APPLICATION OF RAY TRACING WITH HORIZONTAL GRADIENT TO MONOSTATIC BOUNDARY REVERBERATION

by

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ABSTRACT

Presented is the current work being done at the Naval Research Laboratory (NRL) on the development of a series of computer programs to predict long range, low frequency, monostatic boundary reverberation. The emphasis is on the ray tracing technique and its application to the special problems of estimating the transmission loss of rays that hit the boundaries of the medium. The ray tracing technique is to increment a ray from point to point along its ray path by evaluating Taylor series expansions in arc length of various ray parameters such as range, depth, travel time, and range angle, which are based on the ray equation. Possible horizontal variations in sound speed are accounted for by allowing multiple input profiles. Also, a linearly segmented ocean bottom and a flat surface are assumed. Monostatic boundary reverberation is estimated by means of a range dependent formulation developed at NRL. An underlying assumption of this formulation is that when a ray encounters a boundary, it continues to propagate in the direction of specular reflection, while a small amount of the incident radiation is scattered in all directions. By reciprocity, scattered energy will return to the source-receiver back along those ray paths emitting from the source and passing through the boundary point at which the hit occurred.

INTRODUCTION

The purpose of this talk is to present the current work being done at the NRL on the development of a series of computer programs to predict long range, low-frequency monostatic boundary reverberation. The emphasis will be on the ray tracing technique and its application to the special problems of estimating the transmission loss of rays that hit the boundaries of the medium, especially the bottom.

The tracing of rays utilizes an iterative technique first described in Hudson Laboratory Report 150 by W.A. Hardy et al in 1968. This ray tracing model accounts for the possibility of a horizontal sound speed gradient by allowing for multiple input profiles, and assumes a linearly segmented ocean bottom. This model has been programmed in FORTRAN by J.J. Cornyn of NRL as part of a series of ray tracing and transmission loss programs. The ray tracing model has since been extracted from this series of programs, modified, and then adapted to the purpose of calculating monostatic boundary reverberation. Also, computer programs have been written which organize the ray tracing results and estimate reverberation by means of a formulation developed by J.T. Warfield of NRL.

I. ORGANIZATION OF PROGRAMS

The operation of the series of programs proceeds in four basic stages [Fig. 1]. First, since the modelling of the velocity field is a problem unto itself because of the possible presence of horizontal variations in sound speeds, it is tackled separately. Therefore, first a magnetic tape is generated on which is written the multiple profiles located at discrete ranges along the track of interest. Each profile consists of the sound speed and its first two derivatives with respect to depth, specified at discrete depths and the ocean bottom as discrete depth versus range points.

This tape serves as input to the ray tracing program. This program employs the iterative ray tracing technique to determine all boundary hits for the rays of interest. These points as well as the associated travel times, ray angles at the boundary, and transmission losses are written onto an output tape. An initial ray trace may not furnish enough information to adequately describe where the boundary hits occur. If this is the case, additional rays will be traced and additional ray tracing output tapes created.

The next computer program uses as input the output tapes from one or more ray traces and reorganizes this information onto another magnetic tape. By reorganizing is meant the creation of "order contours", which will be discussed in a moment. This reorganizing procedure is done separately, due to computer storage requirements and for the convenience of handling multiple ray traces.

Finally, the reorganized tape of ray tracing information is the input to reverberation calculating program, which computes the boundary reverberation function at discrete times.

II. REVERBERATION FORMULA

Before proceeding to a description of the ray tracing model, I would like to briefly outline the model used for predicting monostatic boundary reverberation.

Equation 1 is the formula used for determining the reverberation from a boundary which is detected at the source-receiver at time t.

$$R(t) = I_1 \, \Phi \int_0^{\infty} \Big[\sum_{p \in m(x)} \; \sum_{r \in m(x)} C_{\substack{x \\ pr}}^{(x)}(t) \cdot I(p,r,x,z) \Big] \, x dx \qquad \text{[Eq. 1]}$$

I_1 is the source level relative to 1 yd and the remaining expression represents an integration over the three-dimensional boundary, assuming separable source-receiver beam patterns and an azimuthal symmetry of the boundary.

An underlying assumption in the development of the reverberation formula, is that when a ray encounters a boundary, it continues to propagate in the direction of specular reflection, while a small amount of the incident radiation is scattered in all directions. By reciprocity, scattered energy will return to the source-receiver back along those ray paths emitting from the source and passing through the boundary point at which the hit occurred.

