THEORETICAL CALCULATION OF TRANSMISSION LOSS IN THE OCEAN

by

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SUMMARY

The topic that is dealt with in this paper is a method for computation of acoustic transmission loss by sound propagation in the ocean. The method is based on the ray theory which is used in connection with an acoustic model of the ocean, the parameters of which are approximated by mathematical expressions. On this basis formulae for ray path coordinates and intensity are developed. Some computer programs have been written to carry out the numerical calculations. Results of this mathematical method are compared with practical measurements.

INTRODUCTION

Prediction of sound transmission conditions in the sea is frequently required, be it for high sonar frequencies and short ranges as well as for the low frequencies used in long range sonar and in long range passive detection systems. To meet this requirement it is first of all necessary to construct a model of the ocean comprising all the parameters which are significant for sound propagation. This work has been concerned with the task of construcing such a model. Its parameters are expressed mathematically in order to simplify the calculations. Some approximations are thereby introduced because the ocean is too complex to be completely described by mathematical laws. There are parameters which cannot be taken into account at all.

From a transmission point of view the basic parameters of a sea area are the bathymetry and the hydrographic conditions expressed by the sound velocity distribution. The acoustic properties of the ocean bottom are also of considerable interest because transmission by reflected ray paths plays an important part in many cases. Equally important for reflected paths is the scattering effect of the corrugated sea surface. Finally, high frequency signals are considerably attenuated due to energy absorption in the water volume.

According to the points of view briefly outlined above the calculation of sound propagation and sound intensity is carried out under consideration of the following laws and parameters [Fig. 1]:

a. The law of geometric spreading formulated by the ray theory.

b. Vertical and horizontal sound velocity distributions.

c. Bottom topography.

d. Energy loss by bottom reflection dependent on frequency and angle of incidence.

e. Energy absorption in the water volume dependent on frequency.

f. Energy scattering loss by reflection from the sea surface dependent on frequency and sea state.

The effects of reverberation and short-time variation of the hydrographic conditions are ignored.

<u>BASIC FORMULAE</u>

At this point the basic formulae for propagation and intensity ought to be reviewed [Fig. 2]. The ray path coordinates and the intensity are expressed by the well-known formulae [Ref. 1]

$$x_i = \int_{z_i}^{z_{i+1}} \frac{dz}{\sqrt{\frac{c_m^2}{c^2(z)} - 1}} , \qquad \text{[Eq. 1]}$$

where $c_m = \text{constant} = \frac{c(z_i)}{\cos \Phi_i}$,

and where x_i is the step in the horizontal direction taken by the ray between the depths z_i and z_{i+1} .

Sometimes the propagation time is of interest. It may be expressed by

$$t_i = \frac{\cos \Phi_i}{c(z_i)} \cdot \left(x_i + \int_{z_i}^{z_{i+1}} \sqrt{\frac{c_m^2}{c^2(z)} - 1} \, dz \right) \qquad [\text{Eq. } 2]$$

Referring to Fig.2, if there is a source at the point $(0, z_i)$ and the intensity of the source at unit distance is P, then the intensity I at the point (x_i, z_{i+1}) may be expressed by

$$\frac{I}{P} = \frac{\cos \Phi_i}{x_i \cdot \left| \frac{\partial x_i}{\partial \Phi_i} \right| \cdot \left| \sin \Phi_{x_i} \right|} \qquad [\text{Eq. } 3]$$

In the case of a number of n layers z_i and consequently $(n-1)$ steps x_i between two depths z_1 and z_2 , Eq. 2 may be developed in the form

$$\frac{I}{P} = \frac{\cos^2 \Phi_i}{\sum_1^{n-1} x_i \cdot \left| \sin \Phi_1 \right| \cdot \left| \sum_1^{n-1} \left[x_i + \int_{z_i}^{z_{i+1}} \frac{dz}{\left(\sqrt{\frac{c_m^2}{c^2(z)} - 1} \right)^3} \right] \right| \cdot \left| \sin \Phi_2 \right|} \qquad [\text{Eq. } 4]$$

MATHEMATICAL APPROXIMATION TO SOUND VELOCITY PROFILE

Considering Eq. 1, it may be integrated if the integrand, for example, can be brought into the form

$$\frac{1}{\sqrt{A + 2Bz + Cz^2}} \quad .$$

The horizontal step x_i, the intensity and propagation time can then be expressed by fixed integrals. Here the function $c(z)$ expresses the vertical sound velocity distribution. It is not possible, in general, to describe a complete depth-velocity profile by means of a single mathematical expression. The method used here [Refs. 2 and 3], is to divide the profile into segments and to make a curvilinear approximation to each segment by means of a quadratic equation, viz.,

$$\frac{1}{c^2(z)} = A1_i \, z^2 + A2_i \, z + A3_i \quad . \qquad \text{[Eq. 5]}$$

The coefficients are valid for the segment between the levels z_i and z_{i+1} [Fig. 3]. Differentiation of Eq. 5 with respect to z gives the sound velocity gradient g_i :

$$-2 \frac{g_i}{c^3(z)} = 2A1_i \, z + A2_i \qquad \text{[Eq. 6]}$$

where $g_i = \frac{dc(z)}{dz}$ at the depth z_i .

The approximation is performed in such a way that the sound velocity gradient at the levels z_i and z_{i+1} is preserved. The velocity at z_i is chosen as the third parameter. To determine the coefficients A1, A2 and A3, then, one equation of the form [Eq. 5] and two of the form [Eq. 6] are used. The velocity at the level z_{i+1} is computed and used as the known sound velocity for the next segment.

One result of this procedure is presented in Fig. 4. It shows a measured sound velocity profile and the corresponding calculated approximation. The total depth is divided into 17 layers. A maximum error of 0.50 m/s occurs at 15 m depth. For the rest of the curve the error is less than the accuracy with which the sound velocity is measured.

