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CALCULATION OF REVERBERATION AND AVERAGE INTENSITY OF
BROADBAND ACOUSTIC SIGNALS IN THE OCEAN BY MEANS OF
THE RAIBAC COMPUTER MODEL

by

WOLFGANG BACHMANN and BERNARD DE RAIGNIAC

1 APRIL 1976

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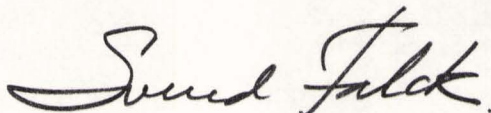
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Calculation of reverberation and average intensity of broad-band acoustic signals in the ocean by means of the RAIBAC computer model

Wolfgang Bachmann and Bernard de Raigniac .

NATO, SACLANT ASW Research Centre, Viale San Bartolomeo, 400, I-19026 La Spezia, Italy
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Based on the conventional ray-tracing principle, a fast numerical algorithm is developed that calculates averaged propagation loss and reverberation of underwater sound. The averaging is performed by summing incoherently the contributions of individual ray bundles to each of the rectangular cells into which the range/depth plane is divided. The resulting matrix description of the average intensity field is used to calculate reverberation. The propagation-loss algorithm is preliminarily tested against mathematical test functions and measured data.

Subject Classification: [43] 30.20, [43] 30.40; [43] 85.84.

INTRODUCTION

This paper deals with numerical models of underwater sound propagation. These models are computer programs that predict the sound field for arbitrary values of signal and ocean parameters. Deterministic propagation modelling nowadays is close to perfection in the sense that there is hardly any characteristic of the ocean structure that could not be taken properly into account; for example, variation of the sound-speed profile with range,¹ or even with azimuth,² water density profiles,³ or surface and bottom elevation profiles.⁴⁻⁶

All these models rely strictly on the assumption of a deterministic medium whose structure is known with any desired precision. In the real ocean, all parameters exhibit random temporal and/or spatial fluctuations superimposed to some mean value, which in turn varies, with the width of the observation window. In the early days of modeling,⁷ the accuracy of the computations was so low that the randomness of the medium appeared to be negligible. Today, in contrast, the degree of mathematical accuracy and resolution of the most used modern propagation models (for example, NISSM II⁸ or FACT⁹) is so good that it can be misleading when applied to the prediction of expected propagation conditions. These models predict sound-field-fine structures that are necessarily only one moment's representation of a stochastic process. A more adequate use of such models to their full capability would be to represent the variable medium in a Monte-Carlo-type simulation of sound propagation.

The new goal in the development of propagation models is bound to be the *stochastic propagation model* (often referred to, in a too narrow sense, as "statistical ray tracing"). There have already been several promising theoretical attempts to tackle a part of the problem.¹⁰⁻¹² Specially noteworthy is the numerical algorithm of Schneider,¹³ where scattering coefficients are interpreted as probability density functions. But we still seem to be far from having an operational stochastic propagation model available. Certainly, the minimum requirement for a sound propagation model to be called "stochastic" is that it can provide statistical mean values for the sound-field intensity. As oceanic

sound propagation is unstationary, both in time and space, the "mean" values have to be expressed in terms of the temporal and spatial cell sizes for which they are calculated.

Following this definition, the RAIBAC algorithm cannot yet be called "stochastic." It treats only the partial problem of algebraic averaging over given spatial cells. Though this step looks almost trivial, it already shows some useful features which distinguish it from really deterministic models.

(a) When analyzing propagation loss experiments, the computer predictions can be adapted to the actual averaging cell size of the data.

(b) The intensity predictions are less sensitive to "high-frequency" perturbations of the sound-speed profile. With increasing averaging cell size, the false caustics and false shadows disappear more and more and the prediction becomes representative for a wider class of sound speed profiles and for a larger region in space.

(c) The numerical effort is proportional to the requested amount of information: the larger the cell size in a given range and depth region, the less computing time is needed. In nonaveraging propagation loss algorithms, the numerical effort cannot be reduced because of under-sampling problems.

I. THEORY

A. Conventional ray theory

The loss factor L is defined by the following relation:

$$I_p = I_0 L, \quad (1)$$

where I_0 is the sound intensity at reference distance from the sound source; I_p , the sound intensity at a point P ; and "sound intensity" is understood as the square of the pressure amplitude. The loss-factor L_ν of the ν th eigenray joining source and receiving-point P is composed of four different contributions:

$$L_\nu = SARD, \quad (2)$$

where

S is the geometrical spreading loss factor (called spreading loss in the following);

$A = \exp[-\epsilon r]$, absorption loss factor for path length r and absorption coefficient ϵ ;

$R = \rho_b^{W_b} \times \rho_s^{W_s}$, boundary reflection loss factor composed of reflection losses ρ at surface on bottom: ρ_b, ρ_s = bottom, (or surface) reflection loss; W_b, W_s = number of bottom (or surface) bounces already made; and

D is the deviation loss of vertical beam pattern, $D \leq 1$.

