SACLANTCEN Report SR - 56

SACLANTCEN Report
SR – 56

# SACLANT ASW RESEARCH CENTRE REPORT

## RANGE AND DEPTH ESTIMATION BY LINE ARRAYS IN SHALLOW WATER

by

RICHARD KLEMM

15 JANUARY 1982

NORTH ATLANTIC TREATY ORGANIZATION

LA SPEZIA, ITALY

*Published by*

STI

SACLANTCEN

NORTH ATLANTIC TREATY ORGANIZATION

SACLANT ASW Research Centre
Viale San Bartolomeo 400, I-19026 San Bartolomeo (SP), Italy.

tel: national 0187 560940
international + 39 187 560940

telex: 271148 SACENT I

# RANGE AND DEPTH ESTIMATION BY LINE ARRAYS IN SHALLOW WATER

by

Richard Klemm

*(Reprinted from Signal Processing 3, 1981: 333-344)*

15 January 1982

*This report has been prepared as part of Project 02.*

APPROVED FOR DISTRIBUTION

RALPH R. GOODMAN
Director

Signal Processing 3 (1981) 333–344
North-Holland Publishing Company 

# RANGE AND DEPTH ESTIMATION BY LINE ARRAYS IN SHALLOW WATER

R. KLEMM
*Forschunginstitut für Funk und Mathematik, Abt. Elektronik, Königstrasse 2, D-5307 Wachtberg-Werthhoven, W.-Germany*

Received 14 August 1980
Revised 1 December 1980

**Abstract.** In shallow water sound propagates in terms of normal modes. The interference between the modes can be utilized to estimate range and depth of an acoustic source in shallow water by applying high resolution power estimators to a hydrophone array. The paper presents results obtained from a theoretical study based on a normal mode model. Various aspects are covered such as depth dependence of resolution, influence of surface fluctuations, comparison of horizontal and vertical line arrays and mismatch between processing and the acoustic field. Conclusions are drawn which give some more insight in the problems.

**Zusammenfassung.** In flachem Wasser breitet sich Schall in Moden aus. Die Interferenz zwischen Moden kann zur Schätzung der Entfernung und Tiefe einer akustischen Quelle genutzt werden indem man hochauflösende Schätzverfahren auf die Ausgangssignale einer Hydrophongruppe anwendet. Die vorliegende Arbeit berichtet über eine Reihe von Ergebnissen einer theoretischen Untersuchung, die mit Hilfe eines Modenmodells durchgeführt wurde. Eine Reihe von Gesichtspunkten wie z.B. die Tiefenabhängigkeit der Auflösung, Einfluss von Fluktuationen der Oberfläche, Vergleich von horizontalen und vertikalen Hydrophongruppen und Fehlanpassung zwischen Prozessor und Schallfeld, wurden berücksichtigt. Es werden Schlussfolgerungen gezogen, die einen besseren Einblick in die Problematik eröffnen.

**Résumé.** Par petits fonds le son se propage en modes normaux. Les interférences entre les modes peuvent être utilisés pour evaluer la distance et la profondeur d'une source sonore en eau peu profonde en appliquant des méthodes de traitement du signal a haute résolution. L'article présente les résultats d'une étude théorique basée sur un modèle de propagation en modes normaux. On y aborde plusieurs aspects de la question, tels que l'influence de la profondeur sur la résolution, les effets des vagues, la comparaison entre les antennes lineáres horizontales et verticales et la difficulté d'adapter le traitement du signal au champs acoustique existant. L'auteur tire enfin des conclusions qui devraient conduire à une compréhension plus approfondie de la problèmatique de ce type de recherche.

**Keywords.** Shallow water, high resolution, array processing, depth estimation, range estimation, fluctuations, sensitivity, horizontal array, vertical array.

## Introduction

The response of a homogeneous shallow water sound channel to a monochromatic point acoustic source can be described by a sum of normal mode functions. As opposed to wave propagation in free space (for example, radar and deep water sonar) where the arrival angles (azimuth and elevation) of a plane wave can be estimated by simple beamforming, we are faced in shallow water with the problem of analyzing the modal interference to extract information, such as source depth and bearing. In addition to free space propagation, however, the modal interference is also range dependent and, therefore, can be utilized for passive range estimation.

This paper presents results obtained from a theoretical study based on a normal mode sound propagation model [1]. The propagation model has been extended by a surface fluctuation model [2]. Both of the models are used to simulate the sound field and to design the array processors used for detection and locating sources. Most of the specific processor parameters, such as array configuration and processor type are kept constant throughout the paper. The author's interest is focussed on environmental parameters, such as surface fluctuations, source depth and range. In this sense, the paper deals with an environmental study as seen through the eyes of an array processor rather than with a specific study on processor design. However, statistical quantities describing a random field, such as coherence, are meaningful only when being considered in connection with a certain processing scheme.

## 2. Signals in shallow water

In the following we derive briefly the spatial covariance matrix from the normal mode solution of the waveguide. The SNAP-model [1] used for the subsequent investigation calculates the modal wavenumbers $k_n$ and the modal amplitudes $A_n$ for a set of environmental input parameters, such as sound velocity profile, water depth, source-receiver geometry and others. The sound pressure due to a monochromatic source is

$$p(\zeta, z, r, t) = \mathrm{e}^{-\mathrm{j}(\omega t - \pi/4)} \sum_{n=1}^{M} A_n \, \mathrm{e}^{\mathrm{j}k_n r} \tag{1}$$

where

$$A_n = p_0 \frac{\omega \rho^2}{H} \sqrt{\frac{1}{8\pi r}} \times \frac{\mu_n(\zeta)\mu_n(z)}{\sqrt{k_n}} \mathrm{e}^{-\alpha_n r}. \tag{2}$$

The different quantities mean

$p_0$ source strength,
$\rho$ water density,
$H$ water depth,
$r$ range,
$z$ receiver depth,
$\zeta$ source depth,
$\alpha_n$ modal attenuation coefficient,
$\mu_n(\cdot)$ normal mode function,
$M$ number of modes.

