THE EFFECT OF GRAVITY-FORCED OSCILLATIONS AT THE BASE OF THE DUCT ON ITS EFFECTIVE DEPTH AS A CHANNEL FOR ACOUSTIC RAYS

by

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This talk must begin with a justification, since at first sight (and possibly even after further glances) it is difficult to see what place a problem of classical applied mathematics (which was originally chosen as a companion to the author's earlier paper on below duct sound levels due to scattering from the sea surface) can rightfully claim to occupy in this gathering of ray tracers and computing enthusiasts. This is especially so when one realises that, as in the earlier paper, given here six months ago, the object is to formulate the problem in such a way as to obtain the maximum of predictive results with the minimum of computation.

Nevertheless, this account of the effect of gravity-forced internal waves at the base of an isothermal layer on its efficiency as a duct for acoustic energy is opposite to this particular convention; it has the laudable aims of saving effort for the sceptical perfectionists, and saving face for the lazy intuitionists, by demonstrating that it is not necessary to develop three-dimensional ray-tracing programs, nor to solve numerically the wave equation for a duct with one sinusoidal boundary, in order to take account of the horizontal stratification of sound velocity introduced by internal waves.

This is a very worthwhile simplification, but to achieve it it is necessary to have some background as to the generation and acoustic effects of internal waves; we start, as always, with the classical picture of acoustic propagation in the upper ocean, with the isothermal surface duct, and associated underlying shadow zone at long ranges [Fig. 1]. This is the ideal, on which computations of field intensities can be made with relative ease, as the stratification purely in horizontal planes makes a two-dimensional ray analysis valid.

Unfortunately, in the real world, things aren't that simple. Even if we retain the basic framework of ray theory, there are three major perturbing effects to be taken into account when considering the idealised situation as a predictive model for in-duct propagation. The first of these is diffraction which, as shown schematically in Fig. 2, can distribute energy into zones forbidden by simple ray acoustics. This is obviously important as far as energy levels in the shadow zone are concerned; there is a school of thought that claims that, as far as in-duct propagation is concerned, the effect is negligible. The argument runs that the effect of diffraction is merely to alter the effective depth of the duct, and that the "below layer" field is an integral part of the trapped modes. Even so it is clear that there is a net energy leakage by this mechanism.

The second complication to be considered is scattering from the rough sea surface of energy which is seemingly entrapped within the duct, so that it is deflected into the thermocline region and escapes. This is shown schematically in Fig. 3; much work has been carried out on this, both numerical and analytic (the papers by van Ness, Schweitzer and the present author, to name but three), and it is fair to say that the effect of this mechanism is now quantifiably determined, or determinable, for most 'typical' duct conditions.

Finally, among these mechanisms, we come to anomalous refraction due to internal waves, the least well documented and most speculative of the three. Schematically, the way in which acoustic energy which has been trapped in the duct can be guided out of it by the action of internal waves is shown in Fig. 4; the dominant effect is the change from upward to downward curvature on crossing the internal-waves profile. It was on the basis of this type of diagram that Schulkin made his widely accepted estimate of the effect of internal waves, reducing the effective duct depth by the rms height of the typical internal wave; the argument running that any ray which vertexes at a greater distance than this from the surface will in time intersect such an internal wave and be refracted out of the duct.

Unfortunately, this conclusion is invalid, as this diagram is totally misleading — an inevitable consequence of the distortion of vertical and horizontal scales to get the figure on to a conventional slide. What in fact happens, because the curvature of the acoustic rays is so small and they are being propagated almost horizontally, is that the ray path, even taking account of refractive differences, occupies several periods of the internal wave in any transition from in-duct to below-duct propagation. The true schematic is more like the one shown in Fig. 5 — again noting that this is grossly distorted — the true grazing angle to the internal waves is less than 1°; this, however, at least indicates that a ray may penetrate into the internal-wave region and even so re-emerge into the surface duct.

