ACOUSTIC EFFECTS OF INTERNAL MICROSTRUCTURE

by

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(No Abstract received)

Introduction

Since the purpose of this meeting is to promote the interaction of oceanographers and acousticians, I'm not going to talk about some of the very interesting mathematical problems that arise in the theory of the scattering of waves by a randomly-inhomogeneous medium: multiple scattering problems, the details of the controversy surrounding the Born and the Rytov method of approximately solving the wave equation, the question of gross inhomogeneities, etc. These are all quite interesting but, I believe, relatively unimportant in the context of oceanographic measurements and modelling. I will concentrate instead on the two sides of the single question of interest here: what does the microstructure of the ocean do to an acoustic wave traversing it, and what can measurements of acoustic waves tell us about the ocean microstructure. Much of our small successes in this field, as in other fields of physics, have come about because of a suitably narrowly-defined area of interest.

In the first place, let me define the time scale. I am going to talk about oceanic phenomena whose time scale ranges from, say, tenths of a second to tens of seconds. Phenomena which might take place on a shorter time scale are simply averaged out in the process of sampling the data, as would be any "noise". Larger time scale "variations" introduce slow changes in time of the average values of the variables of interest, and can be ignored by defining averages local in time. Of course, if the "noise" or the "variations" have time scales which blend into the time scale of interest, then these phenomena cannot be ignored, but must be treated as part of the problem. The small end of the time scale helps define the frequency of the sound waves with which we wish to probe the ocean. We wish to have the acoustic wave period much smaller than the smallest oceanic time scale of interest so that a pulse consisting of a number of acoustic wave periods views each point of the ocean as time-independent.[1c] From the frequency domain point of view, scattering from a time-dependent phenomenon introduces broadening in the acoustic spectrum of the order of the frequency scale of the phenomenon. As long as the time scale we are sampling is no shorter than, say tenths of a second, the broadening will be of the order of tens of hertz, and is unimportant. The long time scale effects can be separated out since the acoustic signals are normally pulsed to separate out boundary effects from the effects of the body of the medium; thus, moving averages can be easily handled. However, these are not trivial considerations, since the fact that we are treating statistical phenomena means that we must average over long time periods. The theory demands that the measurements sample the entire universe of statistical possibilities, so that the probability that a certain configuration of the medium occurs in nature is reflected in the same probability that it happens in the measurements. Phenomena which change in a time scale of tens of seconds must therefore be sampled over hundreds of seconds to be reasonably sure of having measured an unbiased universe. To be honest, I shouldn't say that we don't seem to have any time scale problems in the measurements, but rather that the crudeness of our measurements probably doesn't allow us to recognize any difficulties.

One can make similar comments about the size of the portion of ocean in which measurements are to be made. The shortest distance from source to receiver is determined, aside from acoustic beam-forming considerations, by the desire to have a large sample of the universe of physical sizes involved in the measurements.[1a,2] In a sense, there is a certain competition between distance and time in the statistical character of the measurements, since physically one would expect that large-scale-in-space phenomena are associated with long times for significant changes. In looking for the effects of large physical sizes of inhomogeneities, it would appear necessary to do averaging over a long time interval. At the other end of the scale, the largest distance for measurements, from a practical point of view, is set by surface reflection interferences, and by gross inhomogeneities in the ocean.