The modal attenuation coefficients include two portions, one due to bottom loss, the other one due to scattering at the rough surface [1, 2, 3]. We extend the signal model by introducing a modal phase $\phi_n$ in order to model phase fluctuations due to the time-varying surface:

$$p(\zeta, z, r, t) = p_0 \, \mathrm{e}^{-\mathrm{j}(\omega t - \pi/4)} \times \sum_{n=1}^{M} A_n \, \mathrm{e}^{\mathrm{j}(k_n r + \phi_n)}. \tag{3}$$

The covariance between two receiver positions $i$, $k$ becomes

$$\begin{aligned} \rho_{ik} &= E\{p(\zeta, z_i, r_i, t) p^*(\zeta, z_k, r_k, t)\} \\ &= E\Bigg\{ \sum_{n=1}^{M} A_n(\zeta, z_i, r_i) \, \mathrm{e}^{\mathrm{j}(k_n r_i + \phi_n)} \\ &\quad \times \sum_{m=1}^{M} A_m(\zeta, z_k, r_k) \, \mathrm{e}^{-\mathrm{j}(k_m r_k + \phi_m)} \Bigg\} \\ &= \sum_{n=1}^{M} \sum_{m=1}^{M} A_n(\zeta, z_i, r_i) A_m(\zeta, z_k, r_k) \\ &\quad \times \mathrm{e}^{\mathrm{j}(k_n r_i - k_m r_k)} E\{\mathrm{e}^{\mathrm{j}(\phi_n - \phi_m)}\}. \end{aligned} \tag{4}$$

If the surface fluctuations are small compared with the water depth the phase fluctuation term can be approximately described by a gaussian function [2]:

$$\begin{aligned} \rho_\phi(n, m) &= E\{\mathrm{e}^{\mathrm{j}(\phi_n - \phi_m)}\} \\ &= \mathrm{e}^{-(k_n - k_m)^2 r^2 \sigma^2 / 2H^2}, \end{aligned} \tag{5}$$

where $\sigma$ is the RMS-wave height of the surface. In addition we assume spatially white noise independent of the signal to model non-directive noise components (ambient, receiver). In the following investigation the signal-to-noise ratio is always constant (10 dB).

## 3. Array processing

Estimation of an unknown parameter $\theta_0$ of a vector process is frequently done by applying a power estimator $P(\theta)$ to the data and varying $\theta$ until the output power becomes maximum (for example, beamforming and spectral analysis). This way of estimation is particularly convenient if the parameter is a non-linear function of the data, as, for example, in case of depth and range estimation in shallow water. Hinich [7] has shown that direct estimation of the source depth from a set of measured data, leads to an iterative estimation procedure.

There are two basic kinds of power estimators [5]. The first one is based on the principle of maximizing the cosine between the received vector of field samples and a steering vector $\boldsymbol{h}(\theta)$ where $\theta$ runs through the whole parameter space. A simple well-known example is steering a beam to identify plane waves. The output power of this process is

$$P(\theta)=\boldsymbol{h}^*(\theta)\mathbf{R}\,\boldsymbol{h}(\theta) \tag{7}$$

where $\mathbf{R}\equiv(\rho_{ik})$ is the covariance matrix of the spatial field samples. If the signal part of the field is random a generalized version of (7) is given by:

$$P(\theta)=\operatorname{tr}(\mathbf{RH}(\theta)) \tag{8}$$

where $\mathbf{H}(\theta)$ is a set of steering matrices. For example, a set of covariance matrices due to eqs. (4) and (5) has to be formed for all interesting values of $\zeta$ if (8) is used for depth estimation. The detection factor used by Bucker [4] belongs to this class of estimators.

The second kind of array processors is based on the principle of orthogonalization, i.e. the field is represented by a vector orthogonal to the signal components in the measured covariance matrix. The output power of such a processor is

$$P(\theta)=\frac{1}{|\boldsymbol{g}^*\boldsymbol{h}(\theta)|^2} \tag{9}$$

if the signal is deterministic, and

$$P(\theta)=\frac{1}{\boldsymbol{g}^*\mathbf{H}(\theta)\boldsymbol{g}} \tag{10}$$

if the signal is random. $\boldsymbol{g}$ may be the eigenvector corresponding to the minimum eigenvalue of $\mathbf{R}$ or a row of a projection matrix (i.e. a matrix that projects the received data on a signal-free subspace) or a row of $\mathbf{R}^{-1}$.

The last method is well-known as the maximum entropy method if the field is homogeneous, that means $\mathbf{R}$ is Toeplitz and is referred to as the approximate orthogonal projection method (AOP) otherwise [5]. In most of the subsequent examples we use an estimator of the generalized AOP-form, i.e. eq. (10) with $\boldsymbol{g}$ being the first column of $\mathbf{R}^{-1}$.