Consider now, the expression within the brackets, which is evaluated at each range x. The set $m(x)$ is a set indexing all possible ray paths from the source-receiver which pass through the boundary at range x. A single term of the double sum represents the contribution to reverberation from energy travelling from the source to the boundary point (x, z) along path p and returning along path r [see Fig. 2].

C is a characteristic function which is equal to one if the reverberation contribution I, attributed to the ordered ray pair, (p,r), will be detected at the source-receiver at time t, and zero otherwise. The times of detection will be those times t satisfying the inequality of Eq. 2.

$$t-D \leq T_p(x,z) + T_r(x,z) \leq t \ . \qquad [\text{Eq. } 2]$$

D is the signal duration and T is the one-way travel time for a signal travelling to the boundary at range x, along the path denoted in the subscript.

The contribution, if detected, is given by Eq. 3.

$$I(p,r,x,z) = \frac{B(\theta_p)\ \overline{B}(\theta_r)}{L_p(x,z)\ L_r(x,z)}\ \sigma(\overline{\theta}_p, \overline{\theta}_r) \ . \qquad [\text{Eq. } 3]$$

B and $\overline{B}$ are the transmitting and receiving beam patterns respectively, and the L's are the transmission losses suffered by travelling along the respective paths and σ is the boundary scattering strength.

III. ORDER CONTOURS

In order to get a visual picture as to how the possible paths between the source-receiver and the boundary at range x are found, an explanation of order contours is necessary. For a fixed ray R such as shown in Fig. 3, each boundary hit and turning point (both of which are called occurrences) is assigned a positive integer order. Surface hits and down turning points (called crests) are defined to be of odd order, while bottom hits and up turning points (called valleys) are of even order. The first bottom hit, or valley, for the fixed ray R is assigned an order of 2. Subsequent bottom hits and valleys are assigned 4, 6, 8 and so on. Surface hits and crests are assigned orders of 1, 3, 5, etc., in such a manner that the orders of the occurrences of the ray R increase with range.

Now consider the contours of curves determined by the boundary hits of identical order, plotted on an initial source angle versus range coordinate system, as shown in Fig. 4. Incidentally, a given contour need not necessarily be a continuous curve, but rather several continuous and disjoint segments.

For the reverberation from a given boundary, only those contours corresponding to that boundary are pertinent. For the purpose of illustration, let us assume that bottom reverberation is being sought. To determine all ray paths between the source-receiver and the bottom at range x, consider a horizontal slice at range x, as shown in Fig. 5. Denote the three paths having initial angles of θ_1, θ_2, θ_3 by 1, 2, 3 respectively. Then the set $m(x)$ consists of the integers 1, 2, and 3, and hence there are 3^2, or 9 possible routes from the source-receiver to the bottom at range x and back again.

In reality, the computer ray tracing program will produce only a finite number of bottom hits, or, in other words, a finite number of points on the order contours. Figure 6 illustrates two order contours of the type produced by the computer program. Here the BT's represent the bottom hits and the V's the valleys determined by tracing rays with initial source angles of θ_a, θ_b, θ_c, etc. The true order contours are approximated by linearly connecting the known bottom hits of the same order. Travel time, transmission loss, and ray hit angle values between computed hits are found in interpolating linearly with respect to range. The valley type turning points are displayed to indicate the end points of the contours.

IV. RAY TRACING PROGRAM

The purpose of the ray tracing then, is to determine the boundary hits and the transmission loss, travel time, and ray hit angle for each hit. The rays traced should be those rays which hit the boundary and are not masked out by the source beam pattern. Also, the resulting boundary hits should approximate the true order contours as accurately as possible. In the search for a ray tracing model, it was felt that such a model should allow for multiple nontrivial sound speed profiles and an irregular bottom contour. Such a model was developed at the Hudson Laboratories at Columbia University of New York. This ray tracing technique is an iterative one in which a ray is incremented from point to point along its ray path. This is accomplished by means of evaluating Taylor series expansions in arc length of various ray parameters such as range, depth, travel time, and ray angle. These expansions are derived from the ray equation shown as Eq. 4.

$$\frac{d}{ds}\left[\frac{1}{v(x,z)} \cdot \frac{d\vec{P}}{ds}\right] = \mathrm{grad}\left[\frac{1}{v(x,z)}\right] . \qquad \text{[Eq. 4]}$$

The sound speed, $v(x,z)$, is assumed to be known at every range x, and depth z, throughout the two-dimensional medium, and ds is a differential increment along the ray path, $\vec{P}$.

