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Passive localization using propagation models

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Passive localization with propagation models

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Abstract: Passive ranging using propagation models can be effected in two different ways: forward modeling and direct inversion. The forward-modeling method performs an exhaustive recomputation of the field over a set of assumed source locations and seeks the best match to the measured data. The direct inversion method inverts a set of linear equations constituted by the normal mode model. An overview of these two approaches is given along with a summary of studies performed to date. This is followed by the presentation of a maximum-likelihood algorithm for the direct inversion method. A discussion of the merits and weaknesses of each method is given. Also, an appendix which discusses some general aspects of the inverse problem is included.

Keywords: bearings-only ∘ deep water ∘ environmental acoustics ∘ multipath ranging ∘ passive localization ∘ propagation models ∘ sonar ∘ target tracking ∘ towed array ∘ wavefront curvature

Contents

1. Introduction

One means of obtaining improved processing performance for sonar systems is to include environmental information in the processing scheme [1–3]. In spite of the potential gains that this would offer however, it has only recently begun to be exploited. There are good reasons for this. One is that in most cases of interest sufficiently accurate environmental information has simply not been available. A second reason is that the processing load demanded by such a scheme can be prohibitive. One area where these obstacles are slowly being overcome is that of passive localization.

The standard localization techniques, which usually concentrate on range and bearing, either do not use environmental information or include it in a minimal sense. These standard techniques fall into three general categories: bearings-only, wavefront curvature and multipath.

The bearings-only approach consists of making at least two independent bearing estimates with a towed array. The range is then computed from these two estimates [4]. More general target tracking schemes based on towed array bearing estimates have been developed which provide more complete target motion analyses. These techniques, however, can be very costly in terms of processing time. The wavefront curvature technique provides an estimate of range as the radius of curvature of the wavefront under the assumption that the wave is cylindrical [5]. This can be done either with a towed array or at least three independent hydrophones in a horizontal line. The quality of the estimate depends strongly on the size of the horizontal acoustic aperture at right angles to the direction of propagation. Both the bearings-only and the wavefront curvature techiques are severely limited by the horizontal acoustic aperture available whereas the third technique, that of multipath ranging, depends on vertical aperture or horizontal aperture along the bearing line, since it relies on the ability to resolve multipath components of the fields [6]. It should be pointed out that this technique does depend on environmental information. In particular, one must know the water depth and the sound velocity profile (SVP). Thus, this technique uses a deep-water propagation model to provide an estimate of range. Like the bearings-only technique, it is based on a plane-wave assumption. The wavefront curvature technique uses the non-planar character of the wave under the assumption that it is a circular wave but uses no environmental information.

The techniques that we wish to discuss in this report constitute the next evolutionary step in that they include the environmental information in a more complete and more consistent manner. They also differ from the above techniques in that the studies carried out up to the present are based on shallow-water models.

2. Theory

2.1. BACKGROUND

If one has a model that describes the acoustic propagation in a given channel, then given sufficient information about an acoustic source in this channel and enough sufficiently accurate environmental information to provide the proper parameters for the model, the acoustic field can be computed. The accuracy of the field as computed from the model depends on two things: the goodness of the environmental parameters and the fidelity of the model. This basically describes what is known as the 'forward' problem. That is, given sufficient information about the source and the medium, compute the resulting field. The localization techniques constitute an 'inverse' problem: Given sufficient information concerning the acoustic field and the medium, compute the position of the source. This we refer to as an inverse problem of the first kind. The inverse problem of the second kind obtains when the field and the source are known and the properties of the medium are to be computed. This would be of interest in the case of active sonar, where one would be concerned with determining certain characteristics of the channel. Here, we are concerned with the inverse problem of the first kind. The inverse problem entails certain mathematical difficulties that do not appear in the forward problem. First, the solution may not be stable to small changes in either the data, the model parameters or the noise. Also, the solution may not be unique. If either or both of these situations occur, the problem is said to be ill-posed [7]. (A more complete discussion of the inverse problem can be found in the appendix.) The importance of this to the passive localization problem is that for each algorithm and model under consideration, a careful sensitivity study should be carried out in order to determine the effects of measurement and parameter errors on the localization estimate.