## 4. Separation of several sources

In order to resolve several sources by one of the estimators (eqs. (7)–(10)) the number of non-zero eigenvalues of $\mathbf{R}$ has to be greater than the number of sources. This requires first of all that the order of $\mathbf{R}$ (i.e. the number of hydrophones) is greater than the number of sources. However, when using mode interference for separation of sources in range and depth, there is another condition on the number of modes. Let us illustrate this by an example. Eq. (4) can be written as a matrix equation

$$\mathbf{R}=(\rho_{ik})=\mathbf{C\Phi C}^*, \tag{11}$$

where $\mathbf{\Phi}\equiv(E\{e^{j(\phi_n-\phi_m)}\})$ is the $M\times M$-covariance matrix of modal phase fluctuations and

$$\mathbf{C}\equiv(c_{nk})=(A_n(\zeta, z_k, r_k)\,e^{-jk_nr_k}$$

is an $N\times M$-matrix containing the deterministic part of the waveguide. Let us assume a vertical line array ($r_k=\text{const.}=r_0$) and consider one mode only with wavenumber $k_0$. Then $\mathbf{C}$ becomes a vector with elements $c=\mu(\zeta)A\mu(z_k)$ where

$$A=p_0\frac{\omega\rho^2}{H}\sqrt{\frac{1}{8\pi rk_0}}\,e^{-\alpha_0r_0}\,e^{-jk_0r_0}.$$

The contributions of two uncorrelated sources become

$$c_k=\mu(\zeta_1)A\mu(z_k)+\mu(\zeta_2)A\mu(z_k),$$

i.e. they are linearly dependent which causes one eigenvalue to be zero. We can draw the conclusion that for separation of sources in depth by means of mode interference the number of modes has to be larger than the number of sources. We conclude furthermore, that horizontal arrays cannot be used in broadside direction for range and depth estimation because then the modal arrivals differ only in amplitude but not in phase and, hence, cause linearly dependent sets of field samples.

The subsequent examples (Figs. 1–10) are based on the parameters in the following Table 1.

Table 1

| | |
|---|---|
| water depth | 50 m |
| SVP in water | isovelocity, 1500 m/s |
| sediment thickness | 2 m |
| SVP in sediment | isovelocity, 1600 m/s |
| subbottom | isovelocity, 2500 m/s |
| frequency | 100 Hz, Figs. 3–6<br>140 Hz, otherwise |
| number of hydrophones | 5 |
| spacing | 10 m |
| depth of the horizontal array | 30 m, source at end-fire direction |
| SNR | 10 dB |
| source depth | 25.5 m (except Fig. 1) |
| source range | 5 km (except Figs. 7, 8) |
| number of modes | 4 (100 Hz) and 6 (140 Hz) |

## 5. Comparison of power estimators

Fig. 1 shows the output powers of two processors, namely the generalized beamformer (GBF, eq. (8)) and the AOP, eq. (10), both for a vertical and a horizontal array of 40 m aperture. The source position is denoted by three asterisks above each other. The field is supposed to be entirely coherent, i.e. $E\{e^{j(\phi_n-\phi_m)}\}=1$, $\forall n, m$. It can be recognized that the GBF obtains a maximum when used with a vertical array; the horizontal array is not capable of locating the source properly, obviously due to a lack of resolution. Applying the highly resolving AOP-method yields a considerable gain in peak-to-sidelobe level. The horizontal array resolves the source even better than the vertical one. Obviously the horizontal array has been positioned at a favourable water depth whereas the vertical array averages over the whole water column.

## 6. Influence of the source depth

In Fig. 2 the vertical array is used to detect sources at different depths (4, 25, 44 m). The AOP-processor (eq. (10)) is applied again. We notice that for sources close to the surface the power response is distinctly broader and the "sidelobe level" higher than for source positions in the middle or at the bottom. The lowest "sidelobe level" is obviously obtained if the source is in the middle of the channel. One may think of the usual bearing estimation by horizontal arrays where the beamwidth increases at endfire direction. This is, of course, not the reason; on the contrary to the broadened beam of an endfire array the effect observed here is a property of the pressure release boundary. Similar experiments with the horizontal array have shown the same properties. We find in fact that the position chosen for the horizontal array in Fig. 1 (30 m) was favourable.

## 7. Fluctuations at the surface

The following four figures (Figs. 3–6) demonstrate the sensitivity to surface fluctuations of vertical and horizontal line arrays used for depth and range estimation. The curves of each plot are calculated for different RMS-surface wave heights ($\sigma=0$, 0.05, 0.1, 13, 1 m). It is noticed first of all that the sensitivity is considerably high; nevertheless, the vertical array appears to be less sensitive than the horizontal one. Range estimates will be always periodic due to the interference wavelength (here: 600 m, see Fig. 4). The field is, however, not strictly periodic because of the range dependent modal attenuation factors $\alpha_n r$ in (2). Therefore, the local maxima of 4.4 km and 5.6 km are smaller than the one at the true source position. The influence of the attenuation factors may be masked by the fluctuation ($\sigma=0.3$ in Fig. 4). Depth estimation works obviously better than range estimation; compare Fig. 3 with Fig. 4 and Fig. 5 with Fig. 6. Using a horizontal array for range estimation may yield a large number of pseudo-periodicities

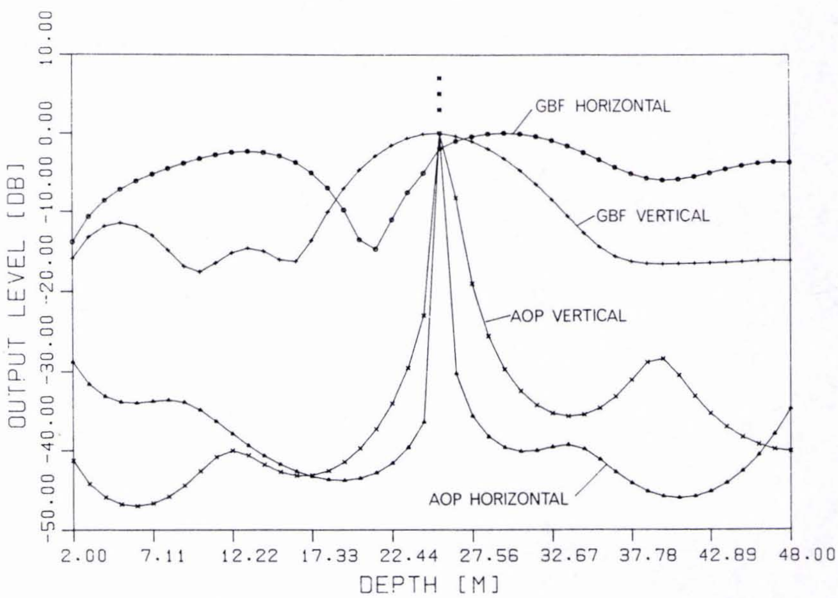


Fig. 1. Comparison of methods for depth estimation.