The present version of this ray tracing model has the following features:

1. Accommodates multiple profiles for possible horizontal variations in sound speed.
2. Each profile is defined at discrete depths with weighted parabolic interpolation used between specified depths.
3. Assumes a linearly segmented ocean bottom and flat surface, with specular reflection applied to both.
4. Uses incremental finite Taylor series approximations to ray paths.
5. Transmission loss, ray angle, and travel time computed at boundary hits as opposed to predetermined ranges.
6. Each ray assigned two subliminal companion rays for computing the amount of geometric spreading.
7. Multiple bottom loss tables are applied to various range intervals.
8. Individual rays are terminated upon exceeding input maximum allowable travel time or bottom loss ceilings.
9. Up to 500 rays may be traced, not including companion rays.

Operationally, rays are selected whose initial source angles are from that part of the source beam pattern which will experience bottom hits. To each of these "primary" rays, is assigned two "companion" rays whose initial source angles tightly bracket that of the primary ray. The rays are then traced through the medium which is described by the multiple profiles, linearly segmented bottom and flat surface. They are traced on an individual basis between predetermined "rectification" ranges. After all the rays are traced to one of these ranges, which necessarily include the ranges at which the profiles are specified, the accumulated boundary hit and turning point information is written onto a magnetic tape. A computer storage area is then reinitialized and the ray tracing continued to the next rectification range or until some predetermined maximum range is reached. However, individual primary rays will be terminated prematurely if they exceed input travel time or bottom loss ceilings. For each boundary hit by a primary ray, six statistics are recorded.

1. Initial angle of ray.
2. Order of hit.
3. Range of hit.
4. Travel time.
5. Ray angle relative to the boundary.
6. Transmission loss.

For the problem to which the present model is applied, transmission loss is assumed to be made up entirely of bottom loss and geometric spreading loss. Bottom loss is input to the ray tracing program as several bottom loss versus incident angle tables which are applied to different range intervals, and geometric spreading loss is assumed to be given by Eq. 5.

$$\frac{1}{L} = \left| \frac{a^2 \cos \theta_0}{x \cos \theta_x} \frac{d\theta_0}{dz} \right| = \left| \frac{a^2 d(\sin \theta_0)}{x \cos \theta_x} \cdot \frac{1}{dz} \right| \qquad \text{[Eq. 5]}$$

where

a = unit distance
x = range of boundary hit
z = depth of boundary hit
θ_0 = initial source angle of ray
θ_x = ray angle at boundary hit

The companion rays [see Fig. 7] are used to calculate the factor dz, by determining their respective depths and directions at the range where the primary ray hits the boundary. The companion ray which has not yet hit the boundary in this region and is still being directed towards it, is used to determine dz. After a primary ray has been specularly reflected off the bottom, the amount of bottom loss will be added to the ray's accumulated bottom loss.

V. RAY TRACING TECHNIQUE

Now let us look at the ray tracing technique itself. When a ray's location, as well as its ray angle and travel time to that point, are known, the sound speed and its spatial derivatives at that position are used to determine the ray's location and associated parameters at a point further along the ray path. A single profile could be employed throughout the track, but often, actually encountered velocity fields require the specification of different profiles along the track of interest. Each sound speed profile is specified at discrete depths and the first two derivatives of sound speed with respect to depth are approximated at the given depths by means of weighted difference equations. Weighted parabolic interpolation is applied vertically and linear interpolation horizontally to make sound speed and its first two depth derivatives functions of both range and depth. Therefore, the first derivative of sound speed with respect to range can be approximated at any range and depth by a linear first difference with respect to range. With the use of this horizontal gradient in the Taylor series expansions, it is hoped to reckon with the effects on ray paths due to horizontal variations in sound speed.