Now we are getting to the core of the problem — if some rays can be refracted back into the surface duct while others are lost from it, how do we calculate what is the effect of the internal waves in quantitative terms? To do this obviously requires a more detailed knowledge of the mechanism of generation and propagation of internal waves and so I must ask you to lay down your acoustics and follow into the uncharted depths of oceanography. The forcing mechanism for internal waves is gravity (salinity can also cause them, but for the purpose of this paper it will be neglected), and buoyancy forces are dominant.

We start with a water mass which was in equilibrium at a depth z below the surface, under a pressure p, its density at that time being ρ, and which by some mischance has been adiabatically displaced a small distance δz from this position. The general definition of the word 'adiabatic' in this context is beyond the scope of this paper; in this context it means that if the same supernatural agency that caused the change in the first place decides to put the water back in its original place, its density and pressure will also revert to their initial values. Even more confusingly, but more importantly, it means that although sea water is a viscous non-Newtonian fluid we can treat it as though it were a perfect gas and obeys the gas laws, $P \alpha \rho T$ and $P \alpha \rho^{\gamma}$. Anyway, in its new position, the density of this element is

$$\delta z \left\{ \frac{P(z+\delta z)}{P(z)} \right\}^{1/\gamma}$$

whereas the local equilibrium density is $\rho(z+\delta z)$. Thus this element has a density deficiency of

$$\rho(z)\ \delta z \left(\frac{1}{\rho}\frac{d\rho}{dz} - \frac{1}{\gamma p}\frac{d\rho}{dz}\right)$$

relative to its surroundings, and experiences a gravity-fed restoring force. The equation of motion is

$$\frac{d^2 s}{dz^2} + N^2(z)\ s = 0 \quad \text{(simple harmonic motion)}$$

with s the vertical displacement and N, the Brunt-Väisälä frequency, the frequency of the oscillations. There are various expressions for N, all equivalent, the most convenient of which is

$$N^2 = \frac{+g}{\theta}\frac{d\theta}{dz} .$$

That has introduced yet another variable, θ, so I had better explain what that is. It is the temperature the element would end up at if it adiabatically was transferred to a position where it was subject to a standard pressure of 1 atmosphere, and it is called the potential temperature.

N is a measure of the speed of reaction of the ocean to a perturbation — in other words of its stability. The bigger N^2 the more stable the ocean. Figure 6 shows the actual distribution of N^2 in the upper ocean in one typical case, together with the normal idealisation of it used by theorists. Normal in this case means

a. that it is the one normally used;

b. that it is a normal distribution;

c. that any ocean to which it is a valid approximation is completely abnormal.

So now we know what happens when a fluid element is displaced vertically; an internal wave, though, has a horizontal particle velocity component, so the full equation of motion must be used. We can simplify them by applying Boussinesq's Approximation, which says that, for slow enough motions (which internal waves are) we can treat the fluid as incompressible <u>except</u> that buoyancy must be taken into account. The equations are the top four in Fig. 7, which I do not intend to go into any detail over, except to say that you can eliminate all the other dependent variables and come up with this general equation for the vertical component of velocity, w

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right) w + N^2(z)\,\frac{\partial w^2}{\partial x^2} = 0$$

which holds no matter what kind of internal motion we are discussing.

However, we are after a propagating internal wave, so we take the most general disturbance which is sinusoidally periodic in time and travelling in the horizontal direction, writing [Fig. 8]

$$w = W(z) \exp\{i(kx - nt)\}$$

and find that W must obey the equation

$$\frac{d^2W}{dz^2} + \frac{N^2 - n^2}{n^2} k^2 W = 0$$

which means immediately that the disturbance decays everywhere except inside the region where its time frequency is less than the local Brunt-Väisälä frequency. So, from Fig. 8 that shows the distribution of N^2 with depth, we can say that internal waves, except the very slow ones, are confined to the region of the thermocline. With that fact established, we can go further and derive a dispersion relation between the time frequency n and the horizontal space frequency k.