(Fig. 6) which is due to the fact that the aperture of the array under consideration is small compared to the interference wavelength. It can be noticed that larger phase fluctuations ($\sigma = 0.3$ m, $\sigma = 1$ m) equalize the short periodicities and replace them by larger pseudo-interference wave-lengths.

## 8. Range dependence

We found already that the detectability of sources depends on the depth of the source and, for horizontal arrays, on the array depth. As the sound field is inhomogeneous in range as well, due

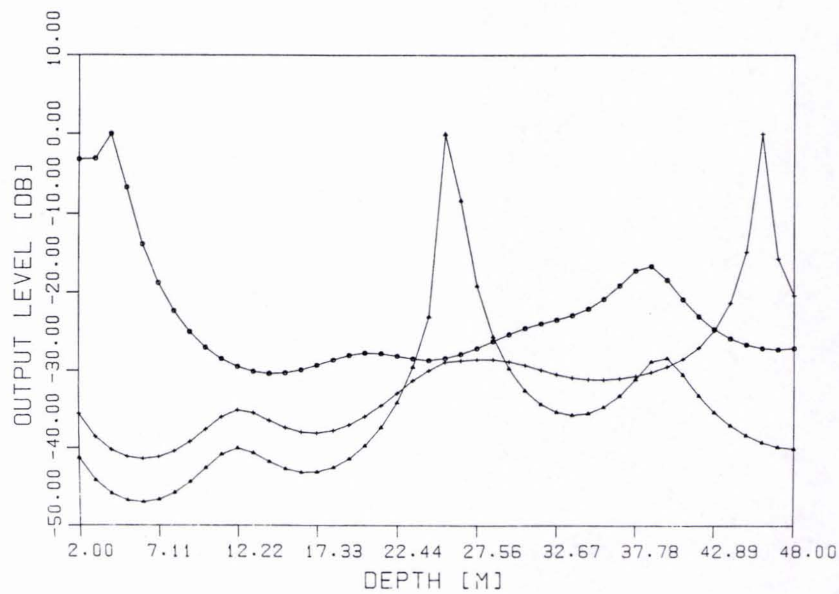


Fig. 2. Influence of the source depth on resolution.

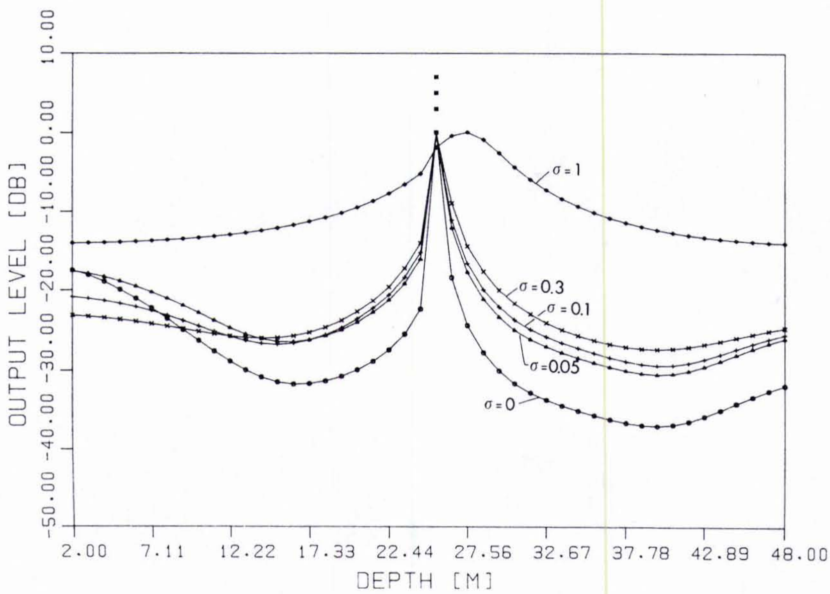


Fig. 3. Influence of surface fluctuations (vertical array).

to mode interference, we can expect that the detectability of sources will also depend on range. The curves in Fig. 7 show depth-curves at various ranges (in km). No surface fluctuations are assumed ($\sigma = 0$). As the signal-to-noise ratio is kept constant we would expect constant PSL if the field were homogeneous in range. However, we notice that there is an irregular relationship between PSL and range due to irregularities of mode interference. Fig. 8 shows the same example, but with a RMS-wave height of $\sigma = 0.3$ m, thus reducing the mode interference. As one can see the PSL goes down, due to phase instabilities caused by the surface fluctuations. Furthermore,

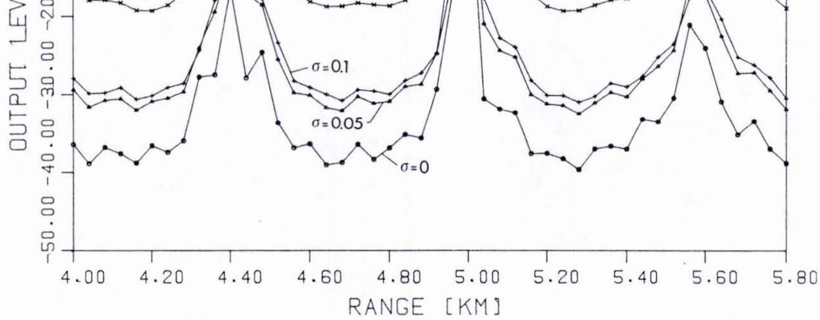


Fig. 4. Influence of surface fluctuations (vertical array).

