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HIGH-RESOLUTION ANALYSIS OF NON-STATIONARY DATA ENSEMBLES

by

RICHARD KLEMM

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HIGH-RESOLUTION ANALYSIS OF NON-STATIONARY DATA ENSEMBLES

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Estimation of an unknown parameter of a random data ensemble is usually done by maximizing a certain power estimator by varying the unknown parameter. This method is convenient where the data are a nonlinear function of the parameter. It is shown for a variety of power estimators that high resolution is always based on the orthogonality between the observed data and a weighting vector that contains the parameter. Some examples illustrate the use of these power estimators to locate sources by means of array antennas.

1. INTRODUCTION

Estimation of an unknown parameter θ of a vector process can be done by applying a weighting vector $\underline{h}(\theta)$ to the data vector and varying θ until the output power becomes maximum (path A in Fig. 1). Typical applications are spectral analysis, beamforming for sensor arrays and matched filters. The resolution (sensitivity to mismatch between signal vector and weighting) is limited by the length of the data window or the aperture of an array. Higher resolution can be obtained by designing a vector orthogonal to the signal component contained in the data (path B in Fig. 1). The latter principle is well known as split-beam technique and is widely used in sonar and radar systems. As these techniques use a weighting vector orthogonal to only one source direction, they will fail if more than one source is present. In the following more general methods based on the principle of orthogonality are discussed.

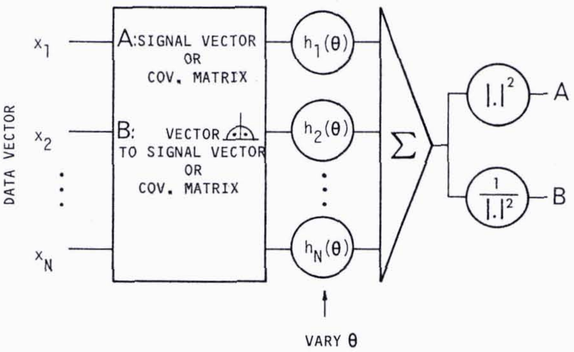

Figure 1: Parameter estimation: Deterministic signal case

2. THE EIGENVECTOR METHOD (EVM)

The idea is to represent the observed vector process by one vector orthogonal to all signal components in the data. The covariance matrix $\underline{R}$ of the observed data vector $\underline{x}$ is hermitian; therefore its eigenvectors are unitary to each other. $\underline{R}$ can be decomposed in a signal matrix plus a white-noise term: $\underline{R} = \underline{S}+\lambda_{min}\underline{I}$, where

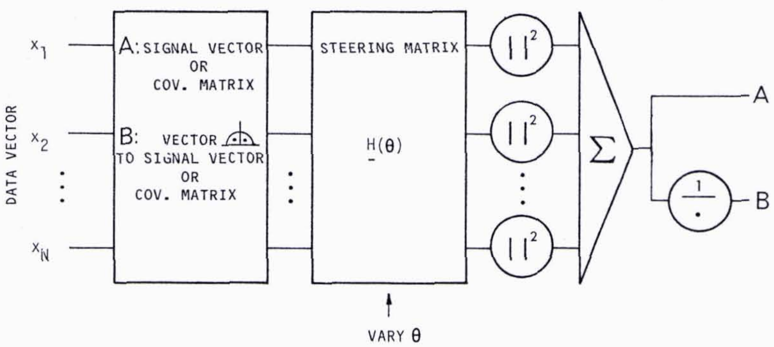

Figure 2: Parameter estimation: Random signal case

the rank of $\underline{S}$ is equal or less than N-1, N being the rank of $\underline{R}$; λ_{min} is the minimum eigenvalue of $\underline{R}$, $\underline{I}$ is the unity matrix. The eigenvector $\underline{g}_{min}$ that corresponds to λ_{min} is orthogonal to the subspace given by $\underline{S}$; therefore high-resolution parameter estimation can be carried out by the expression

$$P(\Theta) = 1 / |\underline{g}^*_{min}\, \underline{h}(\Theta)|^2 \,, \quad \text{(path B in Fig. 1).} \tag{1}$$

$\underline{g}^*_{min}$ is the complex conjugate transpose of $\underline{g}_{min}$. $P(\Theta)$ becomes infinity whenever $\underline{h}(\Theta)$ approaches the signal subspace given by $\underline{S}$.

