliding pulse. If the sign of β were changed no such cancellation would be possible.

The trailing edge of the echo is received at time

$$T + \left(\frac{R + (v - V)\, T}{C - v}\right) \left[1 + \frac{C - V}{C + V}\right]$$

and comparing this with the time of initial arrival the echo length is seen to be

$$T + \left(\frac{v - V}{C - v}\right) T \left[1 + \frac{C - V}{C + V}\right]$$

$$= T \left[1 + 2 \frac{(v - V)}{C}\right]$$

The frequency difference at the end of the echo differs from that at the start of the echo by

$$4 \beta T \left(\frac{v - V}{C} \right)$$

so that the slope of this note is

$$\frac{4 \beta T \left(\frac{v - V}{C} \right)}{T \left(1 + 2 \left(\frac{v - V}{C}\right)\right)}$$

$$= 4 \beta \left(\frac{v - V}{C}\right)$$

The effect of doppler on this FM system may, therefore, be summarized as follows:

1) The time of flight of the pulse will be increased by the factor

$$\left[1 + \frac{v - V}{C}\right]$$

2) The duration of the echo will be increased by the factor

$$\left[1 + 2 \left(\frac{v - V}{C}\right)\right]$$

3) The initial frequency difference between the echo and the generated signal will be increased by an amount

$$2 \left(\frac{v - V}{C}\right) \left(\frac{R \beta}{C} - F\right)$$

4) The difference frequency will no longer be constant but will have a slope

$$4 \beta \left(\frac{v - V}{C}\right)$$

The effect of increased time of flight is unimportant, as is the increased echo duration. The changed initial frequency difference has no effect on the range resolution, but will correspond to an overall range error. By proper selection of operating frequency and frequency slope this range error may be made zero at some pre-determined range - say a weapon firing range - and small in the vicinity of this value. The measured range will then be the range at the instant of transmission.

The effect of the change in difference frequency during the echo is more important. The total change will be

$$4 \left(\frac{v - V}{C}\right) B$$

This is a term proportional to B, and, as we have already seen in the earlier analysis, sets a limit to the signal-to-noise ratio that the system will achieve for a given range resolution. Since the relative velocity might be as high as 50 knots the coefficient $4 \left(\frac{v - V}{C}\right)$, which corresponds to K in the earlier equation, could be as high as 1/15. $S = \frac{1}{4K}$ would then be less than 4, from this cause alone. It is clearly necessary when using this FM system against moving targets to find some method of compensating for this effect. In principle this is not difficult, since it requires that the incoming echo be mixed not with the generated signal but with a signal suitably modified to take account of the changes introduced by doppler. Without discussing at this stage practical methods which might be used, it is of interest to see how far compensation in this way might be expected to improve S.

In addition to being caused by doppler the coefficient K is increased by any non-linearity in the transmitted signal, non-linearity in the signal with which the echo is mixed, and any mismatch in the slope of these two signals. The amount by which each signal might depart from its intended value will be proportional to B, the total sweep. If we call this amount αB cps then the maximum frequency spread this may cause when the two signals are mixed is $4 \alpha B$. Depending on the form of the non-linearity the probable value will be around half this.

The total value of K may then be expressed as

$$4 \left(\frac{v - V}{C} + \alpha \right)$$

Table I shows how K varies with different values of v - V and α. For convenience $\frac{1}{K} = B_o T_o$ has been tabulated, and in addition to (v - V) a figure n is shown indicating the number of different heterodyne signals that will have to be used to cover all relative velocities up to 60 knots. Whether these signals are used sequentially or simultaneously, n is a measure of the complexity, and hence the cost, of reducing K. Values of α of 0.01 are probably easy to obtain, values less than 0.001 are likely to be extremely difficult, if not impossible, in operational equipment. Similarly the complexity of the equipment, as measured by n, increases rapidly as v - V is made less than 3 knots. From the symmetry of the formula for K it is clear that $\frac{v - V}{C}$ and α should be about equal in the optimum case. The table makes it clear that $K^{-1} = B_o T_o$ is unlikely to exceed about 200, and that this can only be achieved if α can be made less than 0.001 and that sufficient channels, 30-60, are used to avoid a doppler mismatch of more than 1 - 2 knots.