The present efforts in model-based passive localization are focussed on two approaches: matched-field processing and direct inversion. These will be discussed in turn, beginning with matched-field processing.

2.2. MATCHED FIELD PROCESSING

Matched-field processing is best described as the solution to an inverse problem by forward modeling. Suppose the range and depth of a point acoustic source are to be estimated from the data received on a vertical array of hydrophones. The field as predicted by the model for a source location at some arbitrary point on a specified range-depth grid is computed. This is then compared to the measured field in some manner. For example, the estimator could be the correlation between the two fields. This estimator is then computed for all range-depth combinations and plotted on a range depth map. The location of the best estimate on this map then constitutes the estimate of the true range and depth of the source.

An example of such an estimator is Bucker's detection factor [8]. This can be written as

$$D(\{\alpha\}) = P_M^\dagger(\{\alpha\})RP_M(\{\alpha\}), \tag{1}$$

where R is the covariance matrix of the measured field and $P_M(\{\alpha\})$ is the field as predicted by the model for source coordinates $\{\alpha\}$. For a vertical array of N phones, $P_M(\{\alpha\})$ is an N-dimensional complex vector and R is $N \times N$. (It should be noted here that in Bucker's original definition the diagonal of R is removed.) Equation (1) can be thought of as the power output of a beamformer 'steering vector' $P_M(\{\alpha\})$.

The detection factor is maximum when the modeled field equals the measured field. This can be seen as follows. Assuming that the estimate of the covariance matrix is obtained by a time average, we have

$$D = P_M^\dagger RP_M = P_M^\dagger \langle PP^\dagger \rangle P_M, \tag{2}$$

where we have temporarily dropped the explicit dependence on $\{\alpha\}$ for clarity. Since P_M is deterministic (for the case of a deterministic model), Eq. (2) can be written as

$$D = \langle P_M^\dagger PP^\dagger P_M \rangle = \langle (P_M^\dagger P)(P_M^\dagger P)^\dagger \rangle. \tag{3}$$

By the Schwartz inequality, the inner product $P_M^\dagger P$ is maximum when $P_M = P$.

More recent studies have considered other forms for the detection factor or 'ambiguity function' as it is called by many authors. These efforts are based on replacing the 'beamformer', P_M, in Eq. (1) with a high-resolution beamformer. In particular, attention seems to be focussing on the so-called maximum-likelihood beamformer of Capon [9]. This is a beamformer whose weights are such that the spatial spectrum is a maximum-likelihood estimate of the power being received as a function of wavenumber (angle) if the noise-only covariance matrix is known [10]. It is equivalent to the beamformer that obtains if the power is minimized everywhere except in the look direction, which is constrained to be a constant. The resulting beamformer weights are given by

$$P_{ML}(\{\alpha\}) = \frac{R^{-1}P_M(\{\alpha\})}{P_M^\dagger(\{\alpha\})R^{-1}P_M(\{\alpha\})}. \tag{4}$$

Substituting Eq. (4) into Eq. (1) yields for the ambiguity function

$$D(\{\alpha\}) = \frac{1}{P_M^\dagger(\{\alpha\})R^{-1}P_M(\{\alpha\})}. \tag{5}$$

Several investigators have used this form to estimate range and depth [11–14]. The results indicate that the lower sidelobes produced by this technique tend to reduce the aliasing problem at the cost of some detection performance.