MATHEMATICAL APPROXIMATION TO THE BOTTOM PROFILE

Propagation of sound energy via ray paths which are reflected from the ocean floor contributes to the energy transmission between two positions. Compared with direct going paths, however, this contribution is reduced because the reflection process is associated with energy loss. The loss is a function of the frequency and of angle of incidence. In order to evaluate the influence of a reflected wave, therefore, it is important to know the slope of the bottom at the point of incidence. It is an advantage if the bottom profile can be described mathematically with sufficient accuracy, because then the slope can be found by simple differentiation of some mathematical expression.

An idea which immediately suggests itself is to divide the bottom profile into segments. Considering Fig. 5 there are 4 parameters at disposal for each segment, viz., the depths and the slopes at distances x_i and x_{i+1}. It is thus possible to use a polynomial with 4 coefficients as an approximation to the measured profile, i.e, a third degree polynomial

$$y = B1_i\, x^3 + B2_i\, x^2 + B3_i\, x + B4_i \quad . \qquad [Eq.\ 7]$$

There will be a set of coefficients belonging to each value of i. The bottom ordinates are denoted by y to avoid confusion with ray path ordinates.

The example in Fig. 6 shows a measured and the corresponding computed profile. The difference between the two curves is insignificant, i.e., it is nowhere greater than about 1 m.

BOTTOM REFLECTION LOSS

The importance of the loss caused by reflection from the bottom has been briefly mentioned earlier. The loss as a function of frequency and angle of incidence (or of grazing angle) depends on the properties of the bottom and will in general vary from one locaility to another. Thus, for example the reflectivities of rock, sand and layered sediments will differ. However, a discussion of the reflectivity as a function of bottom material is outside the scope of this paper. We will study some type of loss curve and try to approximate it by mathematical expressions. Then, at a specified frequency and angle of incidence the reflection loss can be easily found.

We may, for example, consider the loss as a function of grazing angle given in Fig. 7. It represents measured loss in the frequency band 600 Hz - 1200 Hz. A little investigation shows that two parabolas will be a very good approximation to the measured curve, as shown in Fig. 8. The equations for the parabolas are:

$$0^\circ - 40^\circ : \quad \alpha = 11.247\ \Phi^2$$
$$40^\circ - 90^\circ : \quad \alpha = -6.039\ \Phi^2 + 20.072\ \Phi - 5.590 \quad ,$$

where the grazing angle Φ is expressed in radians. Thus, for the frequency band 600 Hz - 1200 Hz and for the sea area where the loss was measured the two parabolas will give the bottom reflection loss per bounce. In an area where the type of bottom varies, several such loss functions may be used.

SURFACE SCATTERING LOSS

In the case when the sound energy is propagating upwards and makes contact with the sea surface the transmission will be influenced by the sea state. The energy will be scattered. The extent to which the scattering takes place is also dependent on the frequency. The problem is dealt with by many authors.

What we want here is a function or a set of functions expressing the surface scattering loss dependence on frequency, wave height and angle of incidence. Such data were not available at the time when this work was done. The only result that seemed to be useful was a curve theoretically computed by Marsh, Shulkin and Kneale [Ref. 4], giving the loss per surface bounce as a function of a frequency-wave height parameter. It does not express any dependence on the geometry of source and receiver. The curve, which is reproduced in Fig. 9, is used in this work although it is based on criticized assumptions. It is easy, in any case, to replace this loss function by more reliable data as soon as they are available. For practical use the curve is redrawn in Fig. 10 with a frequency scale corresponding to wave height h = 3 (sea state 3).

It is seen that the curve approximates a hyperbola. If the frequency is denoted by f and the loss by α_s, then the hyperbola can be expressed by

$$-f^2 + C\alpha_s^2 + 2Df + 2E\alpha_s + F = 0 \qquad \text{[Eq. 8]}$$

In Fig. 10 this expression is valid above the frequency 500 Hz. In the range 0 - 500 Hz the loss is assumed to be zero. For each value of wave height h there is a set of values of the constants C, D, E and F. For h = 3 ft and expression for α_s is

$$\alpha_s = -0.9375 + \sqrt{1.7226 - 3.375\ 10^{-3} f + 3.375\ 10^{-6} f^2}$$

ABSORPTION LOSS IN SEA WATER

The absorption of sound energy in sea water is dealt with by many authors. In Ref. 5, for example it is suggested that the following formula should be used for practical computations of the loss (dB/m):

$$\alpha_{abs} = \left(\frac{S A f_T f^2}{f_T^2 + f^2} + \frac{B f^2}{f_T^2}\right) \cdot (1 - 6.54\,10^{-4} P)\ 8.686 \qquad \text{[Eq. 9]}$$

In Eq. 9, A is constant, $2.34\ 10^{-6}$, S is salinity in parts per thousand, f_T is the temperature-dependent relaxation frequency in kHz at atmospheric pressure, $21.9\ 10^{\left(6 - \frac{1520}{T+273}\right)}$, where T is in °C, f is the acoustic frequency in kHz, B is approximately a constant for the viscosity mechanism, $3.38\ 10^{-6}$, and P is the pressure in kg/cm^2.

It is shown in Ref. 6 that this formula gives too small values for the attenuation at low frequencies.

The absorption problem is extensively treated in Ref. 7 where the loss is expressed by

$$\alpha_{abs} = 0.006\ f^2 + \frac{0.155\ f_r\ f^2}{f_r^{\ 2} + f^2} \qquad [\text{Eq. } 10]$$

where α_{abs} is in decibels per km, the signal frequency f is in kHz and $f_r = 1.7$ kHz. The term $0.006\ f^2$ represents the absorption of magnesium sulphate relaxation. The second term accounts for a relaxation process at 1.7 kHz. In this work the absorption loss is taken into account by using Eq. 10 and expressing the distance in metres and the frequencies in Hz, giving the absorption loss in dB/m as

$$\alpha_{abs} = 10^{-9}\ (0.006 + \frac{155\ f_r}{f_r^{\ 2} + f^2})\ f^2 \qquad [\text{Eq. } 11]$$

NUMERICAL COMPUTATION

General

Numerical computation of ray path coordinates, intensities and transmission loss is carried out by means of a system of 4 FORTAN programs. The first performs the curvilinear approximation to the measured depth-velocity profiles. The second program computes the coefficients of the third degree polynomials which are used as approximation to the bottom profile. The results of these two programs are checked before they are inserted into further calculations.