Once the sound speed, its first two derivatives with respect to depth, and its first derivative with respect to range are determined for a known location on the ray path, various convergence tests are performed to determine how far to increment the ray path location, and associated parameters, by evaluating the Taylor series expansions. Normally, an input incremental step size, Δ_{max}, will be used in evaluating the Taylor series expansions. However, if certain convergence tests fail, the step size will be appropriately reduced. One of these tests require that the ray path does not experience more than some predetermined maximum amount of bending between computed ray locations. By "bending" is meant the change in the trigonometric sine of the ray angle. Another test requires that the chosen step size be used to predict the sound speed at the new ray location within a predetermined accuracy of the "true" sound speed at that point. By the "true" sound speed at a point is meant that value found by interpolating between input profiles. Regardless of these tests, the step size is never allowed to become smaller than some predetermined minimum allowable step size. This is done to expedite the incrementing of the ray path. Therefore, first the initial maximum step size, Δ_{max}, is used to determine a trial sine, given in Eq. 6.

$$\sin \theta_t = \sin \theta_o + \alpha(\Delta_{max}, x_o, z_o, Z_o, D_o, G_o) \qquad \text{[Eq. 6]}$$

The sound speed derivatives Z, D and G,

$$Z = \partial v/\partial z$$

$$D = \partial^2 v/\partial z^2$$

$$G = \partial v/\partial x$$

are all evaluated at the known ray path location, (x_o, z_o). A new step size, Δ', is then determined from Eq. 7.

$$\Delta' = \min \begin{cases} \delta = \sqrt{\dfrac{S}{|\sin \theta_t - \sin \theta_o|}} \cdot \Delta_{max} \\ \sqrt{\dfrac{S\, v_o}{\cos \theta_o\, z_o\, \delta}} \\ \Delta_{max} \end{cases} \qquad \text{[Eq. 7]}$$

where S is an input maximum allowable change in sine, say 0.02. Next, using the new step Δ', a new location (x_t, z_t), is found further along the ray path by evaluating their Taylor series expansions.

By interpolation between two known profiles, a sound speed v_t is determined for this location. Then a computed sound speed, v_c, is found for this location using the Taylor series expansion of Eq. 8.

$$v_c(x_t, z_t) = v_o(x_o, z_o) + Z_o(z_t - z_o) + \frac{D_o}{2}(z_t - z_o)^2 + G_o(x_t - x_o) \qquad \text{[Eq. 8]}$$

If the computed and interpolation sound speeds differ by more than an input accuracy criterion, ϵ, then a new step size is found from Eq. 9.

$$\Delta'' = \min \begin{cases} \dfrac{\epsilon}{|v_t - v_o|} \cdot \Delta' \\ \Delta' \end{cases} \qquad \text{[Eq. 9]}$$

The step size will be further reduced to a predetermined minimum allowable step size Δ_{min} if the cosine of the predicted ray angle θ_t is greater than one, where the cosine is found from Eq. 10.

$$\cos \theta_t = c_t v_t \qquad \text{[Eq. 10]}$$

and where c is found by evaluating a Taylor series in arc length.

The final step size is never allowed to be less than Δ_{min}, except in such cases as approaching a boundary or a known profile location. Once the final step size is determined, a new ray path location and corresponding ray parameters are found by evaluating the different series for this step size. The process is then repeated with the new location being taken as the known ray path location.

Although Δ_{max} and Δ_{min} are inputs to the program, they may be redefined as a ray is traced. If an increase results, Δ_{min} will be taken to be the vertical distance between the known ray depth and the nearest depth, in the direction the ray is headed, at which a sound speed is specified in the input profiles. Δ_{max} will be reduced appropriately, if a straight line projection of the ray direction, of length Δ_{max}, passes outside the medium. Also, when a ray location is determined which is within about 100 m of a boundary, the input Δ_{min} will be used as the iterative step size, without the various tests being conducted.

The boundaries, themselves, are assigned tolerance gates of some vertical distance, such as a half a meter, within which a ray is considered as having struck that boundary. In some cases, interpolation will be employed between computed ray path locations within and without the medium in order to determine the step size necessary to increment the ray to the boundary.

The described ray tracing technique represents an adaptation of the original version to the computation of transmission loss at boundary hits as opposed to predetermined receiver ranges.

CONCLUSIONS

After the reverberation and ray tracing models were programmed, two test cases were run to check the validity of the programs. Both test cases involved the use of a single profile, one case being a constant sound speed profile, the other a constant gradient profile. The computer results for these two test cases compared quite well with the corresponding analytic solutions. The ray tracing technique itself, has been verified for a multitude of cases.

Next, the programs were run on a case for which measured reverberation data existed. This case entailed a track of fairly long range over a relatively rough, upward sloping bottom. The low-frequency signal duration was about 30 s, with the source beam pattern dominated by the main lobe. Input boundary scattering strength tables were based on data appearing in the literature, as were the bottom depth and bottom loss tables.