I'd like to say a word now about laboratory experiments. In performing acoustic measurements on microstructure scattering in a laboratory, one is not trying to reproduce the ocean, but only to reproduce what are believed to be the important aspects of the natural phenomenon to be studied. Thus, if the theory and the laboratory experiments are in good agreement, then one should expect that natural phenomena which can be described in terms of the parameters important to the theory should also be in good agreement with the theoretical predictions. As I shall describe later, we do find that the theoretical predictions concerning the statistics of the sound pressure fluctuations in an acoustic wave agree well with the measurements of such a wave traversing a tank of water heated from below.[3] The theory requires, as the important non-acoustic factor, that the scalar sound speed is a random function of space and time with certain known averages. We may confidently expect that we may use this technique in the ocean and correctly determine certain statistical parameters of the ocean microstructure - <u>if</u> this is the predominent phenomenon. If the speed of sound in the ocean is changing slightly in a random manner (with appropriate time and space scales) due, for example, to changes in temperature or salinity, then we may expect the theory to give correct results. However, if the sound speed changes because the hero of "Jaws" appears on the scene, then the theory is useless. I have stressed this because of two reasons. It has been suggested that the simple idea that the microstructure is due to the intermingling of slowly moving blobs of warm or cool water is incorrect, but that it is due to a turbulent spectrum[4,5] rather than a Gaussian one. If the idea here is that the sound-speed variations are due to temperature variations carried along by a turbulent mixing process, then this change makes no difference to the basic theory, which is given in terms of the microstructure correlation function of the scalar sound speed; anything that affects the scalar sound speed is fair game for the theory. However, if the turbulent mixing is presumed so strong that the change in the local sound speed is due to the actual motion of the water, then the theory cannot be applied because it is developed for a scalar variation, not the vector variation that a fluid velocity would introduce. From measurements in the ocean, the root-mean-square temperature variations producing acoustic fluctuation effects are of the order of 0.02°C; corresponding fluid velocity variations would have to be of the order of 10 cm per second to give acoustic effects of the same order of magnitude. A second point to be noted is that it has been suggested that bubbles might be the cause of acoustic fluctuations.[6] The work, by Professor Medwin, assumes that the scalar sound-speed changes due to changes in the compressibility of the air-water mixture with the random fluctuations coming from fluctuations in the number of bubbles per unit volume or in the resonant frequencies of the bubbles. Again, here the acoustic wave sees an effect of a change in scalar sound speed, and the theory is applicable. In short, one cannot apply the theory without thought as to the assumptions. Differences between theoretical expectations and experimental results may be simply due to incorrectly assumed parameters, such as a wrong form for the refractive index correlation function; or it may be due to the invalidity of the theory, caused by applying it to the wrong physical situation. However, when applicable, I believe that the theory is sufficiently well proven to allow acoustic measurements to be useful as a probe for the measurement of ocean microstructure.

The final point I wish to make in this Introduction, concerns modelling - the topic of this conference. There are many good things to be said about modelling, but there is one very important point to remember, especially when modelling statistical phenomena, where the measurements are, <u>of necessity</u>, incomplete. The point not too often stressed is — don't take the models too seriously. The ocean is a complex medium, and models, to be useful, tend to simplify -- be prepared to abandon a model if the experimental results so demand. Remember that the model is not the phenomenon; it is a substitute for the phenomenon.

General formulation[1,5,7]

Consider an unbounded medium with a sound speed which changes randomly with position and time only slightly from its mean value; the refractive index is:

$$\frac{c_0}{c(\underset{\sim}{r},t)} = 1 + \alpha\, n(\underset{\sim}{r},t)\ ;\ \langle n \rangle = 0\ ;\ \langle n^2 \rangle = 1 \qquad (1)$$

We have chosen c_0 as an appropriate average sound speed, and α is simply a size parameter. The average to be taken (angular brackets) is as a long-time average at any point; by the assumption of statistical homogeneity, this is the same as an average taken over all of space at one instant of time; it may also be looked upon as an average taken over the ensemble of all distributions of refractive index variations. If the acoustic pulse passes any point in space in a time small compared to the characteristic time of change of the refractive index at a point, we may neglect the time-dependence of the refractive index.[1a,c] Each pulse thus traverses a different distribution of refractive index variations, so that a suitably large number of pulses samples the entire ensemble.

We may write the wave equation for the acoustic pressure, to first order in α, under these conditions:

$$\nabla^2 p(\underset{\sim}{r}) + k_0^2 \left[1 + 2\alpha\, n(\underset{\sim}{r}) \right] p(\underset{\sim}{r}) = 0\ ;\ k_0 = \frac{\omega}{c_0} \qquad (2)$$

We shall describe the pressure in terms of its amplitude and phase:

$$p(\underset{\sim}{r}) = |p(\underset{\sim}{r})| \; e^{i S(\underset{\sim}{r})} \tag{3}$$

There are two separate parts of the problem to be concerned with: the statistical aspect and the inhomogeneity aspect. The statistical aspect means that we must describe the pressure in terms of averages, deviations from the mean, and so forth. The quantities we shall work with are the simplest ones: the deviation from the mean of the pressure amplitude, non-dimensionalized by the mean value; and the deviation from the mean of the phase:

$$\delta P(\underset{\sim}{r}) = \frac{1}{\langle |p| \rangle} \left[|p| - \langle |p| \rangle \right] \; ; \; \delta S(\underset{\sim}{r}) = S - \langle S \rangle \tag{4}$$

From these quantities we may form coefficients of variation, two-point correlation functions, etc.

$$V_{PP}^{2} = \langle \delta P(\underset{\sim}{r}) \, \delta P(\underset{\sim}{r}) \rangle \quad ; \quad V_{SS}^{2} = \langle \delta S(\underset{\sim}{r}) \delta S(\underset{\sim}{r}) \rangle \tag{5}$$

$$\Pi_{PP}(\underset{\sim}{r}_1, \underset{\sim}{r}_2) = \left[V_{PP}(\underset{\sim}{r}_1) \, V_{PP}(\underset{\sim}{r}_2) \right]^{-1} \langle \delta P(\underset{\sim}{r}_1) \, \delta P(\underset{\sim}{r}_2) \rangle \tag{6}$$