Generally, for a given number N of bottom reflections, four possible ray paths exist between a source and a receiver located at depth in the sea. In addition to the reflected paths there may be refracted paths. A third program computes the angles of emission at the source corresponding to the values of N, bathymetry and distances between source and receiver.

The fourth and main program, IRATRA, is based on the mathematical expressions dealt with earlier. Some of the variables and parameters used in the main program are obtained from the results of the preceding three programs. In addition, several other data are introduced, for example number of layers, signal frequency, wave height, horizontal variation of the sound velocity. The main program also comprises three sub-programs which, when called, produce the values of absorption, scattering and reflection losses.

The output of the main program may comprise values for the ray path coordinates, the ray angle, the propagation time, the intensity and the bathymetry between source and receiver.
In addition, at each specific distance the program may sum up the amounts of energy which have propagated along all ray paths and present the result as the transmission loss for the signal frequency and sea state in question.

one of the many details of the computing precedure in IRATRA will be discussed below.

Calculation of the Ray Path Coordinates

It was shown earlier that the expression

$$\frac{1}{\sqrt{\frac{c_m^2}{c^2(z)} - 1}}$$

could be brought into the form

$$\frac{1}{\sqrt{A + 2Bz + Cz^2}}$$

by expressing $1/c^2(z)$ as $A1\, z^2 + A2\, z + A3$. It is remembered that $c_m = c_0/\cos\varphi_0$. This gives

$$\frac{c_m^2}{c^2(z)} - 1 = c_m^2\, A1\, z^2 + c_m^2\, A2\, z + c_m^2\, A3 - 1$$

from which we obtain

$$A = c_m^2 A3 - 1$$
$$B = 0.5\, c_m^2 A2$$
$$C \quad 2 \quad c_m^2 A1$$

The integration

$$\int \frac{dz}{\sqrt{A + 2Bz + Cz^2}}$$

may have the following results:

$$\text{(i)} \quad \frac{1}{\sqrt{C}} \ln(B + Cz + \sqrt{C} \cdot \sqrt{A + 2Bz + Cz^2}) + K, \quad C > 0$$

$$\text{(ii)} \quad \frac{1}{\sqrt{C}} \operatorname{arsinh} \frac{B + Cz}{\sqrt{AC - B^2}} + K, \quad (AC - B^2) > 0$$

$$\text{(iii)} \quad \frac{1}{\sqrt{C}} \operatorname{arcosh} \frac{B + Cz}{\sqrt{B^2 - AC}} + K, \quad (B^2 - AC) > 0$$

$$\text{(iv)} \quad \frac{-1}{\sqrt{-C}} \arcsin \frac{B + Cz}{\sqrt{B^2 - AC}} + K, \quad C < 0$$

The program computes the coefficients A, B and C between the levels z_i and z_{i+1} for each value of c_m and selects the appropriate expression out of (i) to (iv).

COMPUTATION OF TWO TRANSMISSION LOSS CURVES

General

The transmission conditions have been studied along two different tracks some miles apart in the same sea area. Only one single measurement of bottom reflection loss by oblique incidence has been done in this area. One result of this measurement is the bottom loss function shown in Fig. 7 for the frequency band 600 - 1200 Hz. The computation has been undertaken in order to learn what result may be expected when it is based on this very limited knowledge.

It is interesting because little information on the acoustical properties of the sea floor is the situation in the majority of cases. Acoustic measurements on the tracks permit a direct comparison.

Track No.1

Along the bottom profile of track No. 1, shown in Fig. 11, the transmission loss was measured in several frequency octave bands using calibrated explosive charges which were detonated at 40 m depth. The sea state was about 3 during the time of the measurement. The hydrographic conditions are described by the sound velocities in Fig. 12, which were measured along the track. The sound velocity distributions denoted by the numbers 77, 73, 74 and 75 are valid within the distance intervals indicated in Fig. 11. Thus, a 2-dimensional sound velocity variation is taken into account. This means, however, that there is a discontinuity of the velocity by passing from one interval to another.

Computation of the transmission loss under these conditions was carried out, assuming a wave height of 3 ft. The bottom loss function in Fig. 7 was used and was supposed to be valid for all distances. Surface reflection loss and absorption loss were calculated for a frequency of 900 Hz. The result is presented in Fig. 13, curve No.2. Also a curve (No.3) was computed on the assumption of no bottom loss.

It is seen that the agreement between the measured and computed transmission loss is good. The greatest difference between the two curves, some 2.5 dB, occurs at distances 9300 yd between source and receiver. The measured increase of loss may be caused by unknown irregularities in the bottom profile.

Track No. 2

The bottom profile of track No. 2 is given in Fig. 14. Acoustic measurements were done in a similar manner to those on track No.1. Average wave height was estimated to be about 4 ft. The sound velocity varied along the track as shown in Fig. 15.

Approximations were computed for the sound velocity profiles, as shown in Fig. 16 for profile No. 1. The difference between the two curves nowhere exceeds 0.15 m/s. The measured and computed transmission loss for the frequency band 600 - 1200 Hz is presented in Fig. 17, again using the bottom loss function in Fig. 7. The computation has been done in the distance interval 10 to 40 km.

The difference between the two curves is almost constant 4.5 dB. there may be several reasons for this deviation. The surface reflection loss may be incorrect and/or the bottom reflection loss may be considerably greater than expressed by the loss function of Fig. 7. The possibility of an inaccuracy in the measurements cannot be completely disregarded.

CONCLUSION

The computing procedure described seems to be a useful tool for predicting sound transmission conditions in the sea, provided that the following parameters are known:

a. The bathymetry.
b. The vertical and horizontal sound velocity distribution.
c. The bottom reflection loss.
d. The absorption loss in the water volume.
e. The surface scattering loss.
f. The sea state.