Incoherent addition of all arrivals gives an estimate of the mean loss factor, free of Lloyd-mirror effects:

$$L = \sum_{\nu=1} L_{\nu} . \quad (3)$$

At low enough frequencies there might be stable interference patterns with periods much larger than the usually chosen averaging cell sizes of 10 to 1000 m. In such cases, the incoherent addition is strictly misleading. At the expense of a more complex calculation, Spofford introduced the "semicoherent addition" (in the model FACT), an auxiliary algorithm to automatically change from incoherent to coherent summation in such critical zones.

The geometrical spreading loss factor of each eigenray is calculated on the assumption that, in an otherwise loss-free medium with $A = R = D = 1$, energy initially confined to a narrow bundle of rays will continue to be confined within that bundle throughout its propagation.¹⁴ Following Lewis and Keller¹⁵ and keeping in mind our simplistic "intensity" definition, Eq. 1, the geometrical result $S_{\nu} = \sigma_0 / \sigma_p$ is corrected by the ratio n_0 / n_p (equals $\cos \beta / \cos \alpha$ in a layered medium, see Fig. 1) of refraction coefficients to take account of the depth variation of sound speed:

$$S_{\nu} = \frac{I_{p,\nu}}{I_0} = \frac{\sigma_0 \cos \beta}{\sigma_p \cos \alpha} , \quad (4)$$

where

σ_0 / σ_p is the ratio of ray bundle cross sections at reference distance and at point P ;

α , the ray angle at the source; and

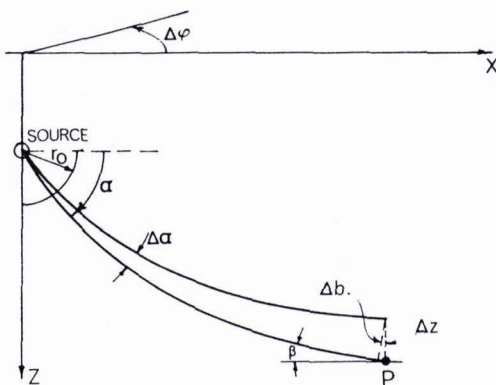


FIG. 1. Ray geometry.

β , the ray angle at point P .

Choosing the z axis to be vertical and assuming cylindrical symmetry about it (see Fig. 1), we find

$$\begin{aligned} \sigma_0 &= \Delta \varphi \Delta \alpha r_0^2 \cos \alpha , \\ \sigma_p &= \Delta \varphi \Delta b x , \end{aligned} \quad (5)$$

where $\Delta \alpha$ is the vertical angular ray spacing; $\Delta \varphi$, the horizontal beamwidth; r_0 , the reference distance; x , the horizontal distance from source to P ; and Δb , the length of the arc representing a part of the wavefront at P . To calculate the spreading loss S at a given point in the x, z plane, the opening $\Delta \alpha$ of the ray bundle is made infinitesimally small. Inserting Eq. 5 in Eq. 4 gives

$$S = \frac{r_0^2 \cos \beta}{x (\partial b / \partial \alpha)_{\beta = \text{const}}} = \frac{r_0^2}{x (\partial z / \partial \alpha)_{\beta = \text{const}}} . \quad (6)$$

Equation 6 assumes that the wavefront at the point of intensity calculation is well behaved, i.e., that it does not "fold," as it would at caustics and focal points.

The main purpose of this paper is to obtain a simple and rapid procedure of calculating spatially-averaged intensity. Averaging, while necessarily decreasing the resolution, decreases also the required amount of output information. In principle, we can thus afford a certain amount of inaccuracy (or, more precisely, unbiased quantization errors), which will be smoothed out. This simplify-and-average strategy is applied in the following four steps: (1) averaging in the vertical direction, (2) averaging in the horizontal direction, (3) averaging in rectangular cells, and (4) smoothing of quantization errors.

B. Averaging in the vertical plane

To numerically obtain an average of the spreading-loss factor over a vertical distance Δz one would usually calculate Eq. 3 for a large number of points in this interval and then compute the mean value. Our first "simplify-and-average" step is to return to ray bundles with finite opening (Eqs. 4 and 5):

$$S_{\nu}^{(1)} = \frac{r_0^2 |\Delta \alpha|}{x |\Delta z|} . \quad (7)$$

As Δz tends to grow with increasing range, the averaging is performed over larger and larger vertical intervals.