There tends to be some confusion in the literature as to just what Eq. (5) constitutes. That is, since it is based on a maximum-likelihood beamformer, does it constitute a maximum-likelihood estimate of the source position? What Eq. (5) does constitute is a maximum-likelihood estimate of the beamformer output power for any parameter set $\{\alpha\}$. The matched-field technique then, is a graphic or numerical technique which estimates $\{\alpha\}$ by finding the maximum of this power estimate. It is by no means clear, however, that this criterion delivers a maximum likelihood estimate of source position.

2.3. DIRECT INVERSION TECHNIQUE

The direct inversion technique is based on the fact that the normal mode model of propagation permits a set of linear equations to be written that can be inverted. The source position coordinates are then extracted, usually as a second step. The technique is also sometimes referred to as a modal filter.

The normal mode model is expressed as follows:

$$P(r,z) = \omega^2 \rho_0^2 \frac{1}{\sqrt{8\pi r}} \sum_{n=1}^{N} \frac{\phi_n(z)\phi_n(z_0)}{\sqrt{k_n}} e^{-\alpha_n r + ik_n r - i\pi/4}, \tag{6}$$

where $P(r,z)$ is the acoustic pressure at depth z and range r from a point source at depth z_0, ω is the circular frequency, ρ_0 is the water density, α_n is the loss factor for mode n, and k_n is the horizontal wavenumber which satisfies $\omega^2/c^2 = k_n^2 + \gamma_n^2$. Here, c is the speed of sound and can depend on depth. γ_n is the vertical wavenumber. Equation (6) is the solution by separation of variables to the Hemholtz form of the wave equation. Using a far-field assumption in shallow water, the field is considered to be a cylindrical wave. Thus the total field is considered to be a function of r and z only. Upon separation, the equation in z becomes an eigenvalue equation given by

$$\frac{d^2\phi_n}{dz^2} + \left(\frac{\omega^2}{c^2} - k_n^2 \right) \phi_n = 0 \tag{7}$$

whose eigensolutions are the so-called modal functions. Upon solving the equation in r, which yields a Hankel function as the range-dependent part of the total solution, and taking the far-field approximation to the Hankel function, Eq. (6) obtains.

A more comprehensive discussion of the normal-mode propagation model can be found in [15].

For our purposes, knowledge of the absolute value of the field is not necessary, thus we normalize Eq. (6), yielding

$$P(r,z) = \sum_{n=1}^{N} \frac{\phi_n(z)\phi_n(z_0)}{\sqrt{k_n}} e^{-\alpha_n r + ik_n r}. \tag{8}$$

Suppose now that we wish to determine the position coordinates of the point acoustic source by means of a vertical array of hydrophones. Defining

$$M_{m,n} = \phi_n(z_m)/\sqrt{k_n}, \tag{9a}$$

$$A_n = \phi_n(z_0)e^{-\alpha_n r}, \tag{9b}$$

$$\chi_n = A_n e^{ik_n r}, \tag{9c}$$

Eq. (8) becomes

$$P_m = M_{m,n}\chi_n. \tag{10}$$

Inversion of Eq. (10) yields the N functions χ_n which in principle can be solved for the range and depth, since $M_{m,n}$ and k_n (and also α_n) can be determined from knowledge of the depth of the water, the bottom conditions and the sound velocity profile; that is, without knowledge of the source coordinates. The problem can be generalized somewhat by the addition of noise. Defining a noise vector ε_n, we have

$$P_m = M_{m,n}\chi_n + \varepsilon_m, \qquad 1 \leq n \leq N,$$
$$1 \leq m \leq M, \tag{11}$$

where M is the number of hydrophones and N is the number of modes supported by the acoustic channel.

Solving Eq. (11) by the method of maximum-likelihood, the maximum-likelihood estimation of χ_n is, for the case of $M > N$, given by [16]

$$\dot{\chi} = (M^\dagger R^{-1} M)^{-1} M^\dagger R^{-1} P, \tag{12}$$

where we have dropped the matrix and vector indices for convenience. Equation (12) is valid for gaussian noise and is sometimes referred to as the generalized least-squares estimator. R is the noise covariance matrix and is given by

$$R = E\left\{\varepsilon\varepsilon^\dagger\right\}. \tag{13}$$

Hinich used this approach to develop a technique for the estimation of the depth of a point source with a vertical array [17]. This is probably the first time that a sophisticated propagation model was used for source localization.