It should be stressed, however, that the calculation is carried out on the following assumptions:

(i) The ray theory must be valid, i.e., the depth of the sea ought to be at least ten times the wave length of the sound.

(ii) The effect of reverberation is not taken into account.

(iii) The amounts of energy having propagated along the various ray paths are added at the end of their travel.

Energy addition takes place if short impulse signals are used, but it does not take place for CW or noise signals in which case the signals having travelled along different paths will interfere. Therefore, the theoretical computation indicates the transmission loss values which will be obtained by measurements using explosive charges as acoustic sources.

REFERENCES

1. C.B. Officer, "Introduction to the Theory of Sound Transmission with Application to the Ocean", McGraw-Hill, New York, 1958.

2. M.A. Pedersen, "Acoustic Intensity Anomalies Introduced by Constant Velocity Gradient", J.Acous.Soc.Am., Vol. 33, 1961, pp. 465-474.

3. K.R. Stewart, "Ray Acoustical Model of the Ocean Using a Depth/Sound-Speed Profile with a Continuous First Derivative", J.Acous.Soc.Am., Vol.38, 1965, pp. 339-347.

4. H.W. Marsh, M.Schulkin and S.G. Kneale, "Scattering of Underwater Sound by the Sea Surface", J.Acous.Soc.Am., Vol. 33, 1961, pp. 334-340

5. M. Schulkin and H.W. Marsh, "Sound Absorption in Sea Water", J.Acous.Soc.Am., Vol.34, 1962, pp. 864-865.

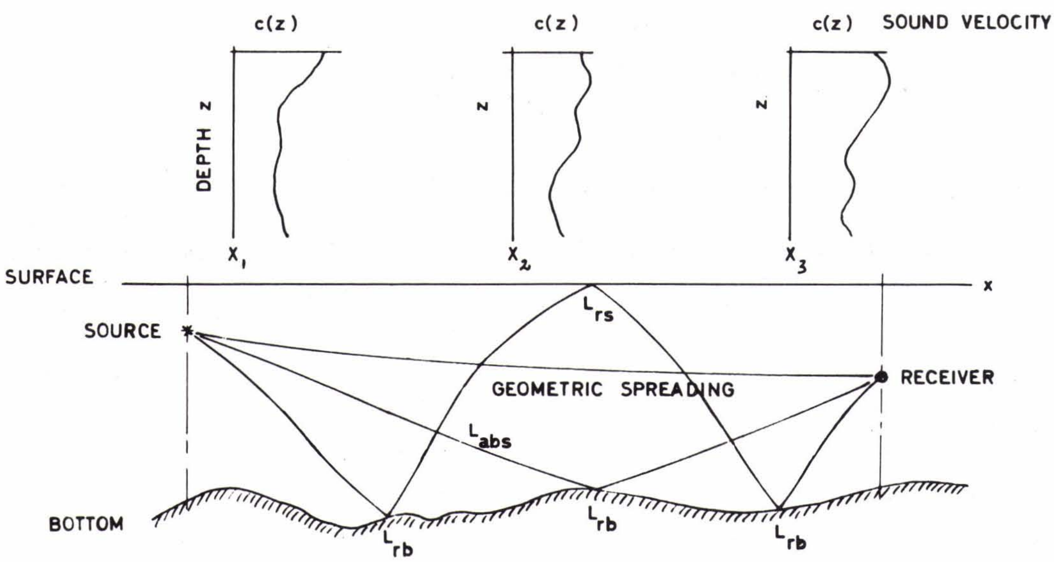
6. W.H. Thorp, "Deep-Ocean Sound Attenuation in the Sub-and Low-Kilocycle-per-Second Region", J.Acous.Soc.Am., Vol. 38, 1965, pp. 648-654.

7. C.C. Leroy, "Sound Attenuation between 200 and 10 000 cps Measured along Single Paths", SACLANTCEN Technical Report No.43, 1965, NATO UNCLASSIFIED

DISCUSSION

The author clarified the point that when a ray crossed the velocity discontinuity between two regions described by different velocity profiles, it was made to do so with its direction unchanged.

In answer to a question, the author acknowledged that when a ray was reflected from the bottom, only the slope of the bottom — and not the curvature — was used for determining the intensity of the reflected ray.