The inhomogeneity problem comes about because we can't solve the wave equation exactly for a general refractive index; in fact, we can't even solve it exactly for practically any refractive index. Two approximation techniques have been used, the Born or single-scattering approximation, and the Rytov approximation. For our purposes, where we will work only to first order in α , we needn't concern ourselves with the difference, but it is of significance for higher-order approximations. I should note that I originally thought that the Born approximation had as large a region of validity as that of Rytov, although some people claimed that Rytov was better. I now suspect that the Rytov approximation is the better one; in fact, I'm writing a paper illustrating that it is so — although these statements are certain to start an argument in some quarters! However, I must admit that I don't know why the Rytov approximation has a larger region of validity, and that bothers me.

If we solve the wave equation to first order in α , using the single-scattering approximation, we may write for the acoustic pressure

$$p(\underset{\sim}{r}) = \frac{e^{ik_0 r}}{r} + \frac{k_0^2 \alpha}{2\pi} \int n(\underset{\sim}{r}') \frac{e^{ik_0|\underset{\sim}{r}-\underset{\sim}{r}'|}}{|\underset{\sim}{r}-\underset{\sim}{r}'|} \frac{e^{ik_0 r'}}{r'} d\underset{\sim}{r}' \quad (7)$$

We find, therefore, as might be expected:

$$\langle |p| \rangle = \frac{1}{r} \quad ; \quad \langle S \rangle = k_0 r \quad (8)$$

One may solve for the deviations from the mean, most simply written in terms of prolate spheroidal coordinates:

$$\delta P = (k_0^2 \alpha r^3/4\pi) \int d\underset{\sim}{\xi}' \; n(\underset{\sim}{\xi}') \; \cos[k_0 r(\xi'-1)] \tag{9}$$

$$\delta S = (k_0^2 \alpha r^3/4\pi) \int d\underset{\sim}{\xi}' \; n(\underset{\sim}{\xi}') \; \sin[k_0 r(\xi'-1)] \tag{10}$$

These results involve no approximations other than the stated ones: scalar sound-speed variations, single scattering, and the characteristic time for change in the refractive index small compared to the pulse length.

The statistics of the medium are described in terms of the refractive index variation average (of zero), mean-square average (of α^2) and its correlation function:

$$N(\underset{\sim}{r}'-\underset{\sim}{r}'') = N(x'-x'', y'-y'', z'-z'') = \langle n(x',y',z')\, n(x'',y'',z'') \rangle \tag{11}$$

$$\eta(\underset{\sim}{k}) = \frac{1}{(2\pi)^{3/2}} \int d\underset{\sim}{r} \; N(\underset{\sim}{r})\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}} \tag{12}$$

Isotropic case:

$$N(\underset{\sim}{r}) = N(r) \quad ; \quad \eta(\underset{\sim}{k}) = \eta(k) \; ; \quad \Phi(k) = 4\pi k^2 \eta(k) \tag{13}$$

where $\eta(\underset{\sim}{k})$ is the Fourier transform of the correlation function, and $\Phi(k)$ is the spectral density of the refractive index correlation. I will not restrict myself to the isotropic case until later. Since δP and δS are in terms of the refractive index variations at an arbitrary point, the coefficients of variation and the two-point acoustic correlation will involve averages over products of these variations, that is, the refractive index correlation function. By various devious methods, one may carry out some of the integrations, and transform others into looking relatively harmless; under the condition $k_o r >> 1$, we obtain what might be considered the general form for the coefficients of variation:

$$V_{pp}^2 = (8\pi\sqrt{2\pi})^{-1} \alpha^2 k_o^2 r^2 \int d\underset{\sim}{k}\, \mathcal{N}(\underset{\sim}{k}) \int_0^{\pi} \sin\theta' d\theta' \int_0^{\pi} \sin\theta'' d\theta''$$

$$\times \exp\left[\frac{1}{2} i k r \cos\theta\,(\cos\theta' - \cos\theta'')\right] \sin\left[\frac{k^2 r}{8k_o} \sin\theta'\right] \sin\left[\frac{k^2 r}{8k_o} \sin\theta''\right] \qquad (14)$$

The results for V_{SS} , the coefficient of variation of the phase, is the same, except for the substitution of $\cos\left[\frac{k^2 r}{8k_o} \sin\theta'\right]$, etc., at the end of the integral. These results are valid under the previously mentioned physical conditions (scalar variations, single-scattering, appropriate time scale), and $k_o r >> 1$; the single-scattering approximation may be written as $\alpha^2 k_o^2 a r << 1$, where a is a characteristic size parameter of the refractive-index correlation function. The models which can be developed are all to be contained in the form of the correlation function, $N(\underset{\sim}{r})$, or its Fourier transform, $\mathcal{N}(\underset{\sim}{k})$.