More recently, Shang, Clay and Wang developed a technique for range estimation based on Eq. (10) [18]. Their work is based on the observation that, since the modal matrix $M_{m,n}$ is real, it is only necessary to know the phase of the modal functions χ_n to determine range. Thus, Eq. (10) is solved directly for the functions χ_n. From Eq. (9c), one then has

$$\arg(\chi_n) - \arg(\chi_m) = \Phi_{n,m}, \tag{14}$$

where the modal phase difference $\Phi_{n,m}$ is given by

$$\Phi_{n,m} = (k_n - k_m)r - L_{n,m}2\pi. \qquad (15)$$

The term $L_{n,m}2\pi$ arises since $\Phi_{n,m}$ only contains information concerning the principal values of the phases.

A third mode is then introduced permitting the calculation of a second modal phase difference, say $\Phi_{n,p}$, where

$$\Phi_{n,p} = (k_n - k_p)r - L_{n,p}2\pi. \qquad (16)$$

The range is then eliminated between Eqs. (15) and (16) resulting in

$$\frac{\Phi_{n,m}}{2\pi} + L_{n,m} = \left(\frac{k_n - k_m}{k_n - k_p}\right)\left[\frac{\Phi_{n,p}}{2\pi} + L_{n,p}\right]. \qquad (17)$$

A search is then made to find all integers $L_{n,m}$ and $L_{n,p}$ that satisfy the condition that the integer parts and the fractional parts of Eq. (17) are simultaneously equal. All of the solutions of Eq. (17) are not valid since the sign of $\Phi_{n,m}$ is not known. Shang et al. discuss a method to deal with this problem.

A natural generalization of the work of Shang et al. is effected as follows. Starting from Eq. (12), a solution can be formally written as

$$\chi = VP_0 + V\epsilon = VP, \qquad (18)$$

where P_0 is the noise-free signal and

$$V = (M^{\dagger}R^{-1}M)^{-1}M^{\dagger}R^{-1}. \qquad (19)$$

Equation (19) is taken from Eq. (12). That is, Eq. (18) is the generalized least-squares solution to the problem. A 'solution matrix' is then defined by

$$S = \langle \chi\chi^{\dagger} \rangle, \qquad (20)$$

where the brackets indicate the time average. For the case of noise uncorrelated with the signal, substitution of Eq. (18) into Eq. (20) yields

$$S = V(P_0 P_0^{\dagger})V^{\dagger} + VRV^{\dagger}, \qquad (21)$$

where we have used Eq. (13).

This formulation has several advantages associated with it. First, it allows the data to be introduced in terms of the covariance matrix, which eliminates the time dependence from the problem. Second, it permits numerical studies of various types

of noise to be directly carried out in terms of the noise covariance matrix. Third, the arguments of the elements of the solution matrix directly provide the phase differences necessary for the solution.

Numerical studies of the algorithm of Shang et al. (Eq. (17)) indicate that it is quite sensitive to errors in the positions of the hydrophones. A more stable solution can be found as follows. From the ijth element of the solution matrix S, one can write

$$\arg(S_{ij}) = (k_i - k_j)r - N\pi. \tag{22}$$

The term $N\pi$ arises from the fact that $\arg(S_{ij})$ is only the principal value of the phase, and the true sign of χ_n is not known.