Instead of $\underline{g}_{min}$ in (1) a matrix of all "white noise" eigenvectors can be used:

$\underline{G} = \sum_{i=M+1}^{N} \underline{g}_i\, \underline{g}_i^*$ if the rank M of the signal matrix $\underline{S}$ is smaller than N-1. Instead of (1) we use the power estimator

$$P(\Theta) = 1 / (\underline{h}^*(\Theta)\, \underline{G}\, \underline{h}(\Theta)) \,. \tag{2}$$

This method may offer advantages in some practical cases. It has been used for spectral analysis in [1].

3. ALTERNATIVE APPROACH: THE ORTHOGONAL PROJECTION METHOD (OPM)

This method creates a vector orthogonal to the signal components of the data covariance matrix without eigenvector decomposition. The following steps have to be carried out:

a. Remove the white-noise portion from the covariance matrix in order to get $\underline{S} = \underline{R} - \lambda_{min}\underline{I}$ (subtract iteratively diagonal matrices $\alpha_i \underline{I}$ from $\underline{R}$ until the determinant becomes zero).

b. Factorize $\underline{S} = \underline{M}\underline{M}^*$ by the Cholesky-algorithm. The method breaks off whenever any eigenvalue of $\underline{S}$ becomes zero. The number of columns of $\underline{M}$ is equal to the number of eigenvalues unequal to zero.

c. Calculate the projection matrix

$$\underline{P} = \underline{I} - \underline{M}(\underline{M}^*\underline{M})^{-1}\, \underline{M}^* \,. \tag{3}$$

Notice that $\underline{P}$ is orthogonal to $\underline{S}$, i.e. $\underline{P}\,\underline{S} = \underline{0}$.

d. Use $P(\Theta) = 1 / |\underline{v}^*\, \underline{h}(\Theta)|^2$ for estimation of Θ. $\underline{v}^*$ is any row of $\underline{P}$.

An estimator similar to (2) can be obtained by taking the whole projection matrix into account:

$$P(\Theta) = 1 / (\underline{h}^*(\Theta)\, \underline{P}\, \underline{P}^*\, \underline{h}(\Theta)) \tag{4}$$

4. APPROXIMATE ORTHOGONAL PROJECTION (AOP)

The eigenvector $\underline{g}_{min}$ in (1) satisfies the minimization problem $\min_{\underline{g}} \underline{g}^*\underline{R}\underline{g}$ with $\underline{g}^*\underline{g} = 1$. Using instead of $\underline{g}^*\underline{g} = 1$ the constraint $g_k = 1$, $1 \le k \le N$, one obtains

$$\underline{g} = \underline{R}^{-1}\, \underline{p}_k \,, \quad \underline{p}_k = \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ P_k \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \tag{5}$$

where P_k is proportional to λ_{min}. $\underline{R}^{-1}$ can be re-written as follows:

$$\underline{R}^{-1} = (\underline{M}\,\underline{M}^* + \lambda_{min}\,\underline{I})^{-1}$$
$$= \frac{1}{\lambda_{min}}\,(\underline{I} - \underline{M}(\lambda_{min}\,\underline{I} + \underline{M}^*\,\underline{M})^{-1}\,\underline{M}^*).$$

Hence we get

$$\underline{g} \sim \underline{I} - \underline{M}(\lambda_{min} + \underline{M}^*\,\underline{M})^{-1}\,\underline{M}^*\,\underline{p}_k \tag{6}$$

In the noise-free case ($\lambda_{min} = 0$) we obtain

$$\underline{g} \sim (\underline{I} - \underline{M}(\underline{M}^*\,\underline{M})^{-1}\,\underline{M}^*)\,\underline{p}_k \tag{7}$$

which is the k-th row of the projection matrix (3).

Replacing $\underline{v}^*$ in (1) by $\underline{g}$, as given by (6), we obtain a noise-dependent approximation of the orthogonal projection method. If the data vector $\underline{x}$ contains equidistant samples of a stationary time sequence (or a homogeneous field) $\underline{R}$ becomes Toeplitz. Choosing $\underline{g}$ to be the first column of the Toeplitz matrix $\underline{R}(k = 1)$ we obtain the well-known maximum entropy method [3]. A power estimator corresponding to [4] is given by

$$P(\Theta) = 1 / (\underline{h}^*(\Theta)\, \underline{R}^{-1}\underline{R}^{-1}\, \underline{h}(\Theta))$$
$$= 1 / (\underline{h}^*(\Theta)\, \underline{R}^{-2}\, \underline{h}(\Theta)) \,. \tag{8}$$

The formula looks similar to the well-known maximum-likelihood method (MLM), [5],

$$P(\Theta) = 1 / (\underline{h}^*(\Theta)\, \underline{R}^{-1}\, \underline{h}(\Theta)) \tag{9}$$

which is a solution of the optimization problem $\min_{\underline{b}} \underline{b}^*\,\underline{R}\underline{b}$ under the constraint $\underline{b}^*\,\underline{h}(\Theta) = 1$.