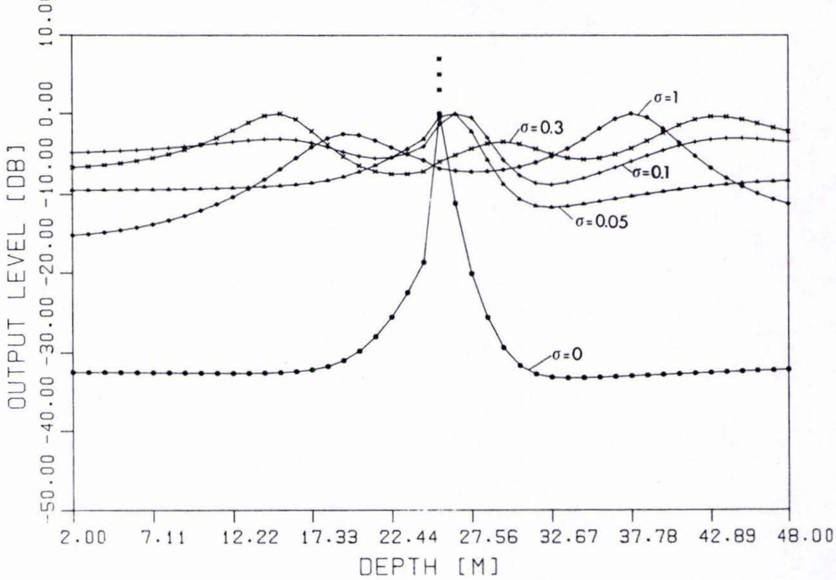


Fig. 5. Influence of surface fluctuations (horizontal array).

the curves get closer together, which means that the randomness of the surface partly equalizes the range dependent irregularities of the field. In fact, if the modes are entirely uncorrelated, the field is homogeneous in range, (except for transmission loss).

Then the intermode phase correlation (5) approaches for large values of $\sigma$ $\rho_\phi(n, m) = 1$, $m = n$, and $\rho_\phi(n, m) = 0$, $m \neq n$. Hence, (4) becomes

$$\begin{aligned}\rho_{ik} &= \sum_{n=1}^{M} \sum_{m=1}^{M} A_n(\zeta, z_i, r_i) A_m(\zeta, z_k, r_k) \\ &\quad \times e^{j(k_n r_i - k_m r_k)} \delta_{nm} \\ &= \sum_{n=1}^{M} A_n(\zeta, z_i, r_i) A_n(\zeta, z_k, r_k) \\ &\quad \times e^{j(k_n (r_i - r_k))} \end{aligned} \tag{12}$$

with $\delta_{nm}$ being the Kronecker symbol.

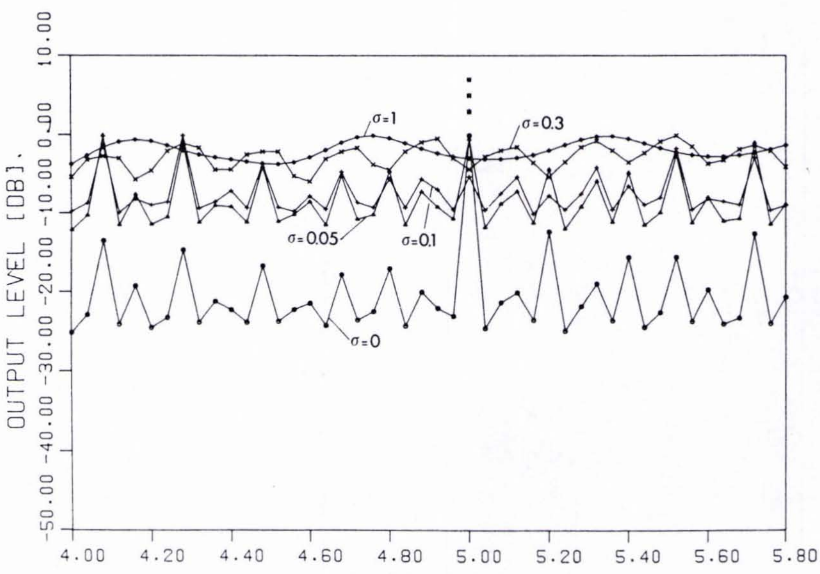


Fig. 6. Influence of surface fluctuations (horizontal array).

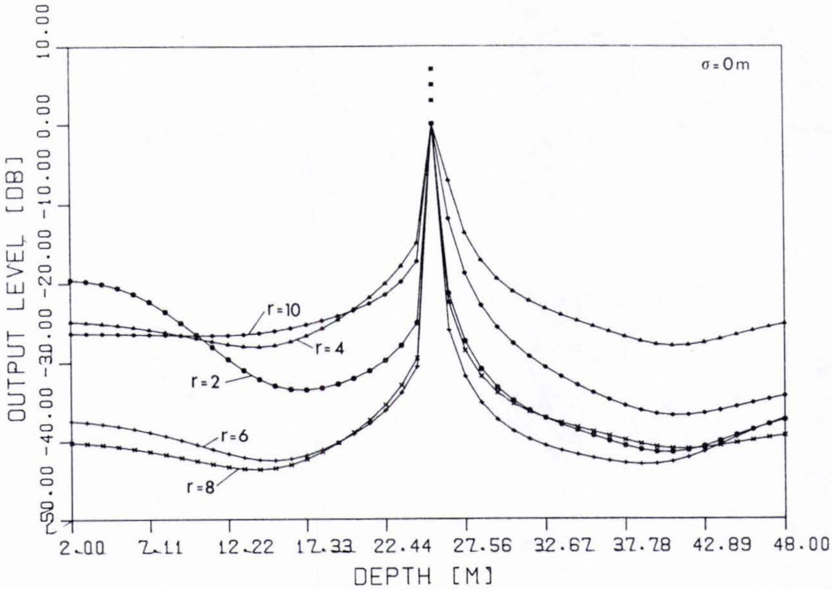


Fig. 7. Range dependence of depth estimation.