$$n^2 = \frac{\delta\rho}{\rho} \frac{kg}{1 + \coth kD} \qquad (kD \gg 1) .$$

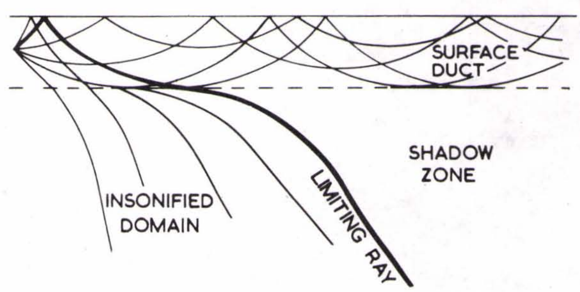
In this D is the depth at which the thermocline is situated and $\delta\rho$ is the change in density across it. Now in this, an increase in k causes an increase in n, but n cannot be greater than the Brunt-Väisälä frequency, so we can find a maximum value of k for each value of N. That defines the minimum wavelength for a wave of this frequency; the wave's amplitude is limited by the depth of the region in which the Brunt-Väisälä frequency is large enough to support it, so we have a relation between amplitude and wavelength.

More usefully, we can plot from Fig. 9, which you can read as either the maximum slope a wave of given amplitude can have, or the maximum amplitude for a wave of given slope on the mean thermocline plane.

These will be the ones which have the greatest effect on acoustic propagation, and Fig. 10 shows the model adopted for calculating this effect. Sinusoids make the calculation too difficult, so they have been replaced by truncated prisms of the same wavelength and maximum slope, cut off so that the area of "intrusion" is the same as that under the sine wave. The assumed velocity profiles are the same as the ones used for the undisturbed situation with the rays in the region of intrusion being straight lines, as the perturbation occurs without change of sound velocity. To calculate the ray path we just use Snell's Law so the calculation is straightforward but tedious.

Because it is so tedious, all that the author has considered are those rays which would be horizontal at the base of the undisturbed duct. He has not finished even these, but says that no matter what the internal wave, more than 90% of the time the ray ends up back in the duct. So it seems that everyone can go away happy in the knowledge that their previous neglect of internal waves, whether due to ignorance or indolence, was and still is justified.

FIG. 1



CLASSICAL UPPER OCEAN RAY PICTURE FOR VELOCITY STRUCTURE WITH SURFACE ISOTHERMAL LAYER

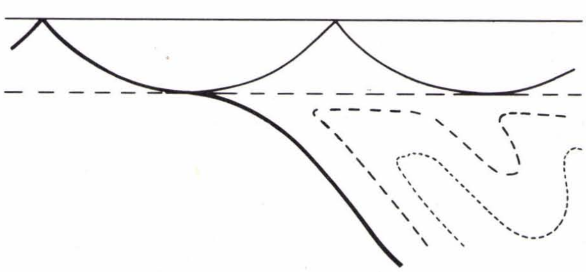
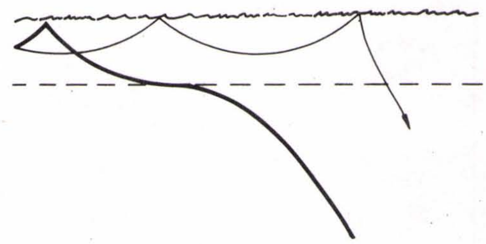


FIG. 2

INSONIFICATION OF SHADOW ZONES BY DIFFRACTIVE PROCESSES. EQUAL LOSS CONTOURS (SCHEMATIC)

FIG. 3



INSONIFICATION OF THE SHADOW ZONE BY SCATTERING AT THE SEA SURFACE (SCHEMATIC)