A comparison of the predicted bottom reverberation versus the total measured reverberation is shown in Fig. 8. For this low frequency study, surface reverberation is assumed to be doppler shifted. The vertical, or reverberation, axis has a scaling of 10 dB between the indicated tick marks. The relative level of the two curves do not agree as well as had been hoped, but the appearance of the peaks at the recorded times seem to correspond fairly well. The peaks of the predicted curve correspond to the main beam of the source striking the bottom. However, the relative level of these peaks are largely determined by a small angular span of rays whose initial source angles usually vary by less than 1°. These so-called "crucial" rays are those rays which experience both valley type turning points and bottom hits with very low grazing angles as they propagate down range. Some of these rays will be completely RSR until an initial bottom hit occurs far down range, at which point the intensity of the given ray will dominate those of other rays hitting the bottom nearby which will have accumulated bottom loss by that range. These "crucial" rays will also tend to encounter the bottom at very low grazing angles where the bottom loss tables are the most questionable. Another problem is the occasional focusing of the "crucial" rays near the bottom. That is, they are sometimes in near-caustic regions as they approach the ocean bottom. Because of the so-called "crucial" rays, the predicted bottom reverberation curve is sensitive to slight changes in bottom depth definition, as was discovered when different approximations to the ocean bottom were used.

The linear segmentation of the bottom is also a problem area. The abrupt changes in slope between linear segments tend to artificially break up the wavefronts. This shows up as radically jagged sections in the order contours, which tends to weaken one's belief in the computer approximation of the contours and the validity of the interpolation performed between computed bottom hits. Finally, averaging over reverberation estimates for several different headings did not significantly improve the results.

Despite these apparent problem areas the main difficulty seems to be in predicting the level of the reverberation curves. Work is continuing on this problem and the extension to the bistatic case. Investigation is underway at NRL to approximate ray intensities in near caustic regions and to more efficiently model the ocean bottom. Also, the ray tracing program is being modified to improve the iterative procedure and the spreading loss estimation.

DISCUSSION

The author said that the rough-surface reflection coefficient was an empirical function of the angles of incidence and reflection.