OCEAN MODEL FOR THE COMPUTATION OF TRANSMISSION LOSS

FIG. 1

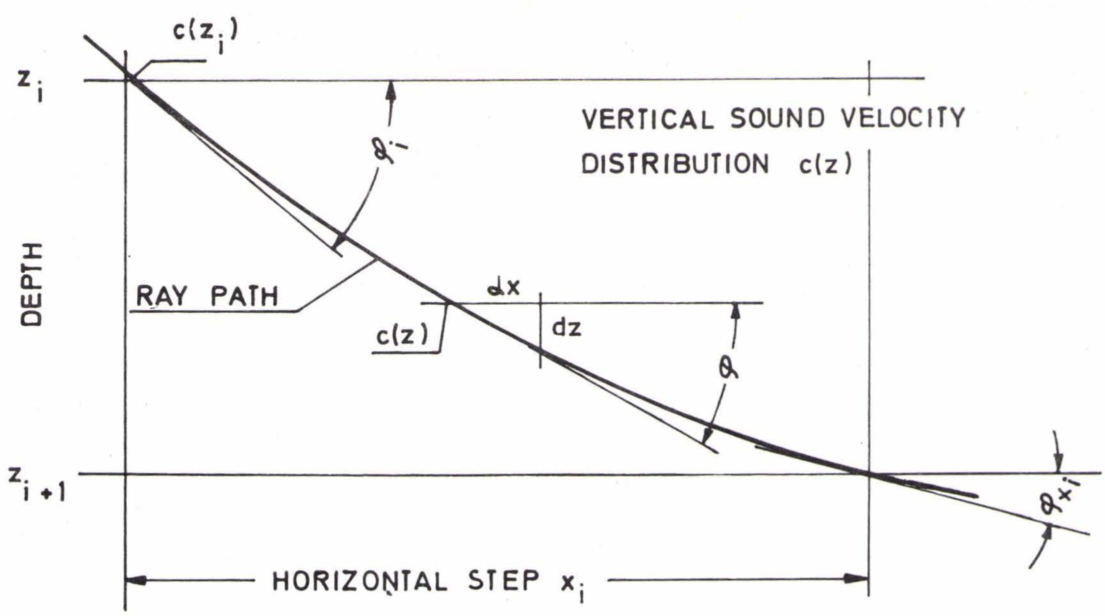


FIG. 2

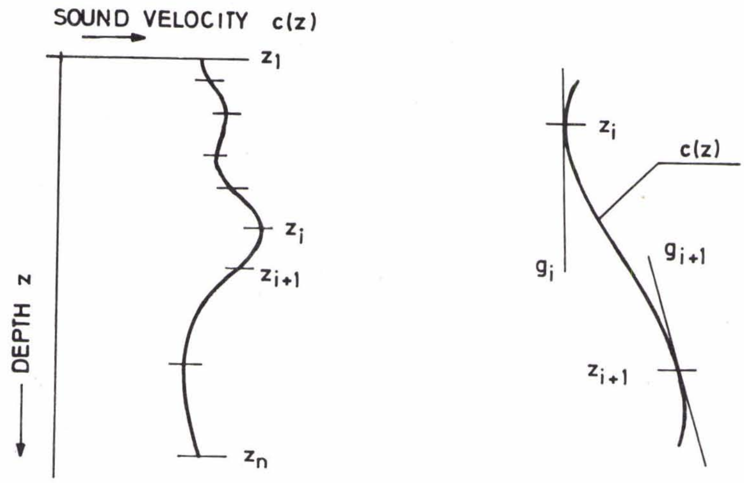


FIG. 3

DIVISION OF THE SEA DEPTH INTO LAYERS

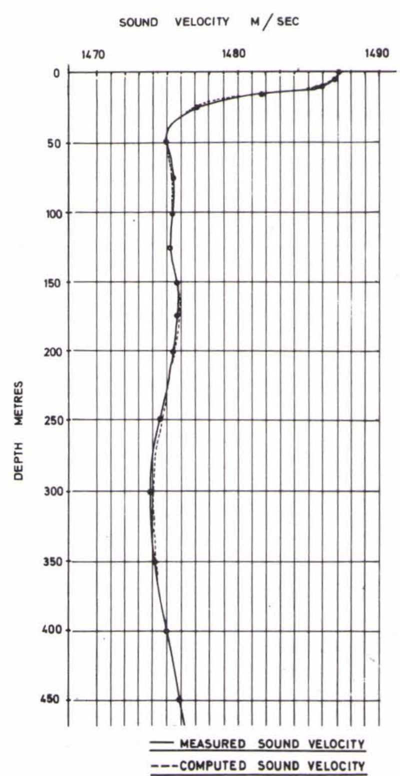
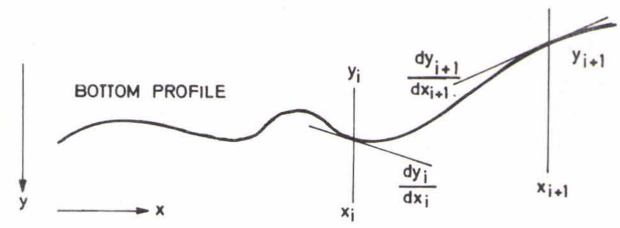


FIG. 4

FIG. 5



DIVISION OF BOTTOM PROFILE IN SEGMENTS

APPROXIMATION BY

$$y = B1_i \cdot x^3 + B2_i \cdot x^2 + B3_i \cdot x + B4_i$$

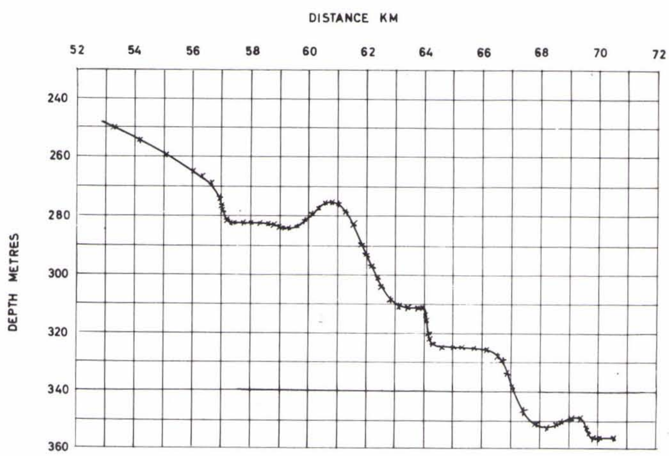
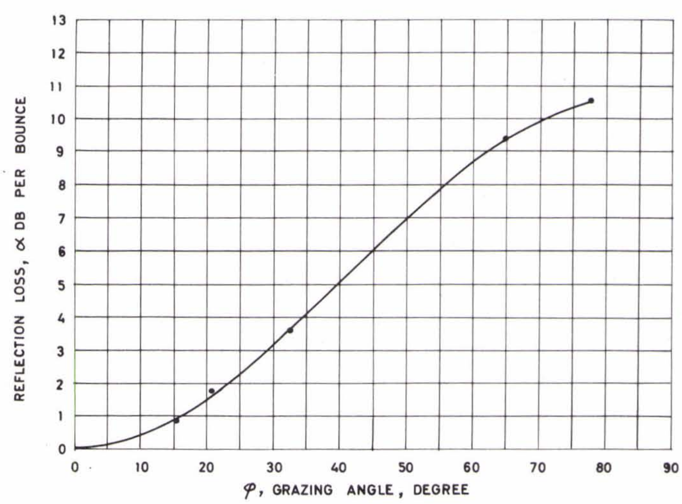