C. Averaging in the horizontal direction

Having performed the above-described averaging along a vertical line of length Δz , the next step is to average these results along a horizontal interval of length u (see Fig. 2). Again, one could do this by calculating a number of Δz averages $S_{\nu}^{(1)}$ in the interval u and then computing their mean value. A less accurate, but much simpler procedure is to replace $|\Delta z|$ in Eq. 7 by the mean of $|\Delta z|$ at the left and right side of the interval u ,

$$\overline{\Delta z} = \frac{1}{2} (|\Delta z_{\text{left}}| + |\Delta z_{\text{right}}|) . \quad (8)$$

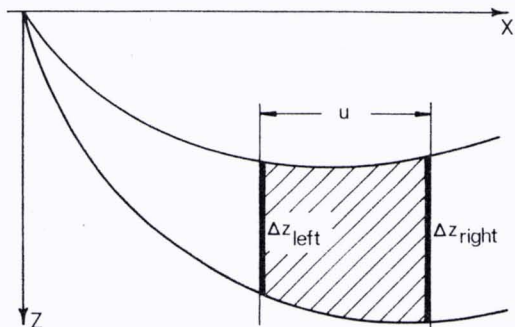


FIG. 2. Averaging over horizontal distance u .

Thus the average loss factor for a certain area in the range/depth plane (see shaded area in Fig. 2) is

$$S_{\nu}^{(2)} = \frac{r_0^2 |\Delta \alpha|}{x \Delta z} \tag{9}$$

D. Averaging in rectangular cells

In the previous step the averaging cells had a constant width u and variable upper and lower boundaries that are segments of rays. This is still inconvenient for two reasons: (1) Besides the averaged loss factors, a computer would need to remember the associated shape and position of the cells; and (2) The vertical ray-spacing can become very small near caustics and focussing points, hindering the averaging process exactly where it is most needed.

These problems are met by introducing a system of rectangular averaging cells, with constant size, u by v (see Fig. 3). For each cell we calculate and ultimately store only one number, namely the average loss factor estimate for this cell. The sound-intensity field in the range/depth plane is described by a two-dimensional matrix, each element being the averaged intensity for that cell.

The new problem arising from having cells of constant shape is how to calculate the contribution of rays that intersect a cell. The most logical way seems to be to weight the contribution of each ray pair by the area μ it covers in a cell divided by the total area uv of the cell. Then the loss factor contribution of the ν th ray pair to cell $\{k, m\}$ is

$$S_{\nu}^{(3)} = \frac{\mu_{\nu}}{uv} S_{\nu}^{(2)} = \frac{\mu_{\nu} r_0^2 |\Delta \alpha_{\nu}|}{uvx \Delta z_{\nu}} \tag{10}$$

where μ_{ν} is the area of ν th ray pair cut out from the cell $\{k, m\}$. The area μ_{ν} can be approximated as

$$\begin{aligned} \mu_{\nu} &= uv, & \text{case 1,} \\ &= \frac{1}{2} uv, & \text{case 2,} \\ &= u \overline{\Delta z} / N_{\nu}, & \text{case 3,} \end{aligned} \tag{11}$$

assuming straight-line ray segments within a cell.

After expressing the horizontal distance to the center of a column by the column number m , we find

$$S_{\nu}^{(3)} = \frac{r_0^2 |\Delta \alpha|}{(m - \frac{1}{2}) u d_{\nu}^*} \tag{12}$$

where

$$\begin{aligned} d_{\nu}^* &= \overline{\Delta z}, & \text{in case 1,} \\ &= \overline{\Delta z}, & \text{in case 2,} \\ &= N_{\nu} z, & \text{in case 3,} \end{aligned}$$

N_{ν} is the number of boxes in the same column m that are intersected by the ν th ray pair (see Fig. 3);

v , the height of cell;

u , the width of cell.

Equation 12 is the basis of the RAIBAC model. During the tracing of a ray, one has only to calculate $S_{\nu}^{(3)}$ for each box $\{k, m\}$ that is enclosed or intersected by the ray being traced and the preceding ray. The result is multiplied with the appropriate loss factors ARD (see Eq. 2) and added to the proper loss matrix element. After having traced all rays, each matrix element \hat{L} contains a sum (see Eq. 3) of individual ray bundle contributions

$$\hat{L}^{(3)}[k, m] = L^{(3)}[v(k - \frac{1}{2}), u(m - \frac{1}{2})] = \sum_{\nu} A_{\nu} R_{\nu} D_{\nu} S_{\nu}^{(3)} \tag{13}$$

E. Smoothing of quantization errors

The spreading loss $S_{\nu}^{(3)}$ is essentially calculated by a counting process. Therefore, the function $L^{(3)}$ which is proportional to the sound-intensity field is discontinuous in range and depth. This is evident from the definition of d^* in Eq. 12. To reduce this quantization error a further averaging process is applied. $\hat{L}^{(3)}[k, m]$ is convolved with a triangular smoothing function extending over one neighbour element to each side:

$$\hat{L}^{(4)}[k, m] = \frac{1}{4} \sum_{\xi=-1}^1 \sum_{\zeta=-1}^1 2^{-|\xi| - |\zeta|} \hat{L}^{(3)}[k - \xi, m - \zeta] \tag{14}$$

This is the final expression used in RAIBAC.