Assuming that the transmission loss inside the array's aperture is negligible we get for a horizontal array with uniform spacing ($z_i = z =$ const., $r_i - r_k = (i-k)d$)

$$\rho_{ik} = \sum_{n=1}^{M} A_n^2(\zeta, z, r)\, e^{j(k_n d(i-k))}$$

which depends, apart from the transmission loss between source and receiver, only on the distances between hydrophones relative to each other. The covariance matrix $\mathbf{R} = (\rho_{ik})$ is Toeplitz. Neither range nor depth estimation can be done.

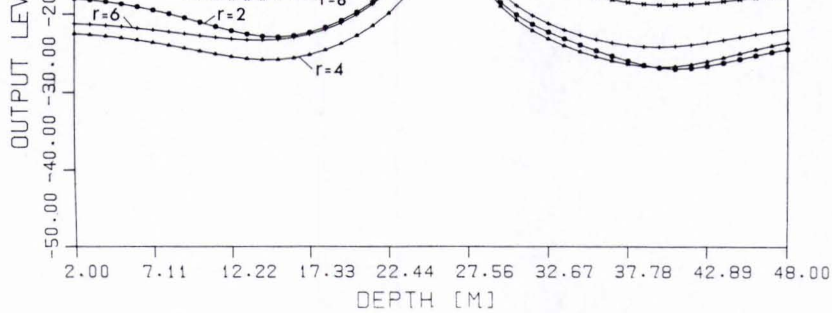


Fig. 8. Range dependence of depth estimation.

## 9. Mismatch between processor and sound field

So far all results were based on the assumption that the sound channel is entirely known. Therefore the steering matrices $\mathbf{H}(\cdot)$ in (10) were perfectly matched to the field parameters. In reality, however, channel parameters are either not well known (for example, bottom and sediment) or are time-variant (SVP, wave height). Let us discuss this aspect by means of a few examples (depth estimation by a vertical array, Figs. 9–11). Fig. 9 and 10 show the influence of imprecise knowledge about the RMS-wave height. In Fig. 9 the processor is based on the assumption that the field be entirely coherent ($\sigma_{\mathrm{PROC}} = 0$); Fig. 10 shows the same example, but with $\sigma_{\mathrm{PROC}} = 0.3$ m. Comparing both of the figures we can draw a few conclusions: It is always better to design the processor on the basis of less fluctuations than those of the actual field, i.e. $\sigma_{\mathrm{PROC}} \leq \sigma_{\mathrm{FIELD}}$; for small fluctuations ($\sigma \leq 30$ cm) the coherent field assumption ($\sigma_{\mathrm{PROC}} = 0$) seems to be appropriate (Fig. 9); for stronger fluctuations ($\sigma = 1$ m) a certain $\sigma_{\mathrm{PROC}} \neq 0$ ($0 < \sigma_{\mathrm{PROC}} \leq \sigma_{\mathrm{FIELD}}$) should be applied.

Fig. 11 shows an example for a mismatch of the deterministic part of the processor and the field. It was assumed on the processor side that the bottom of the channel consists of sandy silt, therefore the processor has been designed on the basis of this type of bottom material. The curves show the power responses of channel with different bottom types (all other parameters being kept constant). As one can see the correct estimate is obtained only if the processor is matched to the acoustic field, i.e. based on the sandy silt assumption. We find that there is basically a considerable sensitivity to unknown bottom parameters. Furthermore, we can expect high sensitivity to variations of other parameters, such as the SVP. Similar effects on high resolution bearing estimation have been observed in [8]. Other calculations have shown that horizontal arrays are more sensitive to any mismatch between processor and acoustic field than are vertical ones.

## 10. Conclusions

A theoretical study based on a normal mode sound propagation model has been conducted in

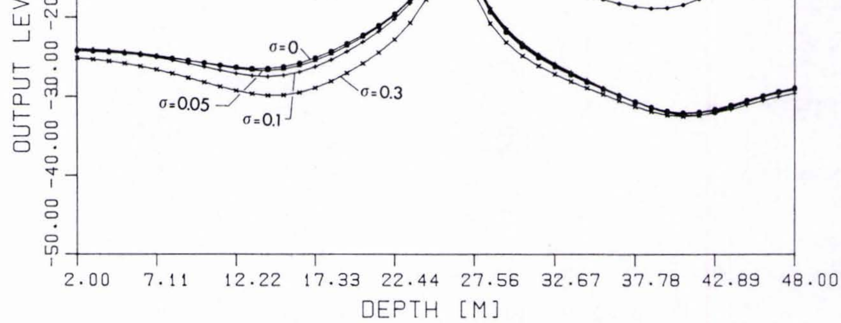


Fig. 9. Mismatch between processing and field.