FIGURE 4.2 —— MEASURED BOTTOM PROFILE
××× COMPUTED BOTTOM PROFILE

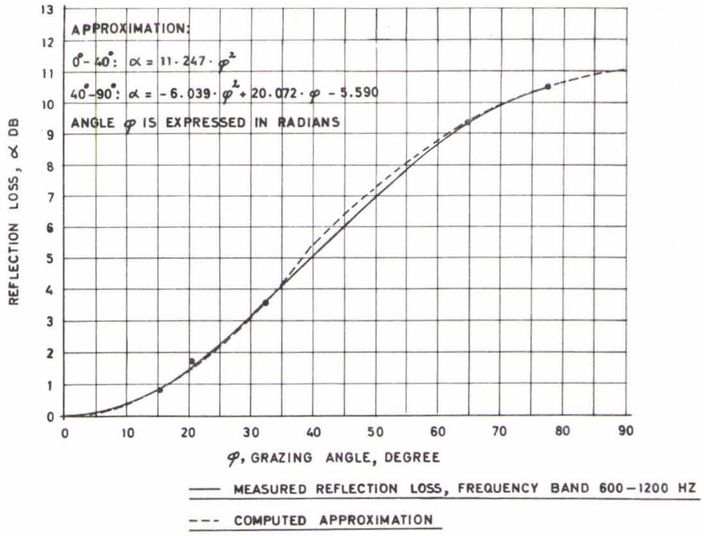
FIG. 6

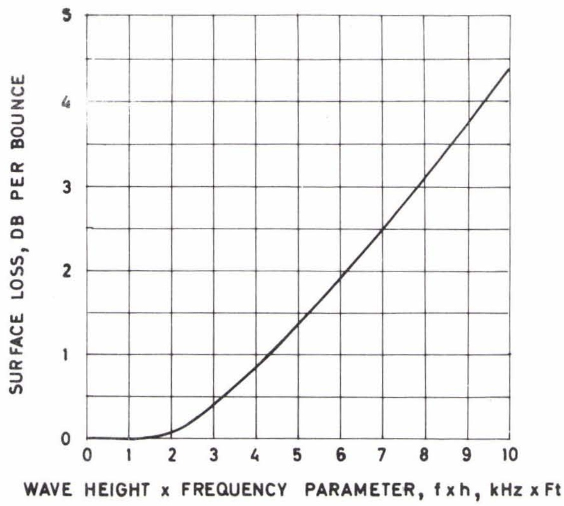
FIG. 7



MEASURED REFLECTION LOSS, FREQUENCY BAND 600–1200 HZ

FIG. 8

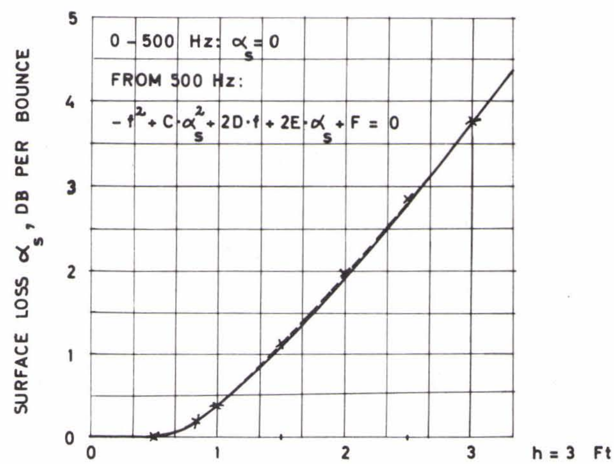




THEORETICAL SEA SURFACE SCATTERING LOSS
ACCORDING TO REFERENCE (4)