Solving for r, the result is

$$r_{ij} = \frac{\arg(S_{ij})}{(k_i - k_j)} + N\frac{\pi}{(k_i - k_j)}. \tag{23}$$

With the definitions

$$r_{ij}^0 = \frac{\arg(S_{ij})}{(k_i - k_j)}, \qquad \Delta r_{ij} = \frac{\pi}{(k_i - k_j)} \tag{24}$$

Eq. (23) becomes

$$r_{ij} = r_{ij}^0 + N\Delta r_{ij}. \tag{25}$$

By introducing a third mode, two more solutions for the range can be written, say r_{ik} and r_{jk}. If now all the multiple solutions for each of the three range solutions are computed out to some maximum range, these solutions can be counted in range bins. At the bin containing the correct range one would expect three solutions. This can be easily generalized to as many modes as described. For example using five modes, 10 pairs can be constructed so that, at the range bin containing the correct range 10 solutions should appear. A histogram can then be constructed indicating the number of solutions in each range bin. The range corresponding to the bin containing the largest number of solutions is the estimated solution. Numerical studies have indicated that this algorithm is reasonably stable to errors in the hydrophone positions.

Once the range is determined, the depth can be found by using Eqs. (9b), (9c) and (12). The procedure is as follows. Equation (9b) is used to eliminate A_n from Eq. (9c). The result is

$$\hat{\chi} = \phi_n(z_0)e^{(k_n + i\alpha_n)\hat{r}}. \tag{26}$$

Here, $\hat{\chi}_n$ is the estimate of χ_n taken from Eq. (12). The complex wavenumbers $k_n + i\alpha_n$ are known and $\hat{r}$ is the range estimate. Thus, the only unknown in

Eq. (26) is z_0, the source depth. It can be seen that the solution of Eq. (26) requires samples of the modal functions for all values of the desired depth mesh. However, the evaluation of the matrix M (see Eq. (9a)) already provides a set of samples at the hydrophone depths. Moreover, the SNAP program [19], as a matter of course, computes the modal functions for up to 501 depths uniformly distributed over the water column.

A solution to Eq. (26) by the method of least squares takes the following form:

$$\sum_{n=1}^{N} [\hat{\phi}_n - \phi_n(z_m)]^2 = J_m.$$ (27)

The quantity J_m can be plotted as a function of m with its minimum providing the estimate of source depth. The vector $\hat{\phi}_n$ is the estimate of $\phi_n(z_0)$ taken from Eq. (26) and $\phi_n(z_m)$ is the matrix of computed values of the modal functions. N is the number of modes. In practice, the quantities $\hat{\phi}_n$ and $\phi_n(z_m)$ must be normalized, since the absolute value of $\hat{\chi}$ is unknown.

3. Experimental work

There have been several successful full-scale localization experiments to date, all using the matched-field technique. As for the direct inversion method, there has been one successful scale-model experiment.

Fizell and Wales [12] report a successful localization using a 980 m array in the upper part of the water column in water of depth somewhat greater than 2800 m. This experiment took place in an arctic environment, where both the array and the source were suspended from the ice cover. The 20 Hz source was correctly located at a range of 270 km. The signal to noise ratio was quoted as 'high'. The propagation model used was a fast field program.

Feuillade and Kinney [13] report on an experiment performed off Panama City, Florida, involving a 450 Hz source at a range of 2.2 km and a depth of 31 m in 31.5 m of water. The array was approximately 26 m in length. The model used was SNAP [19]. The results indicated a biased range estimate (2.42 km) and some ambiguity due to multiple peaks of the ambiguity surface.

Bucker [20] reports a successful localization experiment using a 3 kHz source in water with a depth of 66 ft. He reports good localization out to ranges of at least four times the water depth.

Finally, Shang et al. [18] report on a scale-model experiment with a 3500 Hz source at a range of 1.8 m in air. The acoustic channel was a waveguide formed by two parallel plaster plates with a separation of 0.123 m. The experimentally determined range, using their direct inversion approach, was 1.79£ m. Three modes were used in the calculation. The vertical array was actually a single receiver which was used as a synthetic array.