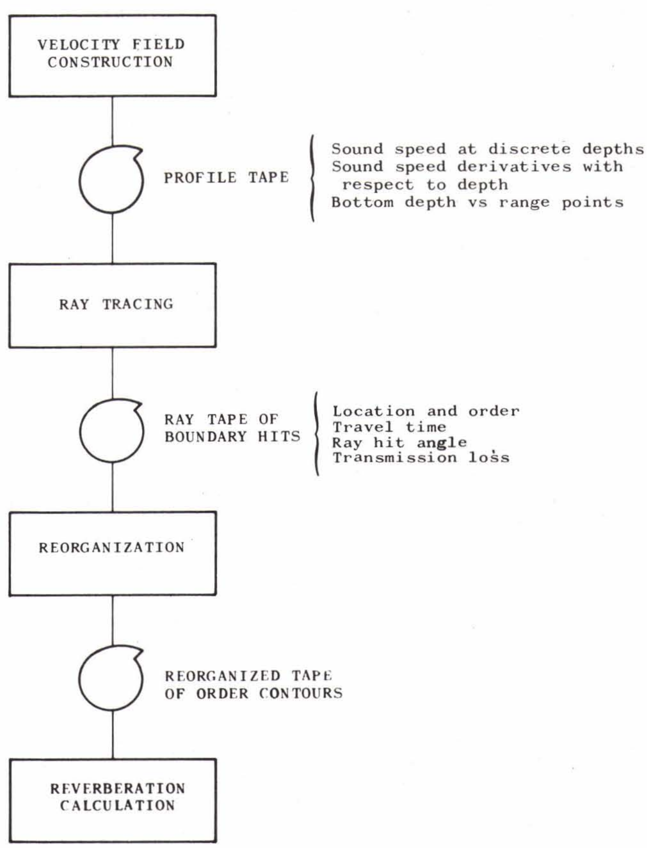

FIG. 1

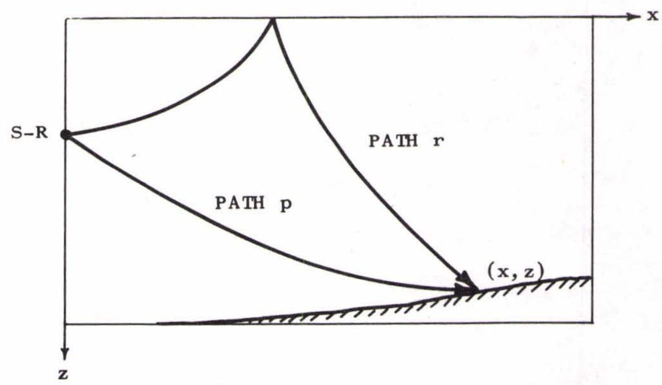

FIG. 2

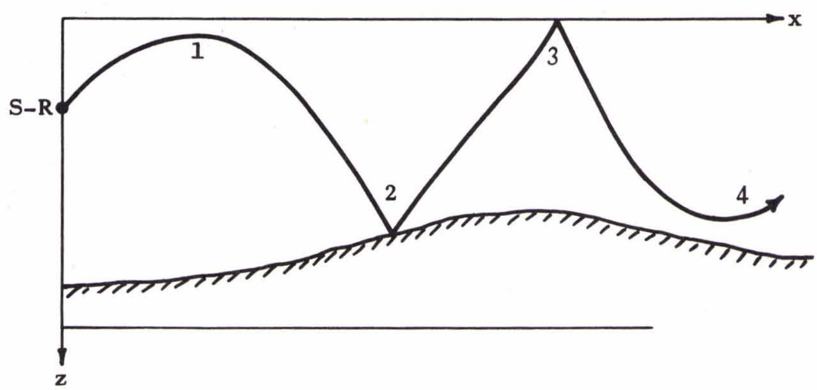

FIG. 3

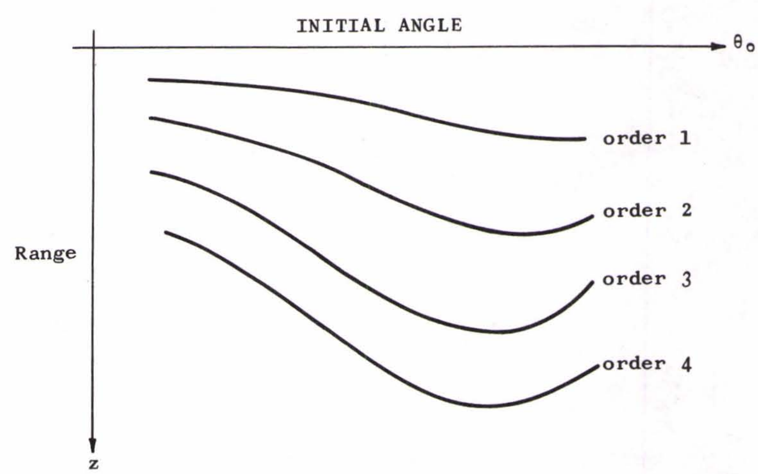

FIG. 4