Results of approximations

I will now only talk about the amplitude fluctuations, although similar comments can be made about the phase fluctuations. Under the condition that $r/k_0 a^2 \ll 1$, we see that $k^2 r/8k_0$ is small for all relevant values of k , and the sines may be approximated by their arguments. This is the high-frequency approximation, or the ray limit. If $r/k_0 a^2 \gg 1$, we may make some transformations to a somewhat different form, which can also be approximated, although the results are different for the cases of $k_0 a \gg 1$ and $k_0 a \ll 1$, that is, for the characteristic length parameter of the correlation function large or small as compared to the sound wavelength. The usual condition is that the correlation length is large compared to the wavelength. The limiting cases are[1a,b,8]

$$V_{pp}^2 = \frac{1}{60}\alpha^2 r^3 \int_0^\infty \nabla^2 \nabla^2 N(\rho)\, d\rho \quad ; \quad r/k_0 a^2 \ll 1 \qquad (15)$$

$$V_{pp}^2 = \alpha^2 k_0^2 r \int_0^\infty N(\rho)\, d\rho \quad ; \quad r/k_0 a^2 \gg 1 \qquad (16)$$

$$k_0 r \gg 1 \;;\; \alpha^2 k_0^2 a r \ll 1 \;;\; k_0 a \gg 1 \;;\; r/a \gg 1$$

The arrow on the integral sign denotes that the integration is taken along the line from source to receiver. If the correlation function is not isotropic, the integration brings in different characteristic lengths depending upon the orientation of the principal axes of the correlation function with respect to the source-to-receiver line. This will be illustrated later.

The interpretation of these results is well known. In the ray theory, the change in intensity in going a short distance along the ray path is proportional to the relative change in the cross-sectional area of a bundle of rays. The cross-sectional change is proportional to the Laplacian of the refractive index, which then gives rise to the form in the ray limit. In the diffraction limit, on the other hand, it is the constructive and destructive interference between scattered waves which causes the sound pressure variations. Such interference depends upon the relative path lengths traversed by the waves, and thus directly upon the path integral over the refractive index variations. In the cases shown here, we are in the limit of $k_0 a \gg 1$; since the scattering angle of a wave impinging upon an obstacle of size a is $\theta \simeq 1/k_0 a$, the scattering angle is small and the predominant effect occurs due to waves scattered along the line from source to receiver. In the case of small $k_0 a$, the form for the coefficient of variation is quite different, but leads to a smaller value than for the large $k_0 a$ case.

The first question one might ask is whether or not the theory and the experiments agree. The answer is not to be looked for, I believe, in ocean experiments because they are simply "not clean": there is usually source and receiver motion, gross inhomogeneities, probably inhomogeneous statistics, and so forth. The primary purpose of laboratory experiments, on the other hand, is simply to check out the theory, since a good laboratory experiment involves the measurement of all of the variables. I'll only use the prettiest slide of the Stone and Mintzer experiments[3c], showing the linearity with frequency of the coefficient of variation in the diffraction limit, and its independence of frequency in the ray limit; corresponding graphs of V vs. distance shows the appropriate square-root dependence and the 3/2 - power dependence for the two limiting regimes. It is, moreover, interesting to note that the acoustic data is internally consistant, even though the data was taken during many separate experiments run over a long period of time. This is to be expected, since the temperature microstructure was formed by the same grid of heating wires each time, approximately the same power inputs were used, and the temperature of the surroundings was about the same; thus the same temperature structure for the body of water could be expected, at least in the statistical sense. Thus, the value for $\alpha a^{1/2}$, which is the combination of temperature microstructure parameters measurable by acoustic data in the diffraction limit, comes out to average $1.25\times10^{-4}\pm2\%$ (cgs units) from a number of different experiments. Data from the ray limit gives closely similar values. Data taken from thermistor measurements of the temperature microstructure gives an $\alpha a^{1/2}$ value of slightly more than double that. Considering the fact that the experimentally-determined temperature microstructure correlation function is not truly Gaussian, and that the r.m.s. temperature microstructure experiments were very difficult to make, the agreement seems quite good. I must note that these data were taken during the Dark Ages, with the received acoustic pulses photographed from an oscilloscope trace and measured by hand and the thermistor readings also recorded by hand from a digital voltmeter, and the analyses all done on an electric desk calculator. With modern electronic equipment, PDP-8's, tape recorders and digital computers, we could have