F. Behavior at reflections and caustics

To avoid a "false" crossing of rays ($\Delta z = 0$) at reflections from the boundaries, the medium (and coordinate system) is "unfolded" in the manner of Barberger¹⁶ (see Fig. 4), with the sound-speed profile ex-

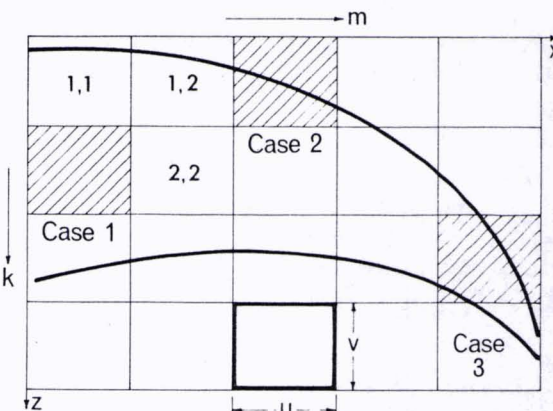


FIG. 3. The three different cases of rays intersecting the rectangular cells $\{k, m\}$ in the range/depth plane.

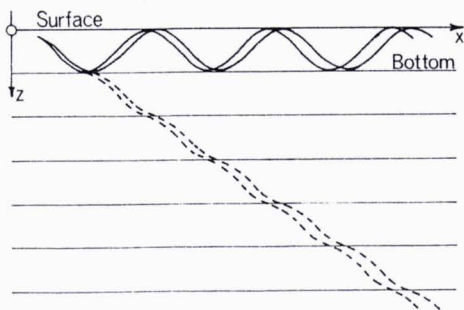


FIG. 4. Unfolding of the medium to avoid ray crossing.

tended by mirror imaging at the boundaries. However, this procedure cannot prevent ray crossing at caustics.

As the assumption that energy remains confined between two rays is not true at caustics, an obvious step would be to introduce here other approximations that take closer account of the wave character of the sound field. This has been already considered in several propagation models (e. g., NISSM II,⁸ FACT,⁹ or MPP¹).

Indeed, these wave correction methods avoid false singularities of the sound field. They introduce, instead, the apparently more realistic rapid fluctuations of the sound field near caustics. For the prediction of expected values of propagation loss one would need to form some kind of a mean over a few fluctuation periods.

The sound field algorithm described in this paper circumvents these problems. As the vertical ray spacing Δz becomes smaller than the chosen height of the cell v (for example, at a caustic) the weighting assigned to that particular ray bundle decreases (see Eq. 10). The final spreading loss factor for the cell where the caustic occurs is determined by the number of rays crossing this cell.

II. COMPUTER PROGRAM

A. Primary sound field

This and the following two chapters describe some features of the RAIBAC computer program (Reverberation and Average Intensity of Broad-band Acoustical Signals). It is an 800-statement FORTRAN IV program that calculates the sound-field intensity and reverberation levels for a monostatic sonar. The field calculations are based on the averaging method described in the previous chapters.

The characteristic feature of this program is the rectangular averaging cell of constant size. The quantization effects in the input data are:

- (a) the sound-speed profile has a constant gradient in each layer,
- (b) the volume-scattering profile has a constant level in each layer, and
- (c) the source depth is an integer number times layer thickness.

1. Ray splitting

At maxima of the sound-speed profile and at boundaries of the medium, rays with grazing incidence may split into an upward and downward going component, leaving a shadow zone between (see Fig. 5).

These critical rays are traced twice, in upward and downward mode separately. Thus, only the first of the periodically repeating ray splits is taken into account (see Fig. 5). The other splits will have to be considered later by a general "leakage" term.

A sound-speed profile specified by N data points can have up to $\frac{1}{2}N$ maxima. To avoid excessive fluctuations of the computed sound field, the profile is smoothed by successive convolutions with a triangular window until less than five maxima are left. It is intended to implement later the more sophisticated smoothing method of Sluyterman,¹⁷ where the characteristic patterns of the sound field are optimally preserved.

Before beginning the ray tracing, the critical source angles α_{crit} are determined from the sound-speed profile $c[z]$:

- (1) c_{crit} = maximum of $c[z]$ curve, or value at the upper or lower boundary of the sea,
- (2) $c_{crit} > \text{preceding } c_{crit} > c_s$; (search for c_{crit} in upward and downward direction separately, starting from source depth), and
- (3) $\alpha_{crit} = \arccos[c_s/c_{crit}]$,

where c_s is the sound speed at source depth.

The maximum angular spacing of ray pairs is limited to one degree by automatically inserting some rays between the critical rays when they are too far apart. However, the influence of maximum ray-bundle spacing on the accuracy has not yet been studied in detail.