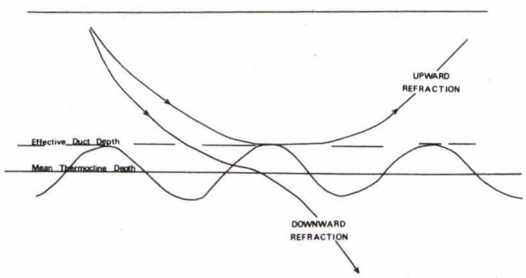
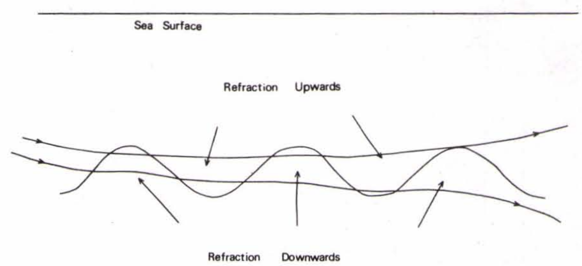


FIG. 4

INTERNAL WAVES AS A FACTOR IN ACOUSTIC DUCT THICKNESS
(after SCHULKIN)

FIG. 5



EFFECT OF INTERNAL WAVES ON RAYS NEAR THE BASE OF THE DUCT (REVISED SCHEMATIC)

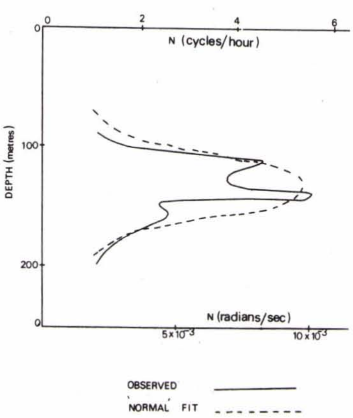


FIG. 6

DISTRIBUTION OF BRUNT - VAISSÄLÄ FREQUENCY ACROSS AN OCEAN THERMOCLINE

BOUSSINESQ's APPROXIMATE EQUATIONS

$$\frac{\partial \sigma}{\partial t} - N^2 w = 0 \qquad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

FIG. 7

$$\frac{\partial u}{\partial t} + \frac{\partial p'}{\partial x} = 0 \qquad \frac{\partial w}{\partial t} + \frac{\partial p'}{\partial z} + \sigma = 0$$

GENERAL EQUATION FOR VERTICAL VELOCITY

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) w + N^2(z)\,\frac{\partial^2 w}{\partial x^2} = 0$$

$$w = W(z)\ \exp\left\{ i(kx - nt)\right\}$$

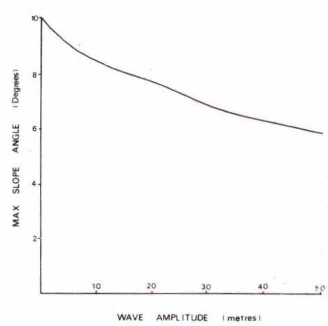
$$\frac{d^2W}{dz^2} + \frac{N^2 - n^2}{n^2} k^2 W = 0$$

$$n^2 = \frac{d\rho}{\rho} \quad \frac{kg}{1 + \coth kD}$$

FIG. 8

GOVERNING EQUATIONS FOR INTERNAL WAVES.

FIG. 9



RELATION BETWEEN INTERNAL WAVE AMPLITUDE AND SLOPE OF WAVE PROFILE

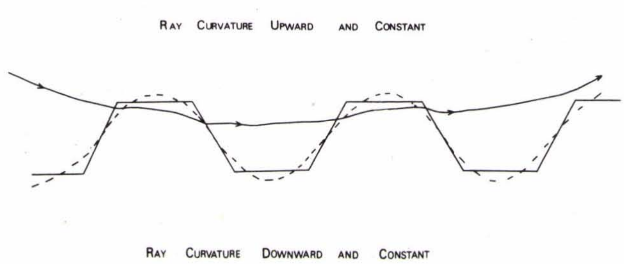


FIG. 10

MODEL ADOPTED FOR CALCULATION OF INTERNAL WAVE EFFECTS