As (9) can be shown to be a sum over N

different power estimators of the form $P(\Theta) = 1/\sum_{i=1}^{N} \underline{g}_i^* \underline{h}(\Theta)$ (where the $\underline{g}_i$ are obtained by triangular factorization of $\underline{R}$ [3]), we conclude that also the resolution properties of the MLM are based on the principle of orthogonal projection.

5. GENERALIZATION TO RANDOM SIGNALS

If the signal is random it can be described by a positive definite covariance matrix [6]. Consequently, estimation of a random signal vector can be done the same way as before by replacing the steering vector $\underline{h}(\Theta)$ by a steering matrix $\underline{H}(\Theta)$, see Fig. 2, path B. The set of steering matrices is obtained by factorization of all possible signal matrices $\underline{S}(\Theta)$ after removing the noise part. As $\underline{S}(\Theta)$ is positive definite we can write $\underline{Q}(\Theta) = \underline{S}(\Theta) - \lambda_{min}\underline{I}$ and $\underline{Q}(\Theta) = \underline{H}(\Theta)\underline{H}^*(\Theta)$, where $\underline{H}(\Theta)$ is a NxN-1-matrix. In case of $\underline{H}(\Theta) = \underline{M}$ the projection matrix $\underline{P}$ in (2) gives $\underline{P}\cdot\underline{M} = \underline{M} - \underline{M}(\underline{M}^*\underline{M})^{-1}\underline{M}^*\underline{M} = \underline{0}$ so that the power estimator $P(\Theta) = 1/|\underline{v}^* \underline{H}(\Theta)|^2$ becomes infinite as before. If the noise part is not removed from $\underline{S}(\Theta)$ the power output will be $1/(\lambda_{min}\ \underline{v}^*\underline{v})$ instead of infinity when $\underline{H}(\Theta)$ is matched to $\underline{M}$. Consequently, the noise power can be used in order to govern a certain finite power response and hence, a certain resolution.

6. ADAPTIVE IMPLEMENTATION OF THE AOP-METHOD

The vector $\underline{g}$ in (6) can be updated directly from the data by a least-squares algorithm of the form

$$\underline{g}'(t+1) = \underline{g}'(t) - \mu\underline{x}'(t)z(t),$$

$$z = x_k(t) - \underline{x}^{*\prime}(t)\underline{g}(t).$$

$x_k(t)$ is the k-th data sequence, $\underline{x}'(t)$ contains the data at time t except for the k-th input channel, and $\underline{g}'(t)$ contains the weighting coefficients except for the k-th channel ($g_k = 1$). The loop gain μ has to be less than $2/\lambda_{max}$ of the covariance matrix of $\underline{x}(t)$. The algorithm is particularly attractive because its convergence does not depend on the condition of $\underline{R}$, i.e. on the S/N ratio.

7. EXAMPLES

Figures 3 to 6 show examples for applying the above resolution methods to array processing. Figure 3 shows estimation of the bearing angle of a plane wave at 40° relative to two different line arrays. Both of the arrays have the same aperture, one of them equally spaced. As the spacing is about 1.2λ the equally spaced array indicates a grating lobe at about 130°

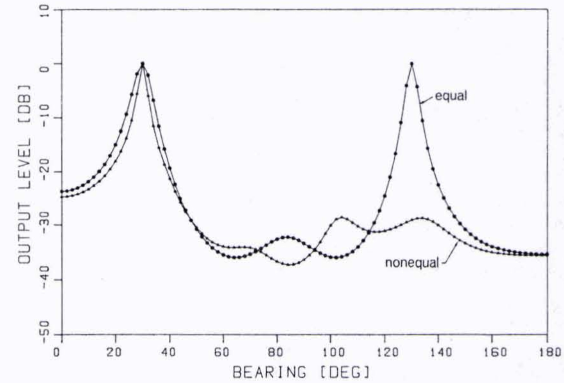

Figure 3: Equal and non-equal spacing

whereas the non-equally spaced arrays shows an unambiguous response. The widths of the peaks are about the same. The example confirms that the resolution properties do not depend basically on the stationarity of the vector process but on how far the signal vector can be made orthogonal to the steering vector. Figure 4 shows a comparison of three methods (matched filter type (MF), AOP, EVM) for range estimation by matching to the circular curvature of an incoming wavefront. An S/N ratio of 10 dB is assumed. The differences in resolution between the methods are remarkable. The conventional kind of matching (MF) is completely useless, and the AOP is considerably degraded by the white-noise level.