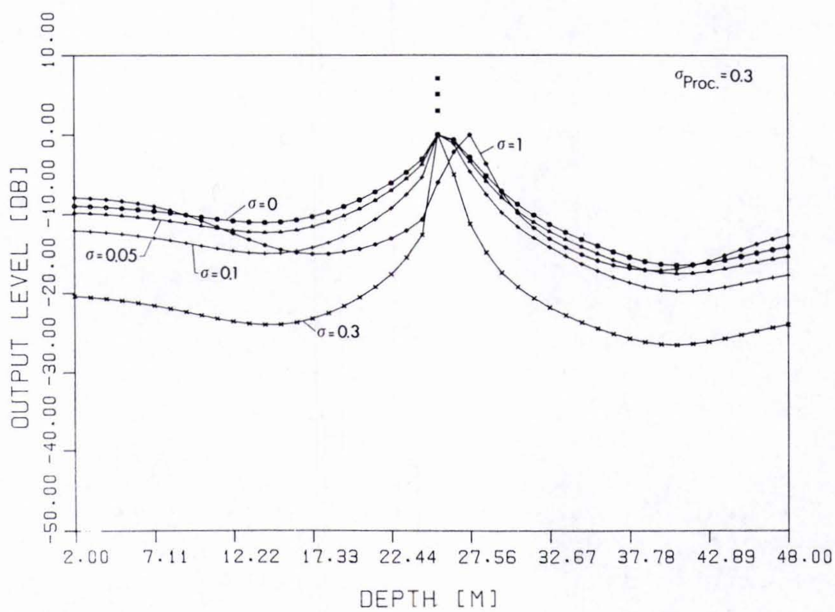


Fig. 10. Mismatch between processing and field.

order to investigate the possibility of locating acoustic sources in shallow water. A generalized high resolution power estimator has been applied to vertical and horizontal arrays for range and depth estimation. The influence of surface fluctuations was of special interest. As the detectability of sources by mode interference depends on a great variety of environmental parameters which, for briefness, have not all been taken into account, no definite conclusions concerning the performance of either vertical or horizontal arrays can be drawn. However, the following statement can be made:

(a) High resolution methods (for example, the AOP-method) obtain higher peak-to-sidelobe than conventional field matching (GBF), particularly for limited apertures (vertical arrays in very shallow water).

(b) Under favourable conditions (array depth, field coherence) a horizontal array may resolve sources better than a vertical one.

(c) Sources are better resolved in the middle of the channel than close to the boundaries, especially at the surface.

(d) Vertical arrays are less sensitive to perturbations and mismatch to environmental parameters than horizontal ones.

(e) Depth estimation works better than range estimation.

(f) The RMS-wave height should not exceed about 1% of the water depth for vertical arrays and 1% for horizontal ones.

(g) The detectability of sources depends on range in an irregular way due to mode interference. If there are surface fluctuations the irregularities are partly equalized, but the detectability decreases.

## 11. Suggestions for experiments

### A. Identification of the channel

The results obtained so far are based on the a priori knowledge of the deterministic part of the channel, i.e. on the matrix **C** in (11). It was assumed that the channel parameters are range independent so that a normal mode propagation

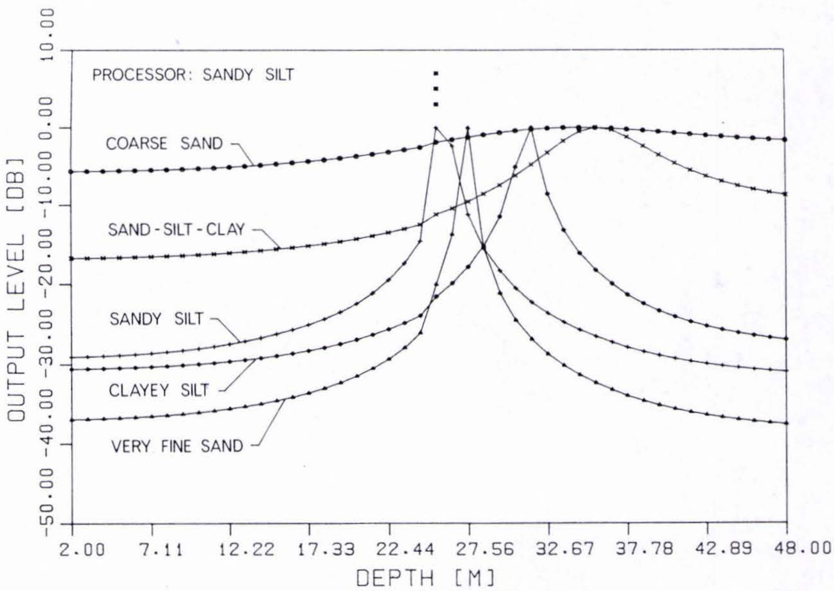


Fig. 11. Mismatch of the deterministic field component.

model is valid. That does not mean that range and depth estimation in shallow water is restricted to homogeneous channels; it is only important that the channel is known so that the matrix $\mathbf{C}(\zeta, r_i, z_i)$ can be calculated. For experimental purposes it is suggested to choose a well-known homogeneous channel and to model the **C**-matrices by means of a normal mode program.

*B. Estimation of the phase fluctuation matrix*

Estimation of the spatial narrow-band covariance matrices according to

$$\hat{\mathbf{R}} = \frac{1}{L}\sum_{t=1}^{L} \boldsymbol{p}_t \boldsymbol{p}_t^*$$

($\boldsymbol{p}_t$ = vector of received signals at time $t$) leads approximately to

$$\mathbf{R} = \mathbf{C}\mathbf{\Phi}\mathbf{C}^* + w\mathbf{I},$$

with $w$ being the white noise power. After removing the white noise part $w\mathbf{I}$ (e.g., by eigenvector factorization) we obtain $\mathbf{R} = \mathbf{C}\mathbf{\Phi}\mathbf{C}^*$ which can be solved for $\mathbf{\Phi}$ provided that the number of sensors is larger than the number of modes: $\boldsymbol{\phi} = \mathbf{C}^{\mathrm{t}}\mathbf{R}\mathbf{C}^{\mathrm{t}*}$, with $\mathbf{C}^{\mathrm{t}}$ being the pseudoinverse of $\mathbf{C}$ ($\mathbf{C}^{\mathrm{t}} = \mathbf{C}(\mathbf{C}^*\mathbf{C})^{-1}$). The matrix $\mathbf{C}^*\mathbf{C}$ is invertible only for vertical or large horizontal arrays due to their capability of resolving the modes. For large horizontal arrays the correlation loss due to (5) becomes relevant inside the arrays aperture and, hence, cannot be described anymore by a simple matrix $\mathbf{\Phi}$. Therefore, the use of a vertical array is suggested. The validity of Clay's formula (5) should be proved for a variety of source-receiver configurations.