2. Ray-termination

Ray-tracing programs usually include a provision for terminating a ray that has been reflected too often. This is justified by the assumption that the cumulated reflection losses cause this ray's intensity to be much smaller than the intensity of other rays at the same dis-

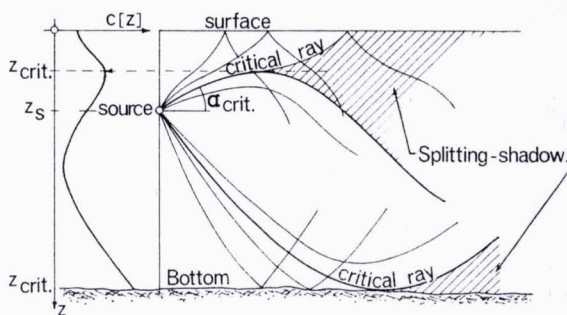


FIG. 5. Ray splitting at maximum of the sound-speed profile and at boundaries.

tance, so that its contribution is comparatively negligible. However, there is always the danger that rays that would eventually illuminate a shadow zone are eliminated. A safer method is applied in RAIBAC by adding the following condition: Starting the ray tracing from source angle 0°, a ray can only be terminated beyond the horizontal distance where the vertical spacing Δz of the preceding ray became equal to the water depth. This method is only efficient for the large ray spacings as used in RAIBAC.

3. Absorption loss

The absorption coefficient ε is calculated from a combination of empirical formulae of Thorp¹⁸ and Shulkin and Marsh¹⁹:

$$\epsilon = (1 - C_1 Z) \left(C_2 \frac{F_2 F^2}{F_2^2 + F^2} + C_3 \frac{F_3 F^2}{F_3^2 + F^2} + C_4 \frac{F^2}{F_3} \right) \frac{dB}{km}, \quad (16)$$

where

$$C_1 = 6.32 \cdot 10^{-5}; \quad C_2 = 0.155;$$

$$C_3 = 2.03 \cdot 10^{-2} Y; \quad C_4 = 2.93 \cdot 10^{-2};$$

$$F_2 = 1.7;$$

$$F_3 = 21.9 \exp[13.82 - 3500/(T + 273)];$$

T is the temperature in degrees centigrade;

Y, the salinity in ‰;

F, the frequency in kilohertz;

and

Z, the source depth in meters.

B. Reverberation

Reverberation is the signal energy scattered back from randomly distributed small inhomogeneities and received at the location of the sound source. Usually the distribution of scattering centers in the ocean and on the boundaries can be considered as a short-time stationary Poisson process. Under this assumption one can interpret the theoretical results of Faure²⁰ so that the ocean behaves like a linear, time-invariant filter with regard to the expected value of reverberation intensity. Hence

$$I_R[t] = \int_{-\infty}^{\infty} I_T[t - t'] p[t'] dt', \quad (17)$$

where I_R is the reverberation intensity;

I_T, the transmission intensity at reference distance r₀;

p[t], the power-impulse response of the medium.

For rectangular-shaped signals,

$$I_T[t] = I_0, \quad 0 \leq t \leq \tau, \\ = 0, \quad \text{otherwise}, \quad (18)$$

Eq. 17 gives

$$I_R[t] = I_0 \int_{t-\tau}^t p[t'] dt'. \quad (19)$$

For signals with very short pulse duration δτ compared to the duration of p[t], the integral, Eq. 19, simplifies again:

$$I_R[t] = I_0 \delta\tau p[t]. \quad (20)$$

From the comparison of the definition of scattering strength,²¹

$$q = \frac{r^2 I_R[t]}{I_i \delta\sigma}, \quad (21)$$

where

q is the backscattering strength of volume q_v or boundary q_b,

r, the distance (path length) from the receiver to the illuminated area or volume,

I_p, the incident plane-wave intensity; and

δσ, the illuminated volume element (δσ_v) or area element on the boundary (δσ_b)