FIG. 9

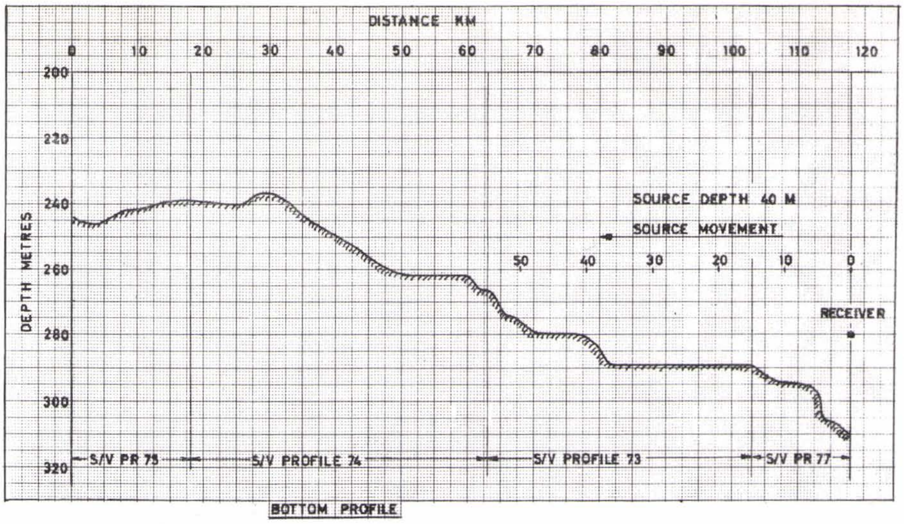
FIG. 10



FREQUENCY, kHz

COMPUTED APPROXIMATION TO THE
SURFACE LOSS FUNCTION ----

FIG. 11



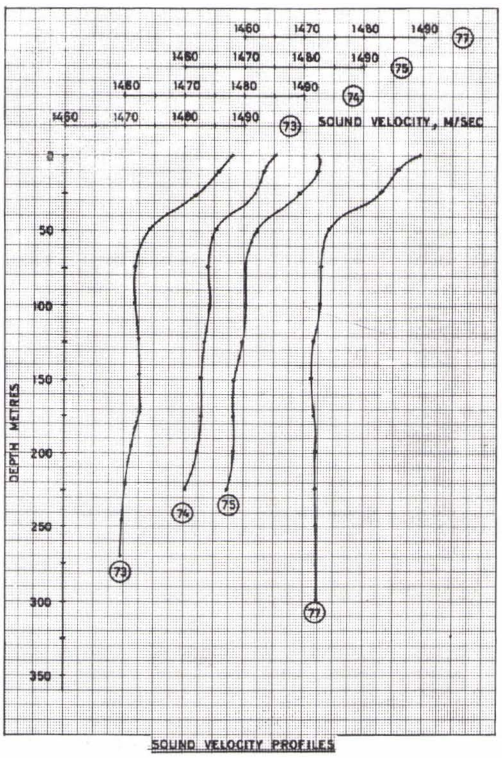
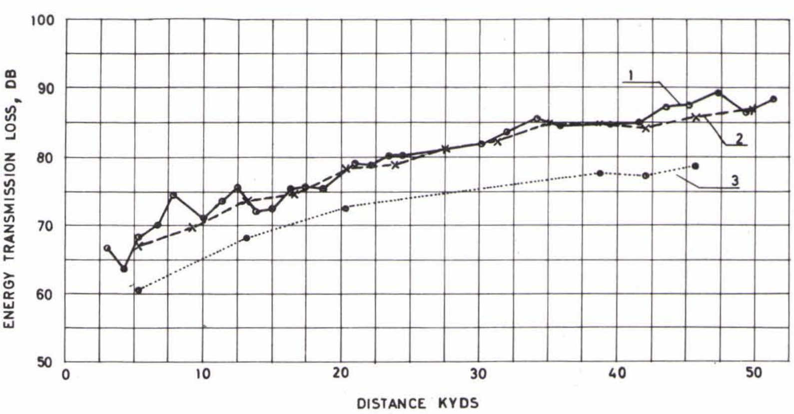