	Values of $B_o T_o$				
	α				
v-V	.0005	.001	.002	005	n
1	300	188	107	47	60
1.5	250	167	100	45	40
3	167	125	83	42	20
4	137	107	75	40	15
6	100	83	62.5	36	10

TABLE I

DISCUSSION

It has been shown that the requirement for a given range discrimination coupled with the feature that the signal-to-noise ratio in each receiving channel should be a maximum, enables a unique FM sonar system to be designed. Only relatively small variations in the values of B and T, the frequency sweep and pulse duration, can be tolerated without a substantial reduction in signal-to-noise resulting. The maximum value of the product BT which can be used to advantage is likely to be about 200. The signal-to-noise gain over the short pulse system will then approach $\frac{1}{4}$ BT, i.e. 17 db. Instrumental errors and propagation inhomogeneities will tend to reduce this gain. In any practical case it is, therefore, necessary to obtain experimental data on these factors in order to assess the value of the FM system. There should, however, be many circumstances in which 14 - 15 db gain can be achieved.

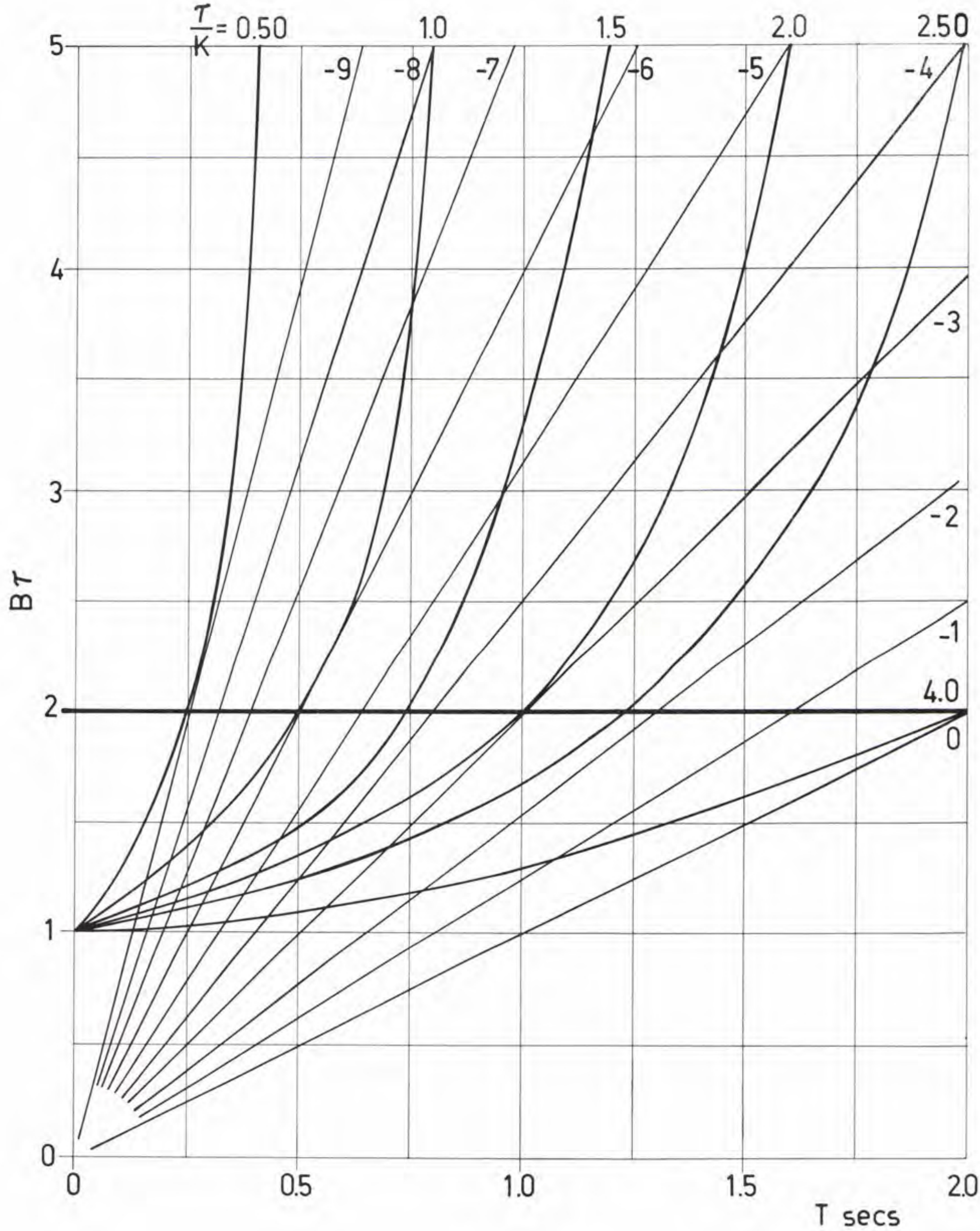
When the predominant background is reverberation the FM pulse will give the same signal-to-background as the short pulse with the same range discrimination. There is no optimum set of parameters provided the range discrimination criterion is satisfied. When designing a system in which the background may be either reverberation or noise it is first necessary to select a range discrimination small enough to make the signal-to-reverberation ratio adequate at the maximum range required. When this has been done the limiting background at maximum range will generally be noise and the optimum system can be determined as shown in this report.