(Eq. 21 is valid only if the mean diameter of δσ is much smaller than r),

we find

$$p[t] = \frac{I_p[t] \delta\sigma q}{r^2 I_0 \delta\tau}. \quad (22)$$

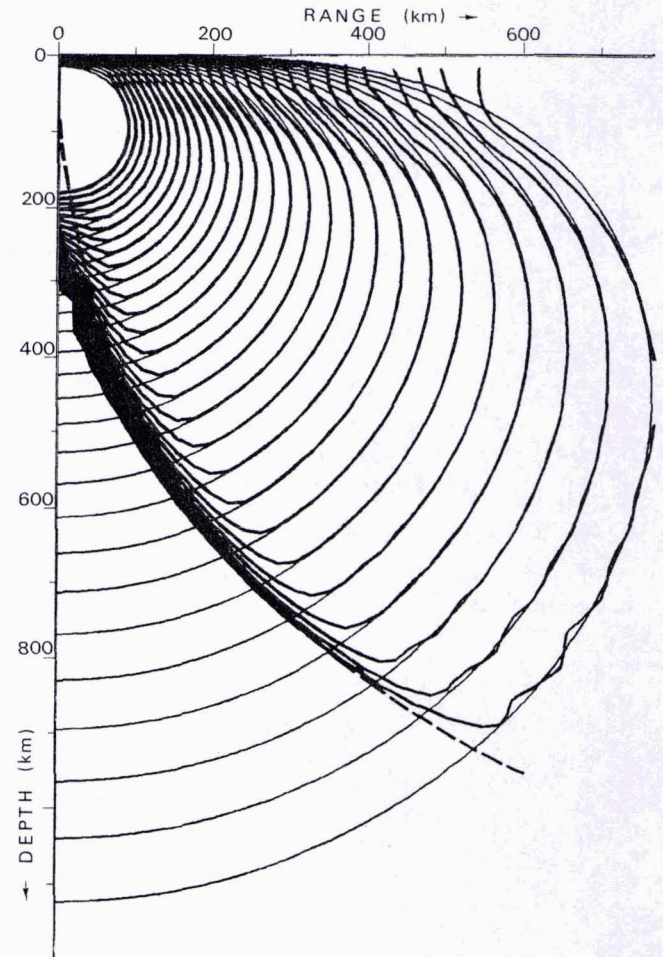


FIG. 6. Comparison of exact solution of wave equation with RAIBAC prediction: Isointensity curves of primary sound field for c[z]=gz, where g=0.017/sec. The contour interval is 2.5 dB.

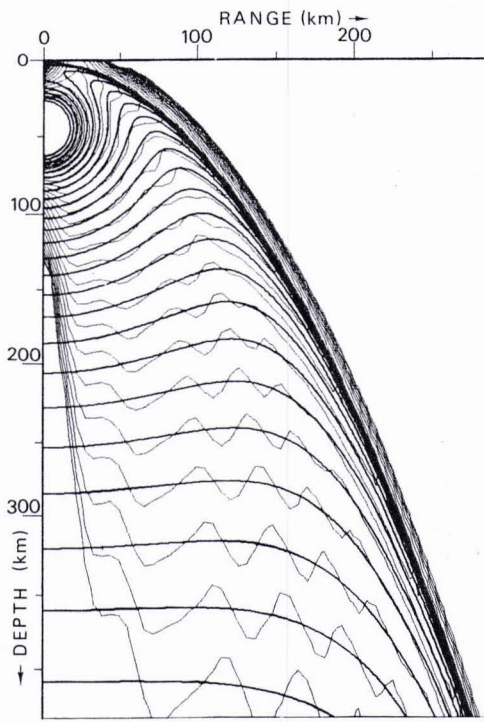


FIG. 7. Comparison of exact ray solution of wave equation with RAIBAC prediction: Isointensity curves of primary sound field for $c[z] = [c_s^2/(2gz)]^{1/2}$, where $c_s = 1500$ m, and $g = 0.017$ /sec. The contour interval is 2.5 dB.

Expressing I_p and r^2 by the propagation loss L and substituting $\delta\sigma$ by $\delta\sigma_b = \frac{1}{2}c\delta\tau x\Delta\varphi$, or $\delta\sigma_v = \delta\sigma_b \cos[\beta]\delta z$, gives

$$p_b[t] = \frac{L^2[t]}{r_0^2} \frac{1}{2} c_s x \Delta\varphi q_b[\beta] \tag{23}$$

and

$$p_v[t] = \frac{L^2[t]}{r_0^2} \frac{1}{2} c_s x \Delta\varphi \int_{(h)} q_v \cos[\beta] dz, \tag{24}$$

where

$\Delta\varphi$ is the azimuthal beamwidth of the transmitter/receiver combination and

h is the illuminated depth.

The averaging principle applied for the propagation loss computation is also applied to the reverberation. Consequently, a one-dimensional matrix $\hat{p}[m']$ is defined

$$\hat{p}[m'] = p[2u(m' - \frac{1}{2})/c_s] \tag{25}$$

to represent the average reverberation power impulse response over time intervals that correspond to the cell width u . The contribution of the cell $\{k, m\}$ illuminated by the ν th ray bundle is

$$\hat{p}_\nu[m'] = \frac{c_s \Delta\varphi u (m - \frac{1}{2})}{2r_0^2} L_\nu^{(3)}[k, m]^2 (\hat{q}_\nu[k] v \cos[\bar{\beta}] + q_b[\bar{\beta}]), \tag{26}$$

where

$$m' = [(k^* - \frac{1}{2} - z_s/v)^2 + (m - \frac{1}{2})^2]^{1/2} + \frac{1}{2}, \text{ rounded to the nearest integer;}$$

k^* is the layer number in the unfolded medium (see

Fig. 4);

$\hat{q}_\nu[k]$, the volume-scattering strength averaged over the k th layer,

$\bar{\beta}$, the arithmetic mean of β of the ν th ray bundle at the range $(m - \frac{1}{2})u$.

