SACLANTCEN REPORT
serial no.: SR-134

SACLANT ASW
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REPORT

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March 1988

Page count for SR-134
(excluding covers)

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Subcritical penetration of narrow Gaussian beams into sediments

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Abstract: Complete wave-theory solutions have been obtained numerically for both the reflected and the transmitted fields associated with a narrow Gaussian beam incident on a water-sediment interface near the critical grazing angle. Interesting reflectivity phenomena are observed, such as subcritical beam penetration into the sediment, nonspecular beam reflection, and lateral displacement along the interface of both the reflected and the transmitted beams. These results are in qualitative agreement with experimental observations using parametrically (nonlinearly) generated beams. It is demonstrated, however, that the fundamental reflectivity characteristics of narrow beams can be entirely explained within the framework of linear acoustics.

Keywords: beam physics ○ finite angular spectrum ○ Gaussian beam ○ grazing angle ○ linear acoustics ○ modelling ○ narrow beams ○ nonspecular reflection ○ parametric beam ○ reflection ○ safari ○ Snell's law ○ subcritical penetration ○ transmissions ○ water-sediment interface ○ wave theory

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This report is reprinted from the *Journal of the Acoustical Society of America*, **82**, 1987: 574–579.

Subcritical penetration of narrow Gaussian beams into sediments

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(Received 9 June 1986; accepted for publication 30 March 1987)

Complete wave-theory solutions have been obtained numerically for both the reflected and the transmitted fields associated with a narrow Gaussian beam incident on a water–sediment interface near the critical grazing angle. Interesting reflectivity phenomena are observed, such as subcritical beam penetration into the sediment, nonspecular beam reflection, and lateral displacement along the interface of both the reflected and the transmitted beams. These results are in qualitative agreement with experimental observations using parametrically (nonlinearly) generated beams. It is demonstrated, however, that the fundamental reflectivity characteristics of narrow beams can be entirely explained within the framework of linear acoustics.

PACS numbers: 43.30.Bp, 43.20.Fn, 43.30.Ma

INTRODUCTION

While aspects of beam physics have been studied in considerable detail over the past 15 years within the fields of optics and ultrasonics, the interest in this subject within the underwater acoustics community dates back to 1979 when Muir *et al.*[1] published results of reflectivity measurements made in a water tank with a narrow parametric beam. Several unexpected features were observed for a beam incident on the water–sediment interface near the critical grazing angle. Specifically, it was shown that for subcritical incidence, where plane-wave theory predicts total internal reflection for lossless media, considerable energy was transmitted into the bottom. It was also found that near critical incidence the transmitted beam was laterally displaced along the interface with respect to the incident beam.

These observations by Muir *et al.*[1] triggered a series of publications[2–6] that dealt with the phenomenon of subcritical penetration of narrow parametric beams into sediments. Apart from one experimental study,[6] where the effect of truncating the nonlinear interaction volume of the parametric source at the water–sediment interface was investigated, the remaining four articles[2–5] all attempt to provide a theoretical description of the observed beam reflection and transmission phenomena. Common to these articles is the fact that in formulating a tractable mathematical model the basic physics is partly obscured, and a fundamentally simple reflection problem, neatly explained in two classical articles by Bertoni and Tamir[7,8] in the early 1970s, now appears extremely complicated. As demonstrated in Refs. 7 and 8, beam problems are most easily solved by plane-wave decomposition techniques.

Three of the theoretical articles[2–4] address the beam reflection/transmission problem within the framework of linear acoustics. Of most interest to the present study are the results by Tjøtta and Tjøtta,[3,4] who solved the boundary value problem approximately for a three-dimensional pencil beam. Their results, derived without doing a proper plane-wave decomposition of the incident, transmitted and reflected fields, are in good qualitative agreement with the experimental findings of Muir *et al.*[1] for the transmitted beam, while we find inconsistencies in their results for the reflected beam.

In the articles by Wingham *et al.*,[5,6] it was concluded that the special (nonlinear) generation mechanism of the parametric beam is responsible for subcritical penetration into the sediment. (Note that Wingham uses angle of incidence instead of grazing angle, so his postcritical penetration is equivalent to our subcritical penetration.) It is repeatedly stated in Refs. 5 and 6 that subcritical penetration is a feature associated solely with parametrically generated beams and that it does not occur with conventional beams.