If the target, or the source, is moving relative to the medium, the processing of the received echo is more complicated if maximum gains are to be achieved. A variety of heterodyne signals must be mixed with the echo, so that at least one of them has a form corresponding to a target doppler within 1 - 2 knots of the actual value for a $B_o T_o$ value of 200. Lower values of $B_o T_o$ will, of course, make the system more tolerant to doppler changes at the cost of some reduction in signal-to-noise ratio. The engineering of such a system certainly represents the most critical feature of an FM sonar.

If we ignore the small correction factors γ and ϵ, the determination of the optimum system and its dependence on the various parameters may be conveniently shown in graphical form as in Figure 2. The abscissa of this graph is the pulse length T, in seconds. Normally this must be made as large as possible in order to increase the total signal energy. The ordinate of the graph is the dimensionless quantity $B\tau$, the product of the frequency sweep and the required time discrimination. The optimum value for this product is 2, as is indicated by the horizontal line through this value. The graphs show the effect of selecting non-optimum values for B. The straight lines radiating from the origin represent lines of constant signal-to-noise ratio. As an arbitrary reference the line which passes through the point $B\tau = 2$, $T = 2$ seconds is taken as having the maximum signal-to noise ratio likely to be achieved. The other lines then show db steps below this value. For example, a system using $B\tau = 4$, $T = 1$ second would be 6 db inferior.

The family of curved lines radiating from the point $B\tau = 1$, $T = 0$, show the values of K necessary to achieve any particular combination of B and T. To make them more general these curves have been labelled in terms of $\frac{\tau}{K}$. It will be noted that the curves and straight lines are tangential at the optimum value $B\tau = 2$. In other words for a given K the best signal-to-noise ratio is obtained at $B\tau = 2$, or, conversely, for a given signal-to-noise ratio the largest value of K may be used at the optimum value of B.

An example will illustrate the application of the graphs. Suppose a time discrimination, τ, of 20 ms is required. The optimum value of B will be 100 cps. If a pulse length of 2 seconds is used a signal-to-noise ratio of 0 db can be achieved if a value $\frac{\tau}{K} = 4$ is used. For $\tau = 20$ ms this corresponds to $\frac{1}{K} = 200$, which, as has been shown earlier, is probably achievable. If the frequency sweep B were increased to 200 cps for the same pulse a value of $\frac{1}{K}$ only slightly greater than 125 would suffice, but there would be a 3 db loss in signal-to-noise. A similar loss would also occur if the pulse length T



$\frac{\tau}{K}$ = 0.50
1.0
1.5
2.0
2.5
0
-9
-8
-7
-6
-5
-4
-3
-2
-1
4.0
0
5
4
3
2
1
0
$B\tau$
0
0.5
1.0
1.5
2.0
T secs

Figure 2

were reduced to 1 second, retaining the optimum value $B = 100$ cps. In this case, however, a value $\frac{1}{K} = 100$ would suffice.

CONCLUSION

It has been shown that an FM sonar system may be designed whose performance against a noise background will be comparable with that of a system using a pure tone pulse of the same length. The optimum parametric relations for this case are derived. The FM system, however, has a range discrimination which may be about one hundred times better than a long pure tone pulse, and the signal-to-reverberation ratio will correspond to this high range discrimination.