*C. Localization of sources*

Different processors according to 5 should be used by applying the steering matrices

$$\mathbf{H}(r, \zeta) = \mathbf{C}(r, \zeta)\mathbf{\Phi}(r)\mathbf{C}^*(r, \zeta)$$

to the measured covariance matrix **R** or a modification of **R**. A comparison of the detectability of sources in presence of different sea states is of special interest.

**Acknowledgement**

The author would like to thank Mr. M. Daintith for carefully revising the text.

## References


[1] F.B. Jensen and M.C. Ferla, "SNAP, the SACLANTCEN normal mode acoustic propagation model", SACLANTCEN SM-121, 1979.

[2] C.S. Clay, "Effect of a slightly irregular boundary on the coherence of waveguide propagation", *J. Acoust. Soc. Amer.*, Vol. 36, 1964, pp. 833–837.

[3] W.A. Kuperman and F. Ingenito, "Attenuation of the coherent component of sound propagating in shallow water with rough boundaries", *J. Acoust. Soc. Amer.*, Vol. 61, No. 5, pp. 1178–1187, May 1977.

[4] Homer P. Bucker, "Use of calculated wave fields and matched field detection to locate sound sources in shallow water", *J. Acoust. Soc. Amer.*, Vol. 59, No. 2, February 1976, pp. 368–373.

[5] R. Klemm, "High resolution analysis of non-stationary data ensembles", *Proc. of the EUSIPCO*-80, Lausanne, Switzerland, Sept. 16–19, 1980 (North-Holland, Amsterdam, 1980).

[6] R. Klemm, "Use of generalized resolution methods to locate sources in random dispersive media", *IEE Proc.*, Vol. 127, No. 1, 1980.

[7] Melvin J. Hinich, "Maximum-likelihood signal processing for a vertical array". *J. Acoust. Soc. Amer.*, Vol. 54, No. 2, 1973, pp. 499–503.

[8] R. Klemm, "The influence of unknown bottom parameters on bearing estimation in shallow water", *Proc. of Bottom Interaction Ocean Acoustics*, June 9–13, 1980, SACLANT ASW Research Centre, La Spezia, Italy.

## INITIAL DISTRIBUTION

| | Copies |
|---|---|
| **MINISTRIES OF DEFENCE** | |
| MOD Belgium | 2 |
| DND Canada | 10 |
| CHOD Denmark | 8 |
| MOD France | 8 |
| MOD Germany | 15 |
| MOD Greece | 11 |
| MOD Italy | 10 |
| MOD Netherlands | 12 |
| CHOD Norway | 10 |
| MOD Portugal | 5 |
| MOD Turkey | 5 |
| MOD U.K. | 16 |
| SECDEF U.S. | 61 |
| **NATO AUTHORITIES** | |
| Defence Planning Committee | 3 |
| NAMILCOM | 2 |
| SACLANT | 10 |
| SACLANTREPEUR | 1 |
| CINCWESTLANT/COMOCEANLANT | 1 |
| COMIBERLANT | 1 |
| CINCEASTLANT | 1 |
| COMSUBACLANT | 1 |
| COMMAIREASTLANT | 1 |
| SACEUR | 2 |
| CINCNORTH | 1 |
| CINCSOUTH | 1 |
| COMNAVSOUTH | 1 |
| COMSTRIKFORSOUTH | 1 |
| COMEDCENT | 1 |
| COMMARAIRMED | 1 |
| CINCHAN | 1 |

| | Copies |
|---|---|
| **SCNR FOR SACLANTCEN** | |
| SCNR Belgium | 1 |
| SCNR Canada | 1 |
| SCNR Denmark | 1 |
| SCNR Germany | 1 |
| SCNR Greece | 1 |
| SCNR Italy | 1 |
| SCNR Netherlands | 1 |
| SCNR Norway | 1 |
| SCNR Portugal | 1 |
| SCNR Turkey | 1 |
| SCNR U.K. | 1 |
| SCNR U.S. | 2 |
| SECGEN Rep. SCNR | 1 |
| NAMILCOM Rep. SCNR | 1 |
| **NATIONAL LIAISON OFFICERS** | |
| NLO Canada | 1 |
| NLO Denmark | 1 |
| NLO Germany | 1 |
| NLO Italy | 1 |
| NLO U.K. | 1 |
| NLO U.S. | 1 |
| **NLR TO SACLANT** | |
| NLR Belgium | 1 |
| NLR Canada | 1 |
| NLR Denmark | 1 |
| NLR Germany | 1 |
| NLR Greece | 1 |
| NLR Italy | 1 |
| NLR Netherlands | 1 |
| NLR Norway | 1 |
| NLR Portugal | 1 |
| NLR Turkey | 1 |
| NLR UK | 1 |
| NLR US | 1 |
| Total initial distribution | 236 |
| SACLANTCEN Library | 10 |
| Stock | 34 |
| Total number of copies | 280 |