We do not agree with those statements, although it is clear that a parametric beam with the nonlinear interaction volume truncated at the interface will generate a different field in the bottom than would a similar conventional beam. Thus it has been experimentally shown[6] that the truncated parametic beam generates secondary arrivals in the sediment. These are, however, higher-order effects whose presence should not obscure the fact that it is the spectral properties of narrow beams that lead to subcritical penetration, independent of whether the beam is generated by a linear or a nonlinear source.

We shall here avoid formulating a new approximate theory for the beam reflection problem, and, instead, apply a general wave propagation model which has been shown to provide full wave-theory solutions to beam problems within the framework of linear acoustics.[9] Emphasis will be on the basic physics of the problem, and we shall, in particular, demonstrate that *the subcritical penetration is a simple consequence of the finite angular spectrum associated with narrow beams*. Hence, the driving factor is the spectral width of the beam, and not whether it is generated by a linear or a nonlinear source.

In Sec. I, we show numerically generated total-field solutions for a narrow beam impinging on a homogeneous sediment near the critical angle. It is seen that subcritical penetration is significant even for incident grazing angles that are 5° below the critical angle. In Sec. II, we perform a

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plane-wave decomposition of beams of different widths and relate the angular spectra to the angular reflectivity characteristics of the interface. This comparison is crucial in explaining the fundamentals of beam physics. The observed deviation from Snell's law (nonspecular reflection) is investigated in Sec. III, while the lateral displacement of reflected and transmitted beams is addressed in Sec. IV. The article ends with a summary and conclusions.

I. FIELD SOLUTIONS FROM NUMERICAL MODEL

We shall here address the problem of reflection and transmission of a two-dimensional Gaussian beam of arbitrary width at a fluid–fluid interface near the critical angle. Full wave-theory solutions are obtained with the SAFARI model,[9] which is a general seismoacoustic model of wave propagation in multilayered viscoelastic media. This model was used earlier to study aspects of beam physics in relation to reflection at fluid–solid interfaces,[9] and the present application is merely an extension to a simpler case in which there is no shear wave propagation.

In the SAFARI model, a beam is generated by a vertical source array composed by a number of equidistantly spaced line sources (Fig. 1). The beam direction is varied by phasing the source elements appropriately, and the intensity distribution across the beam can be selected by applying an amplitude weighting across the array. By varying the array distance from the interface and the number of source elements (at half-wavelength spacing), a beam of arbitrary width can be generated. Moreover, beams can be focused or defocused by phasing the source elements appropriately.

The beam reflection/transmission problem will be solved for an environment similar to the one used by Muir *et al.*[1] We consider a water half-space overlying a fluid–sediment half-space. The sound speeds are 1450 m/s in the water and 1674.3 m/s in the bottom, giving a critical (grazing) angle of 30°. The density ratio of sediment to water is 2.0 and we initially neglect attenuation in the sediment. In such a case, plane-wave theory predicts total internal reflection for grazing angles less than the critical, though with an angle-dependent phase shift. For grazing angles greater than critical, part of the incident energy is transmitted into the sediment. Field calculations are done for a source frequency of 20 kHz, corresponding to an acoustic wavelength (λ) in the water of 7.25 cm. We have chosen a source array of 121 elements with $\lambda/2$ spacing, giving an array length of 4.25 m. The distance of the array above the interface was varied as a function of beam angle so as to ensure constant beamwidth at the interface at all grazing angles. A Gaussian amplitude weighting was applied across the array, producing a Gaussian beam with low sidelobe levels.

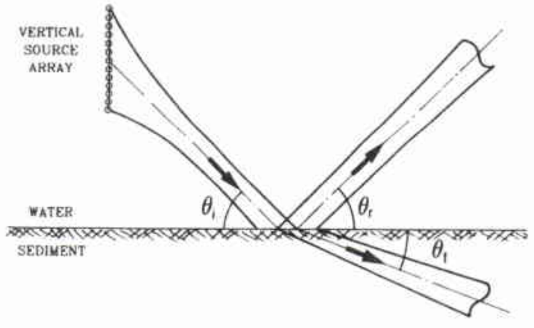

FIG. 1. Beam generation by a vertical source array.