4. Discussion

As pointed out in the introduction, from a logical point of view, both the matched-field processing method and the direct inversion method are considered inverse problems. However, in practice only the direct inversion method is considered as an inverse problem. The reason for this is that much of the literature concerning inverse problems deals with the mathematical issues that are unique to the method of direct inversion. These issues are, to a large degree, concerned with the so-called 'ill-posed' problem. A more complete discussion of this is given in the appendix. It is of concern to us here since it manifests itself in the method of direct inversion. From Eq. (12), we see that the solution requires the inversion of a matrix. Even in the case of white noise where the covariance matrix drops out we must invert the matrix $M^\dagger M$. When the data do not provide a complete enough sample, this matrix becomes singular. In particular, when the array is vertical studies have shown that, at least at low frequencies, nearly the complete water column must be sampled to stabilize the problem [21, 22]. In the matched-field technique this does not occur. A more 'graceful degradation' of the precision of the estimate occurs instead. Another advantage of the matched-field method is that one is not limited to the normal-mode model. In principle any model of any degree of sophistication can be used. In particular, Baggeroer et al. [11] use a full-field model which includes the continuous part of the spectrum as well as the modal solution [23]. The direct inversion method, on the other hand, although limited to the range dependent case, allows a direct solution to be computed without an exhaustive recomputation of the field. In particular, when only the range is desired, the computation consists of a single matrix inversion (Eq. (12)) followed by the evaluation of Eq. (23) for all relevant values of N. The determination of depth however requires a search over source depth. Thus a two-dimensional search is replaced by two one-dimensional searches. Another possible advantage of the direct inversion method is its performance against noise. The matched-field techniques appear to require S/N values as high as 10 dB where a study of the S/N performance of the direct inversion method produced solutions with S/N ratios as low as −13 dB [22].

As pointed out in Subsect. 2.3, the solution of Eq. (12) provides a maximum-likelihood solution to the problem for the case of gaussian noise. This can be seen as follows. Equation (12) constitutes a linear problem in the vector χ, where χ

is given by $\chi_n = A_n e^{ik_n r}$ (Eq. (9c)). Thus, solution of Eq. (12) provides a direct maximum-likelihood estimation of χ, which we designate as $\hat{\chi}$. Maximum likelihood estimators possess the property of invariance under parameter transformation [24]. This means that if α is the parameter of interest and we have the maximum-likelihood estimate of χ, designated as $\hat{\chi}$, then

$$\hat{\chi} = \chi(\hat{\alpha}). \tag{28}$$

This can be seen from the following argument. Given the appropriate likelihood function $L(\alpha)$ (or its logarithm) the estimate of α is the solution of

$$\frac{\partial L(\alpha)}{\partial \alpha} = 0. \tag{29}$$

But this can be written as

$$\frac{\partial L}{\partial \alpha} = \frac{\partial L}{\partial Z} \frac{\partial Z}{\partial \alpha}, \tag{30}$$

where $Z(\alpha)$ is a parameter that permits a more convenient solution. As can be seen by inspection of Eq. (30), that value of α that satisfies

$$\frac{\partial L}{\partial Z} = 0 \tag{31}$$

automatically satisfies Eq. (30). Thus,

$$\hat{Z} = Z(\hat{\alpha}), \tag{32}$$

which is equivalent to Eq. (28). Of course, the question of whether one has a global maximum or not still remains.