FIG. 12

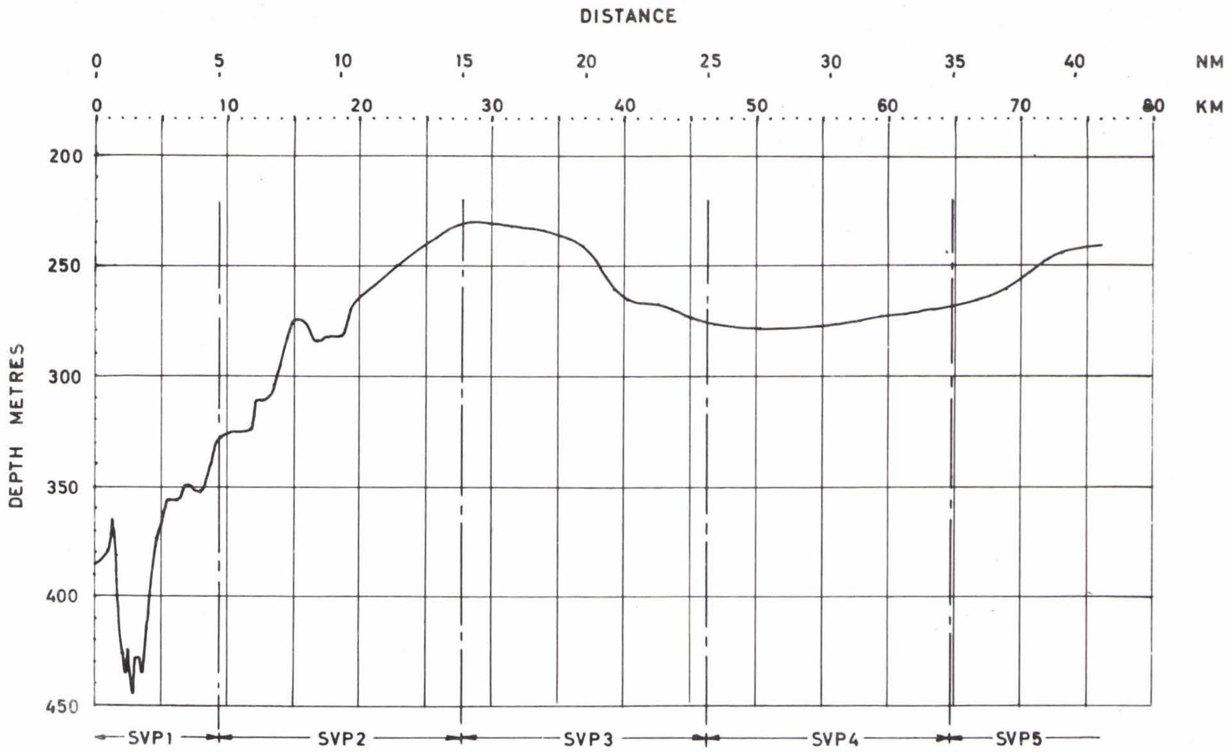
FIG. 13



CURVE 1: MEASURED TRANSMISSION LOSS, BAND 600 - 1200 HZ

CURVE 2: COMPUTED TRANSMISSION LOSS, 900 HZ, WAVE HEIGHT 3 FEET

CURVE 3: LOSS DUE TO GEOMETRIC SPREADING, ABSORPTION AND SURFACE SCATTERING, 900 HZ, WAVE HEIGHT 3 FEET



BOTTOM PROFILE AND HORIZONTAL SOUND VELOCITY DISTRIBUTION

FIG. 14

FIG. 15

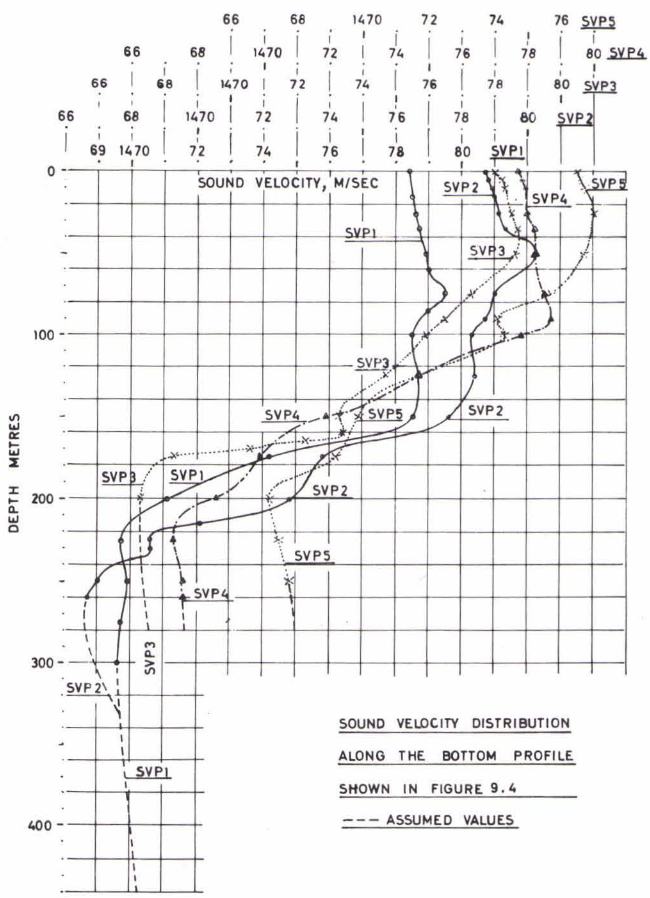
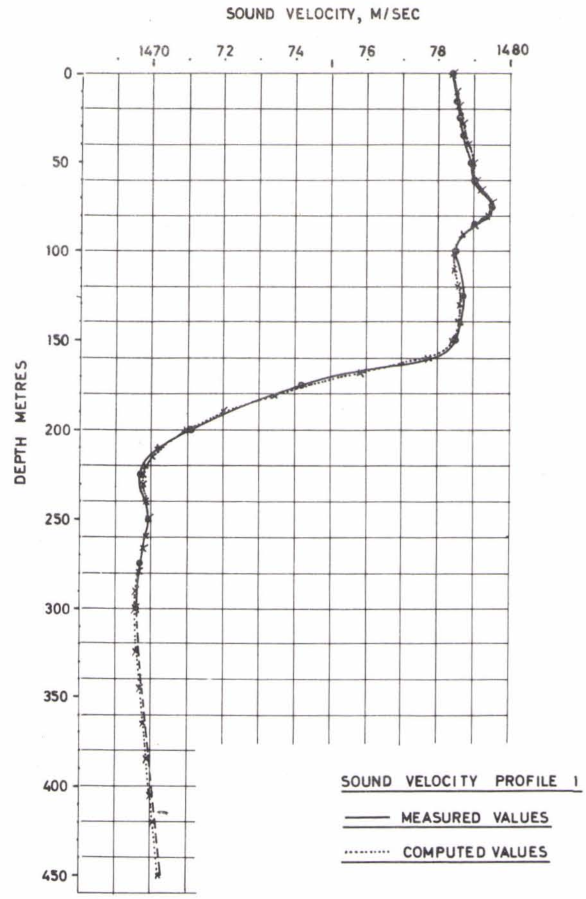
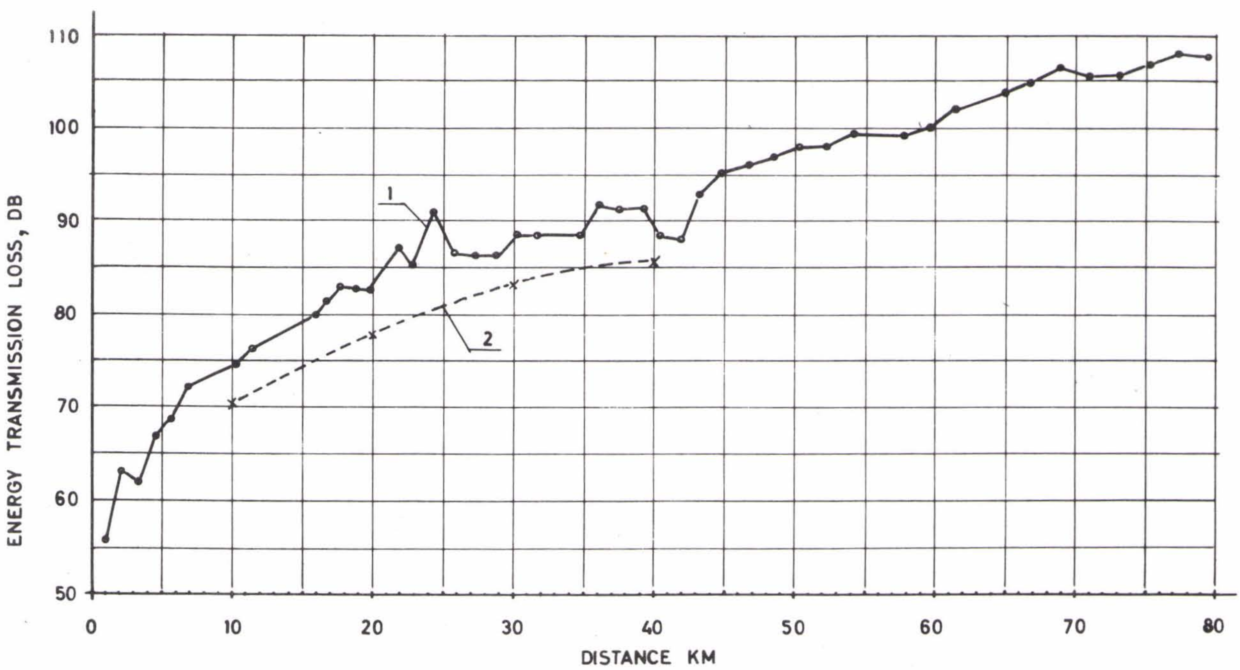


FIG. 16





CURVE 1: MEASURED TRANSMISSION LOSS, BAND 600 - 1200 HZ

CURVE 2: COMPUTED TRANSMISSION LOSS, 900 HZ, WAVE HEIGHT 4 FEET

FIG. 17