We are going to present results for beams of widths between 2.5 and 10λ at the interface, which covers the range of beam widths (2.5–3.5λ) used in the experiment.[1] Only in one respect is our simulation of the experimental setup approximate: Although the experiment was performed with a true three-dimensional pencil beam, the acoustic model solves the two-dimensional beam problem, in which the beam is of infinite extent transverse to the plane of observation (Fig. 1). We do not, however, expect this difference in out-of-plane beam structure to significantly affect the in-plane reflectivity characteristics.

Figure 2 shows computed field solutions for a focused beam that is 2.5 λ wide at the interface. The beamwidth is measured across the beam between the 3-dB down points. The arrows in the upper part of the figure indicate the beam directions; also shown are the angles of incidence (θ_i), reflection (θ_r) and transmission (θ_t)—all measured along the maximum amplitude in a beam with respect to horizontal. Three cases are considered, $\theta_i = 25°$, 30°, and 35°, with the critical angle being 30°. Hence, the upper beam is incident at 5° below the critical grazing angle, and a transmitted beam is still present in the bottom ($\theta_t = 11.5°$). With increasing angle of incidence, more energy is being transmitted into the bottom (contours show losses in arbitrary dBs). We also notice that the angle of reflection is lower than the angle of incidence (nonspecular reflection). These beam reflection and transmission properties are in good qualitative agreement with the experimental results reported by Muir *et al.*,[1] even though we here consider a two-dimensional beam and not a pencil beam, as in the experiment.

The important result here is the observed subcritical penetration for a beam incident at 5° below the critical grazing angle. From plane-wave reflection theory, one would expect the beam to be totally reflected back into the water half-space. However, the key point is that the incident beam is not a plane wave, and, hence, cannot be expected to behave like one. In fact, a narrow beam, such as the one considered here, consists of many plane waves, and it is this wide plane-wave spectrum that causes the observed nontrivial reflection properties of beams.[7,8] This will be explicitly demonstrated in Sec. II, where we perform a spectral decomposition of Gaussian beams of different widths.

It should be emphasized that the subcritical penetration illustrated in Fig. 2 for a conventional linear source array is in close agreement with the experimental results of Muir *et al.*,[1] which were obtained using a nonlinear parametric source. This agreement indicates that it is the beam properties at the interface that determine the reflection/transmission anomalies, and not the beam generation mechanism. Tjøtta and Tjøtta[3,4] arrived at a similar conclusion.

II. SPECTRAL DECOMPOSITION OF INCIDENT FIELDS

A standard approach to the study of bounded beam physics is a plane-wave decomposition of the incident