C. Theoretical verification

The verification of computer models of sound propagation seems to be as difficult as the construction of such models. Before even starting such a procedure, we have to be sure of which aspect of the model we want to verify. Is it

- (1) The validity of the underlying mathematical formulae?
- (2) The validity of the physical assumptions?
- (3) The coding of these formulae into a computer program?

or

- (4) The agreement with some measured data?

Inspired by D. H. Wood of SACLANTCEN, we have adopted the following two-stage test for the central part of the propagation models, the computation of the incident sound field:

Test A. Comparison of model results with exact theoretical examples. This test is performed into extreme and unrealistic regions of sound speed, range, and depth so as to exaggerate the effect of any errors in the mathematical formulae or the computer programming.

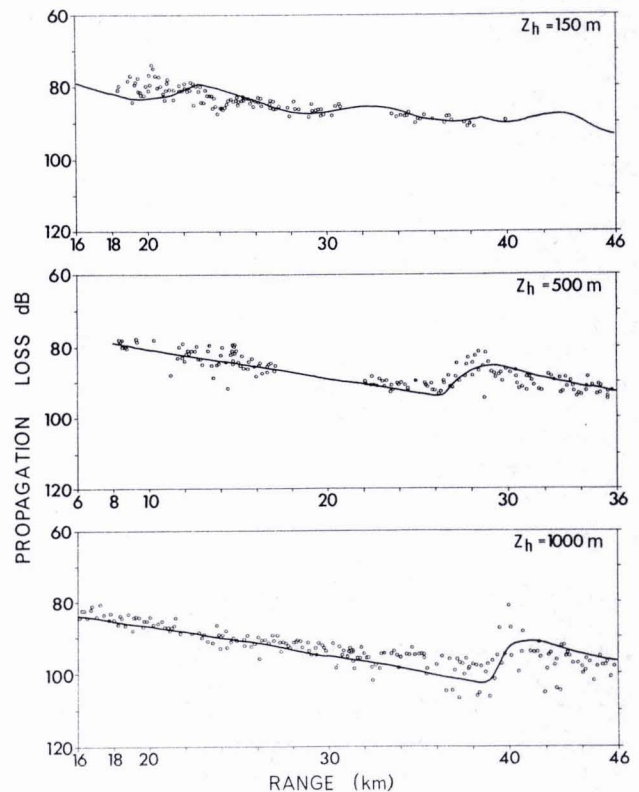


FIG. 8. Comparison of measured data with RAIBAC predictions for deep water. Source depth 50 m; frequency 3500 Hz; hydrophone depth z_h as indicated above.