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Appendix A
Inverse problems

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Generally, acoustic signal processing is concerned with both detection and estimation. Detection is the determination of the existence or non-existence of a signal at the receiver whereas estimation is the determination of certain parameters or descriptors of the signal, the medium or its contents. Certain estimation problems fall into a class of problems commonly referred to as 'inverse problems'. This is not a well-defined concept and is best defined in terms of its related 'forward problem'. The problem under discussion here is the passive ranging problem. The forward problem then, would consist of computing the acoustic field radiated from a source of known location in a known acoustic channel. The concomitant inverse problem in one form is concerned with the determination of the location of the source, based on the measurements of the acoustic field and the known properties of the acoustic channel. The inverse problem will, almost always, entail much more computational labour than the forward problem. Most linear inverse problems can be written in the form of a Fredholm equation of the first kind:

$$f(X) = \int K(X, X')g(X') \, dX', \tag{A1}$$

or in its discrete form

$$f_i = K_{ij}g_j. \tag{A2}$$

If Eq. (A1) represented an acoustic scattering problem, then $K(X, X')$ would represent the acoustic energy scattered from a differential volume dX' or 'source' at the point denoted by the vector coordinate X' and $g(X)$ would represent the distribution of sources and therefore the distribution of scattering medium. The scattered field would be given by the function $f(X)$. The forward problem, then, would be concerned with determining $f(X)$ given some information about $g(X)$ and $K(X, X')$. An example of an inverse acoustic scattering problem would be the determination of $g(X)$ given some information about $K(X, X')$ and measured values of $f(X)$. Sometimes $K(X, X')$ is called Green's function and here would describe the acoustic field scattered from a point 'source'.

In the case of the passive localization problems, Eq. (21) reduces to the form of Eq. (10) where one makes the identification

$$f_i = P_i, \tag{A3}$$

$$M_{ij} = K_{ij}, \tag{A4}$$

$$g_i = \chi_i. \tag{A5}$$

Because of its generality, the inverse problem does not lend itself to a formal structure in the same sense as do detection and estimation. It is probably more realistic

to consider it as a collection of techniques. There are, however, some insights that can be gleaned from examination of Eq. (A1). First, it is clear that it is generally an integral equation problem. Except for rare special cases, solution in closed form will not be possible, so that a numerical solution will be called for. If the data are in discrete form, it becomes a problem in linear equations. Also, there can arise severe mathematical pathologies in the solution of such problems. One of these is that the problem can be what mathematicians refer to as ill-posed. This is a problem that can be due to noisy data, measurement errors, not enough data, or a combination of these. A problem is well posed when a unique solution exists that is stable to changes in the data. The problem of nonuniqueness can, in many cases, be dealt with by introducing a priori information.

The stability problem arises when, given the existence of a solution, the solution is extremely sensitive to small perturbations in the data. As an example, consider a measurement of $f(X)$ denoted by $f(X_1)$. Let this measurement incur an error Δf and let f_0 be the true value. Then,

$$f(X_1) = f_0 + \Delta f = \int K(X_1, X) \left[g(X) + \Delta g \right] dX, \qquad (A6)$$

$$f_0 = \int K(X_1, X) g(X) dX. \qquad (A7)$$

Subtracting Eq. (A7) from Eq. (A6) yields

$$\Delta f = \int K(X_1, X) \Delta g \, dX. \qquad (A8)$$

It can be seen from Eq. (A8) that Δf can be thought of as a weighted average of Δg where $K(X_1, X)$ is the weight. Thus, one is free to select a function Δg whose weighted average is as close to zero as desired, but can produce large errors on $g(X)$.

References

[1] MERMOZ, H. Complementarity of propagation model design with array processing. *In:* TACCONI, G., *ed.* Aspects of Signal Processing with Emphasis on Underwater Acoustics. Proceedings of the NATO Advanced Study Institute held at Portovenere, La Spezia, Italy, 30 August–11 September 1976, part 1. Dordrecht, The Netherlands, Reidel, 1977: pp. 463-468. (NATO Advanced Study Institutes Series: Series C—Mathematical and Physical Sciences, volume 33).

[2] MERMOZ, H. Spatial processing beyond adaptive processing. *Journal of the Acoustical Society of America*, **70**, 1981: 74–79.

[3] BREKHOVSKIKH, L.M. Some problems of oceanic acoustics. *Izvestia Atmospheric and Oceanic Physics*, **4**, 1968: 739–746.