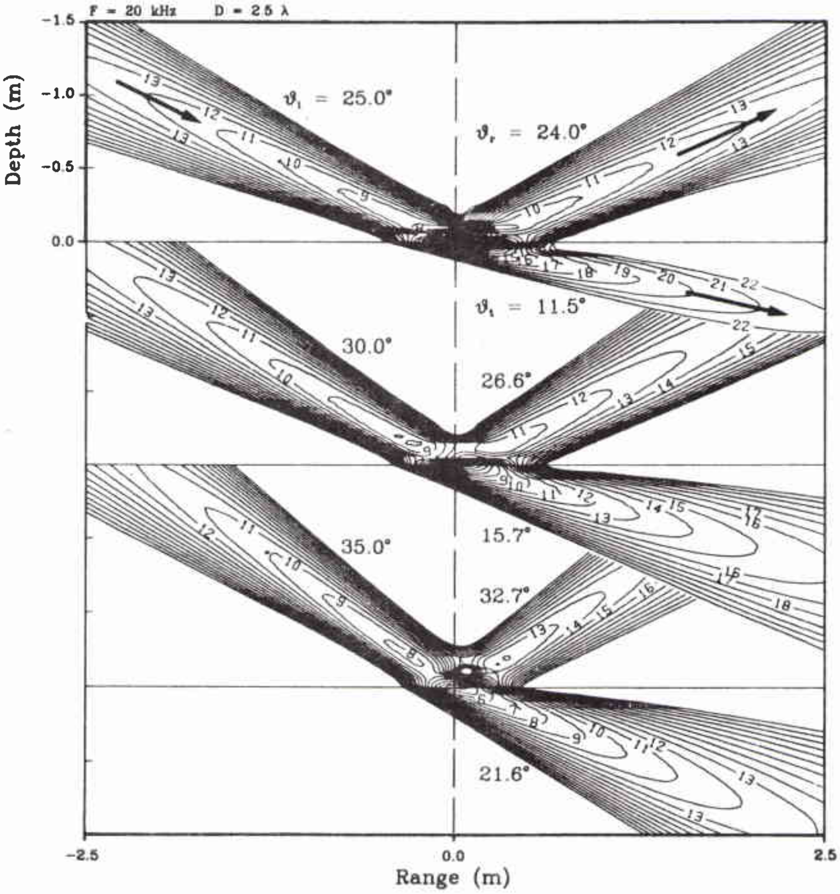

FIG. 2. Reflection and transmission of a narrow beam of sound ($D = 2.5\ \lambda$) at a water–sediment interface. The critical grazing angle is 30°, and there is no attenuation in the bottom.

field.[7,8] Since the SAFARI model[9] provides this type of information as an output option, we can display the spectral content of an incident Gaussian beam and compare it with the angular reflectivity characteristics of the interface, as shown in Fig. 3. The upper part of the figure displays the angular plane-wave spectra of three beams of different width, all incident at the critical angle. Note that the narrow beam ($D = 2.5\ \lambda$) has the widest spectrum, while the spectral width decreases with increasing beamwidth. In this angular representation an infinitely wide plane wave becomes a delta function with only one direction of propagation. Any bounded beam has a spectrum of finite width; the very narrow beams have broad spectra, indicating that energy is propagating over a wide range of angles, i.e., in many different directions. Hence, it is a contradiction to state that narrow beams are highly directional.[1,4]

For a qualitative explanation of the observed reflection and transmission anomalies for narrow beams, we shall compare the angular spectra of Fig. 3(a) with the reflectivity characteristics of the interface, as shown in Fig. 3(b). Note that we have perfect reflection below 30° with an angle-dependent phase shift, and an increasing reflection loss above

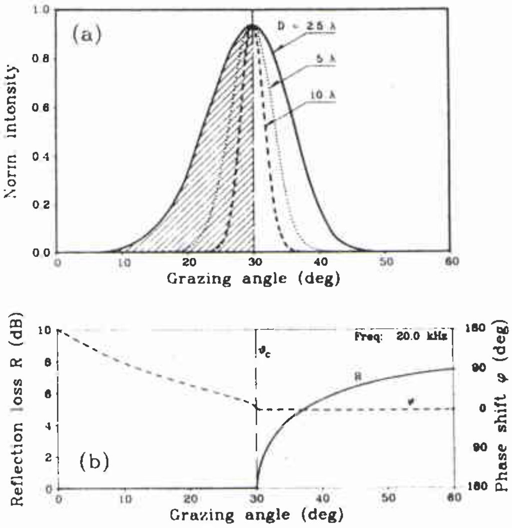

FIG. 3. (a) Angular spectra of incident beams with different half-power widths, and (b) reflectivity characteristics of the water–sediment interface.

30° with no phase shift. We shall, for a moment, concentrate on the wide spectrum in Fig. 3(a) ($D = 2.5\lambda$), which has its peak energy propagating at 30° but also has significant energy propagating at 20° and 40°. Comparison of Fig. 3(a) and (b) shows that the left-(hatched) half of the beam spectrum is perfectly reflected (though with a phase shift) while the right-half is partly transmitted. Thus, in practice, we need to weight the incident-beam spectrum with the reflectivity characteristics of the interface in order to get the spectrum for the reflected or transmitted beam. The above simple argument qualitatively explains what to expect. For example, the left (hatched) part of the spectrum will be entirely reflected, while the right-half will be partly transmitted. The different weightings put on the two half-spectra will result in asymmetry in both the reflected and the transmitted beams, which is also observed in Fig. 2. Furthermore, the peak energy in the reflected beam moves to smaller angles ($\theta_r < \theta_i$), causing nonspecular beam reflection.

It is clear from Fig. 3 that, even though each spectral component is reflected and transmitted according to Snell's law, the total beam, in which phase and amplitude weightings are applied to a broad spectrum, cannot be expected to be so. On the other hand, the beam results should approach Snell's law for increasing beamwidth (decreasing spectral width), and they do. This will be explicitly demonstrated in Sec. III.