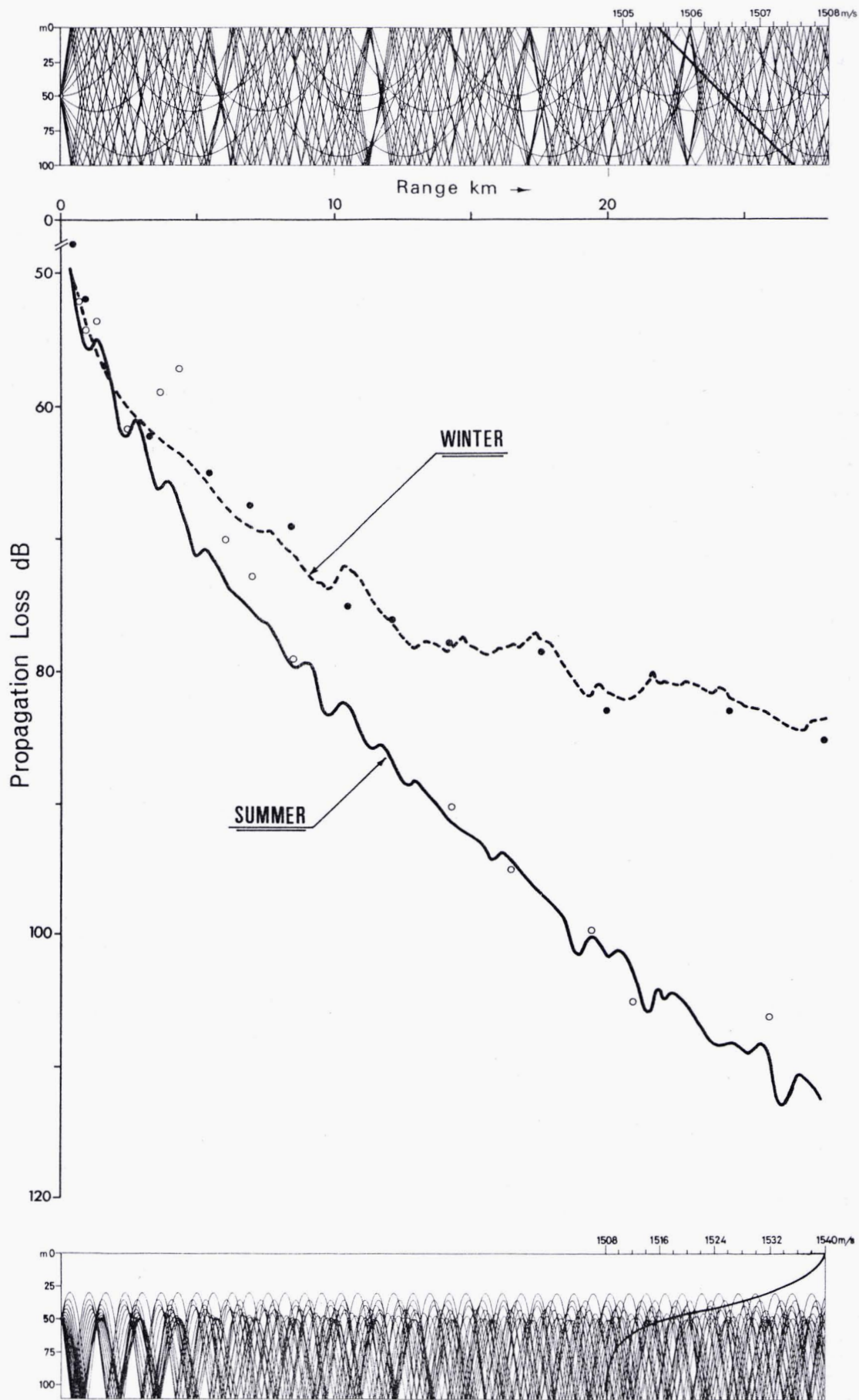


FIG. 9. Comparison of measured data with RAIBAC predictions for shallow water. Bottom depth 100 m; frequency 3200 Hz; receiver depth 60 m; dots and circles are measured data.

Test B. Comparison of model results with experimental data. As Test A is already completed at this stage, the outcome of Test B tells something about the validity of the model's assumptions.

For the application of Test A to RAIBAC, the following two cases were considered:

(1) Sound-speed profile: $c[z] = gz$, where g is a constant sound speed gradient. This type of profile has a known, caustic-free, exact solution of the wave equation²² with intensity independent of frequency. Figure 6 shows the comparison of the two intensity contour plots in the range/depth plane. The source depth is 88 km; the gradient of the sound-speed profile is $g = 0.017/\text{sec}$, corresponding to a constant-temperature, constant-salinity sea. The contour interval is 2.5 dB. Within the insonified region the deviation between the exact solution and the RAIBAC program is always less than 0.5 dB.

(2) Sound-speed profile $c[z] = \{c_s^3/(2gz)\}^{1/2}$: This type of profile gives the simplest known caustic.²³ The theoretical ray solution is compared again with the RAIBAC computations in an intensity contour plot (Fig. 7) using 2.5 dB contour interval. Despite of false caustics, RAIBAC does not deviate from theory by more than 1 dB.

As Test A did not give any evidence of errors, either in the mathematical formulae or in the program, we then performed Test B.

D. Experimental verification

Two well-calibrated sets of measured data were used for the comparison with RAIBAC results:

(1) *Deep water case* (frequency is 3.5 kHz, water depth is 2000 m; Mediterranean Sea). The dots in Fig. 8 represent propagation loss data²⁴ measured with a sound source suspended at a depth of 50 m and with a hydrophone at three different depths. The loss curve predicted by RAIBAC remains always within the cloud of data points.

(2) *Shallow water case* (water depth = 100 m; frequency = 3.2 kHz; Mediterranean Sea). Figure 9 shows the comparison of $\frac{1}{3}$ -octave filtered explosive-source measurements²⁵ at a hydrophone depth of 60 m for summer and winter conditions. The large number of bottom bounces in the summer case requires a very high accuracy for the bottom-reflection loss value (say ± 0.05 dB) when used as input to the RAIBAC program. As our knowledge of bottom reflection at small grazing angles is inadequate (say ± 1 dB), we were forced to fit the input value *within these limits* to match the measured propagation loss curve of the summer case.

Thus, the predicted loss curves for both shallow-water cases lie again within the cloud of data points. However, this test has revealed a very high sensitivity to variations of the bottom reflection loss, a fact which might render the prediction of shallow-water, downward-refracting cases questionable.

III. CONCLUSIONS

A new algorithm for sound-field calculations has been developed. Conventional ray-tracing techniques are ap-

plied to compute spatially-averaged values of sound intensity. The averaging process permits a drastic reduction in numerical effort since only a few rays have to be traced. The result of the computation is a two-dimensional matrix. Each matrix element represents the incoherently averaged intensity value of one of the equally sized rectangular cells in the range/depth plane.

This matrix description is also suited for a rapid calculation of reverberation. The elements for each range column are simply weighted by the corresponding scattering strength. The whole concept has been performed in a computer program called RAIBAC. A preliminary test of the propagation loss part against mathematical test functions and measured data has not shown any errors so far.

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