[4] CHOU, S.J. Anti-submarine warfare (ASW) passive target tracking. NOSC Report 3512. San Diego, CA, Naval Ocean Systems Center, Jan 1980. [AD B 057 688]

[5] CARTER, G.C. Variance bounds for passively locating an acoustic source with a symmetric line array. *Journal of the Acoustical Society of America*, **62**, 1977.

[6] GONG, K.F. Multipath target motion analysis: properties and implication of the multipath process. NUSC TR 6657A, New London, CT, Naval Underwater Systems Center, 1982.

[7] SARKAR, T.K., WEINER, D.D. and JAIN, U.K. Some mathematical considerations in dealing with the inverse problem. *IEEE Transactions on Antennas and Propagation*, **29**, 1981: 373–379.

[8] BUCKER, H.P. Use of calculated sound fields and matched-field detection to locate sound sources in shallow water. *Journal of the Acoustical Society of America*, **59**, 1976: 368–373.

[9] CAPON, J. High resolution frequency-wavenumber spectrum analysis. *Proceedings of the IEEE*, **57**, 1969: 1408-1418.

[10] MARZETTA, T.L. A new interpretation for Capon's maximum likelihood method of frequency-wavenumber spectral estimation. *IEEE Transactions on Acoustics, Speech and Signal Processing*, **31**, 1983: 445–449.

[11] BAGGEROER, B. KUPERMAN, W.A. and SCHMIDT, H. Performance bounds on array processing for source localization using full wave modeling of signal and noise fields. Proceedings of the Symposium on Computational Acoustics, Yale University, 6–8 August 1986.

[12] FIZELL, R.G. and WALES, S.C. Source localization in range and depth in an arctic environment. *Journal of the Acoustical Society of America*, **78**, 1985: S57–S58: CC5.

[13] FEUILLADE, C and KINNNEY, W.A. Source localization in a waveguide using a matched-field technique. *Journal of the Acoustical Society of America*, **78**, 1985: S30: N6.

[14] KLEMM, R. Range and depth estimation by line arrays in shallow water. *Signal Processing*, **3**, 1981: 333–344.

[15] CLAY, C.S. and MEDWIN, H. Acoustical Oceanography. New York, John Wiley & Sons, 1977.

[16] WHALEN, A.D. Detection of Signals in Noise. New York, Academic Press, 1971.

[17] HINICH, M.J. Maximum likelihood signal processing for a vertical array. *Journal of the Acoustical Society of America*, **54**, 1973: 499–503.

[18] SHANG, E.C., CLAY, C.S. and WANG, Y.Y. Passive harmonic source ranging in waveguides by using mode filter. *Journal of the Acoustical Society of America*, **78**, 1985: 172–175.

[19] JENSEN, F.B. and FERLA, M.C. SNAP: The SACLANTCEN normal-mode acoustic propagation model. SACLANTCEN SM-121. La Spezia, Italy, SACLANT ASW Research Centre, 1979. [AD A 067 256]

[20] BUCKER, H.P. Experimental test of acoustic localization in shallow water. *Journal of the Acoustical Society of America*, **77**, 1985: S71: GG9.

[21] WILSON, B.R. and VIDMAR, P.J. Matched mode method of passive source localization. *Journal of the Acoustical Society of America*, **77**, 1985: 571.

[22] SULLIVAN, E.J. and RAMEAU, K. Passive ranging as an inverse problem: A sensitivity study. SACLANTCEN SR-118. La Spezia, Italy, SACLANT ASW Research Centre, 1987.

[23] JENSEN, F.B. and SCHMIDT, H. Review of numerical models in underwater acoustics, including recently developed fast-field programs. SACLANTCEN SR-83. La Spezia, Italy, SACLANT ASW Research Centre, 1984.

[24] COX, D.R. and HINKLEY, D.V. Theoretical Statistics. London, Chapman and Hall, 1982: p. 287.

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