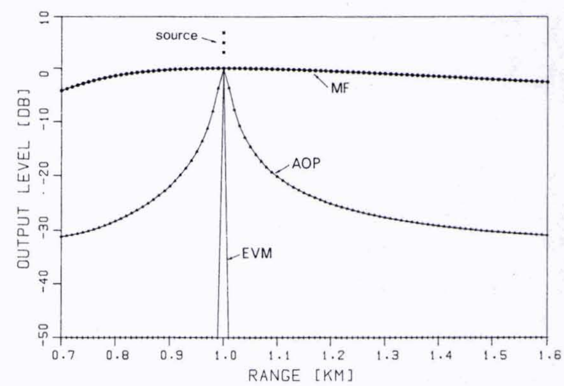

Figure 4: Near field range estimation

Figure 5 shows the problem of estimating the bearing of an acoustic source by a horizontal array in shallow water. In a waveguide like the shallow-water sound channel sound propagates in terms of normal modes. The actual source position is denoted by three asterisks, the vertical lines in the small subfigure denote the modal amplitudes, i.e. the spatial channel response. Modes are supposed to fluctuate slightly in phase. The channel has been modelled by a shallow-water sound propagation model [4]. The AOP (replacing $\underline{g}_{min}$

in (1) by g(7)) looks for the energy maximum and, hence, obtains a considerable bearing error. The generalized version GAOP which uses a steering matrix H according to section 5 yields a bias-free bearing estimate because H contains a-priori knowledge about the channel. The generalized EVM (GEVM) obtains a bias-free bearing estimate as well, but at higher resolution. Figure 6 shows how the GAOP and the GEVM can be used for range estimation of an acoustic source in the shallow-water waveguide by a small horizontal array.

CONCLUSIONS

It has been shown that high-resolution parameter estimation is based on the ortho gonality between a steering vector (or matrix) and another vector representing the signal. A new method based on the principle of orthogonal projection has been proposed (OPM). It has been shown, furthermore, that the maximum entropy method (MEM) is a special case of an approximate orthogonal projection method (AOP). It turns out that the principle of orthogonality is the reason for the high resolution of the MEM; the resolution properties are not constricted to stationary data. The principle of maximum-entropy extrapolation of data records or apertures of line arrays seems to be an additional way of physical interpreting stationary data, rather than the basic reason for high resolution. These statements have been validated by many numerical examples. Typical examples for parameter estimation for nonstationary data are:

a. non-equally spaced arrays, particularly multi-dimensional arrays

b. doppler estimation in staggered-PRF radar

c. estimation of the position of an acoustic source in shallow water (bearing, range, depth) by horizontal, vertical and more dimensional arrays

d. analysis of transformed data (e.g. after beamforming or FFT)

e. arrival time of a known signal

f. high-resolution parameter estimation in the presence of such known system errors as mutual coupling in electromagnetic arrays or tolerances of receiver channels in arrays.

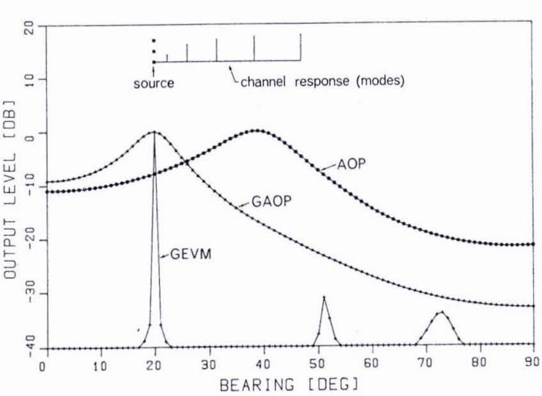

Figure 5: Bearing estimation in shallow water

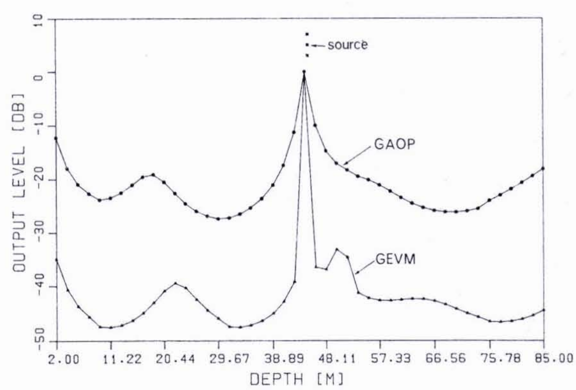

Figure 6: Depth estimation in shallow water

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