There are a couple of additional observations to be made on the basis of Fig. 3. In the first instance, it is clear that pronounced reflection/transmission anomalies are to be expected primarily for beams incident near the critical angle, where the various spectral components are subject to strongly varying reflectivity conditions. Hence beams wider than 10 λ will essentially behave according to plane-wave reflection theory except when incident near the critical angle. On the other hand, very narrow beams, with a width approaching 1 λ, will exhibit nonplane-wave behavior at almost any angle of incidence.

These observations are crucial in explaining the experimental results of Muir *et al.*[1] They compared the transmission characteristics of a conventional beam with a width of 10–15 λ at the interface against those of a parametric beam with a width of 2.5–3.5 λ, finding subcritical penetration only for the narrow parametric beam. This is entirely consistent with our spectral considerations, while others interpreted the difference in the experimental results as being due to the different beam generation mechanisms.[5,6]

III. DEVIATION FROM SNELL'S LAW

It was seen in Fig. 2 that a symmetric Gaussian beam that is incident near the critical angle is reflected nonspecularly with a slightly asymmetric beam profile. In Fig. 4, we display both the angle of reflection and the angle of transmission versus incident grazing angle in order to explore the deviation from plane-wave reflection theory as represented by Snell's law. Results are given for three beams with widths of 2.5, 5, and 10 λ. We immediately notice that the beam results approach Snell's law for increasing beamwidth, which is fully consistent with the spectral considerations given in Sec. II. In fact, beams wider than 10 λ are essentially specularly reflected, even though there is a weak subcritical penetration. For a narrow beam with a width of 2.5 λ, on the other hand, the results deviate considerably from plane-wave theory.

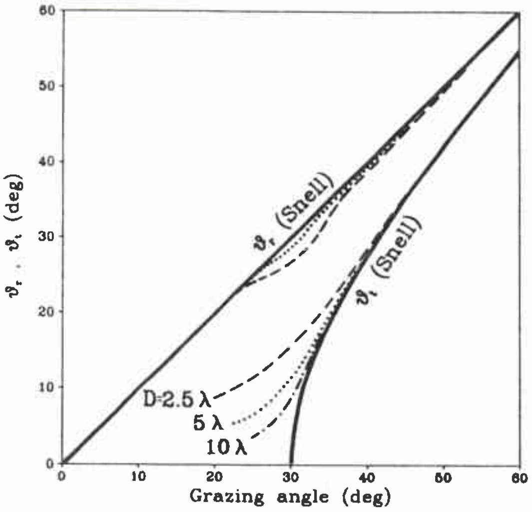

FIG. 4. Deviation from Snell's law for reflection and transmission of narrow beams at a water–sediment interface. The beamwidths are measured between the 3-dB down points of the incident beam at the interface.

We can compare the results of Fig. 4 with similar curves obtained by Tjøtta and Tjøtta[4] from an approximate theory for three-dimensional Gaussian beams. Noting that their displays are in terms of angles of incidence, we have excellent agreement for the transmission angle as a function of beamwidth (Fig. 4 of Ref. 4). Thus, for a Gaussian beam with a width of 2.5 λ, incident at a grazing angle of 14°, we get from both theories a transmission angle of 6°–7°. Comparing this theoretical result with the experimentally determined transmission angle of 25° (Ref. 1), it is evident that the agreement is poor. The reason could be both measurement errors and the fact that the experimental beam had a non-Gaussian profile. In this context, it was shown in Ref. 4 that the transmission angle is particularly sensitive to the incident beam profile.

When comparing the deviation from Snell's law for the reflected beam (Fig. 4), we always find θ_r smaller than θ_i. This result is in direct contrast to that of Tjøtta and Tjøtta,[4] who found θ_r to be larger than θ_i. The properties of the reflected beam were not measured by Muir *et al.*,[1] but the disagreement is probably due to the approximate solution applied in Ref. 4 to the boundary-value problem.

IV. LATERAL BEAM DISPLACEMENT

The lateral displacement of beams in connection with subcritical reflection is a well-known phenomenon in optics and ultrasonics.[7,8] The beam displacement concept has also been applied recently to standard ray theory, yielding ray-based results in close agreement with full wave-theory solutions.[10] In the experiment by Muir *et al.*,[1] it was noted that the incident and transmitted beam axes intersect the inter-