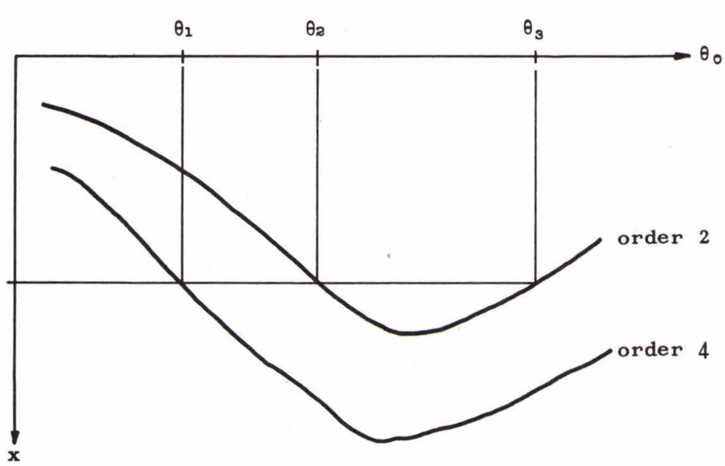

FIG. 5

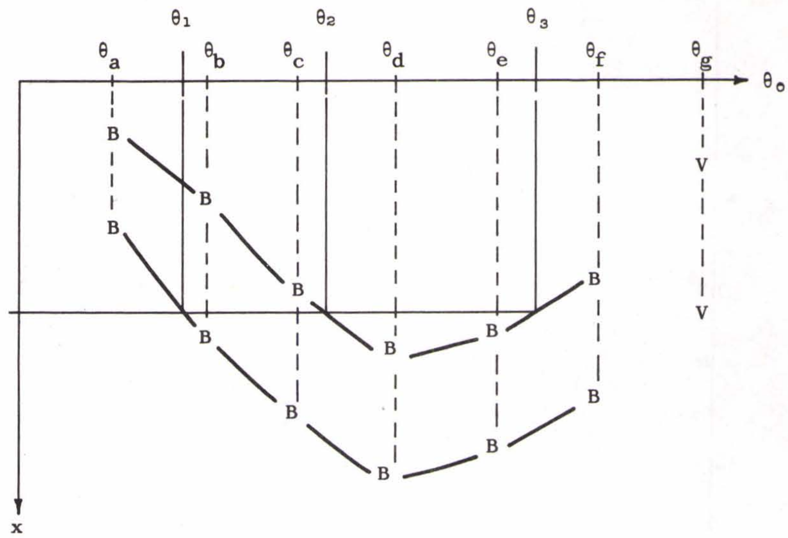

FIG. 6

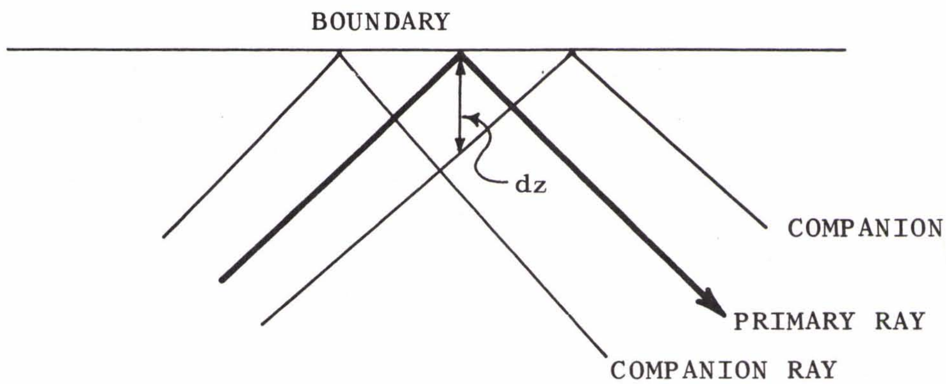

FIG. 7

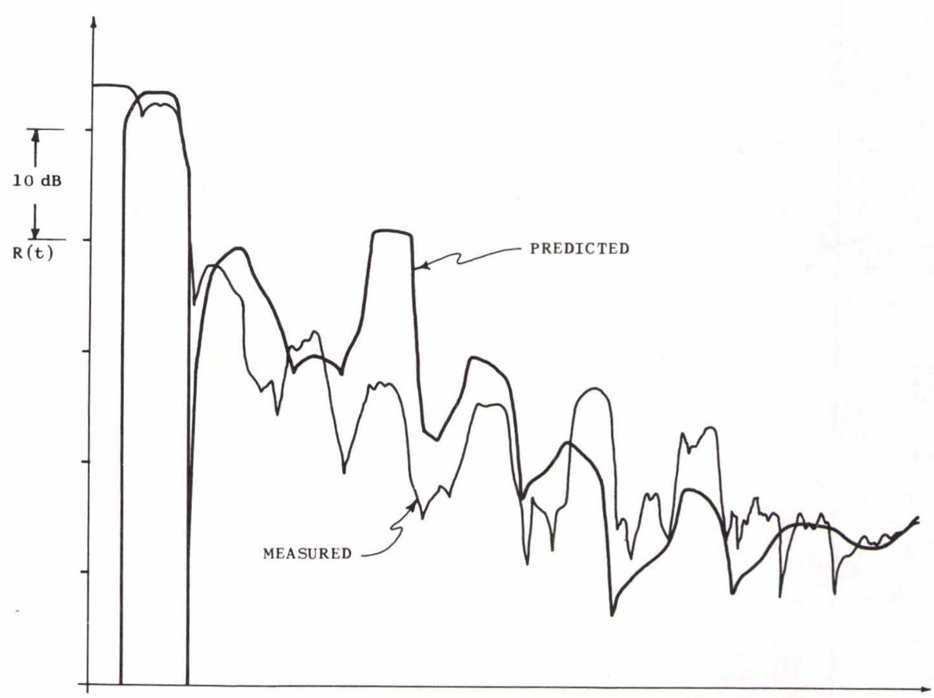

FIG. 8