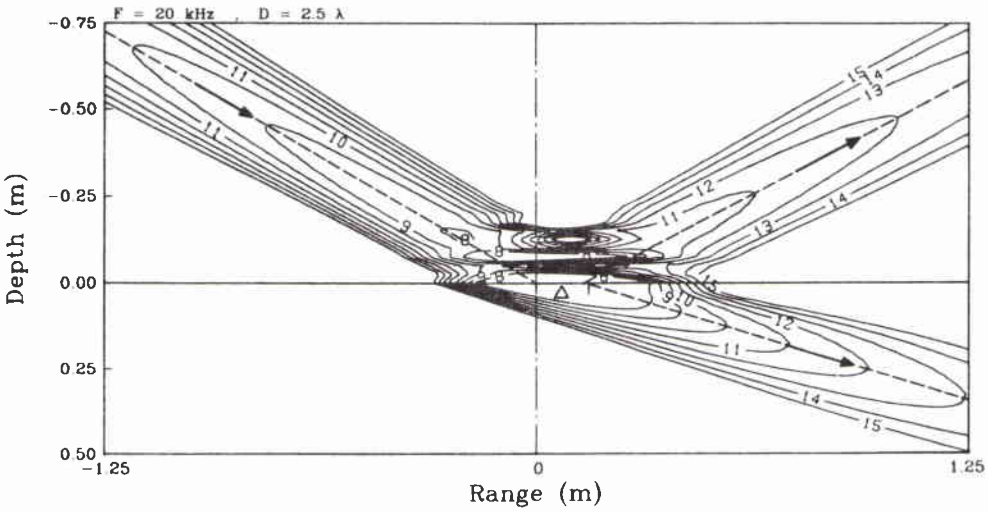

FIG. 5. Blowup of the computed sound field near a water–sediment interface at critical incidence ($\theta_c = 30°$). Note the lateral displacement of both the reflected and the transmitted beam at the interface.

face at different points with a horizontal separation of around 25 cm.

In Fig. 5, we present a blowup of the computed sound field near the interface for a narrow beam with a width of 2.5 λ incident at the critical angle. Inspection of this figure shows that both the reflected and the transmitted beam centers are displaced; this is because a lateral wave is excited when energy is incident on the interface below the critical angle.[7,8] The reflected and transmitted fields are then composed of contributions from both the "specular" beam and the lateral wave field, causing an apparent horizontal displacement of the beams. The displacement can be both forward and backward and is a function of beamwidth as well as of angle of incidence. Furthermore, reflected and transmitted beams generally have different displacements.

We see from Fig. 5 that the lateral displacement is around 15 cm for both the reflected and the transmitted beams. This result is in good agreement with the findings of Tjøtta and Tjøtta[3] for the transmitted beam, since their estimated displacement is 18 cm for an incident Gaussian beam with a width of 2.5 λ. However, they found the reflected beam to have zero displacement, which is clearly inconsistent with both our results and with several studies of beam physics within the optics literature.[7]

The theoretically determined displacement of 15–18 cm for the transmitted beam is somewhat less than the 25 cm measured by Muir *et al.*[1] There is, however, considerable uncertainty associated with a displacement measurement, and, in addition, beams are slightly curved near the interface which makes a nearfield measurement different from a farfield measurement.

Finally, it should be pointed out that we also performed beam calculations with a realistic sediment attenuation of 0.8 dB/λ. This resulted in strong attenuation of the transmitted beam in the sediment, but we noticed no substantial changes in the fundamental beam reflection characteristics, i.e., subcritical penetration, nonspecular reflection, and lateral displacement.

V. SUMMARY AND CONCLUSIONS

We have presented full wave-theory solutions for the reflected and transmitted fields associated with two-dimensional Gaussian beams incident on a water–sediment interface near the critical angle. Compared with plane-wave reflection theory the numerically generated beam results exhibit pronounced reflection and transmission anomalies, such as subcritical penetration into the sediment, nonspecular reflection, and lateral displacement of both the reflected and the transmitted beams. We have demonstrated that these results are a simple consequence of the angular spectra associated with narrow beams. In other words, the observed reflection anomalies occur simply because a beam is not a plane wave: In fact, it is a composite of many plane-wave components that are subject to different reflectivity conditions at the interface.

The beam results presented here for a conventional linear source array are shown to be in good qualitative agreement with experimental results obtained by Muir *et al.*[1] using a nonlinear parametric source. This agreement was obtained merely by matching beam spectra at the interface, which clearly indicates that it is the beam properties at the interface that determine the reflection/transmission anomalies, and not the beam generation mechanism.

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