BEAM SPREADING AND LOSS OF SPATIAL COHERENCE IN AN INHOMOGENEOUS AND FLUCTUATING OCEAN

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ABSTRACT

A theory has been developed that enables estimates to be made of beam bending, beam spreading and loss of spatial coherence. In this paper we concentrate on the loss of spatial coherence due to scattering. Using a minus two power law as representative of horizontal tow data the solution predicts that the law of coherence of a signal received by a horizontally positioned line array behaves as $I\exp(-Ek^{5/2}ZS^{3/2})$. Here I is the signal intensity, E is an environmental parameter, k is the signal wave number, Z is the range and S is the separation distance between receiver hydrophones.

INTRODUCTION

A propagation model for long-range, low-frequency ocean acoustic experiments should ultimately incorporate the effects of diffraction, refraction and both volume and boundary scatter. In this paper we discuss a model that includes the first three of these factors. The large loss of energy for the portion of an acoustic signal that interacts with the ocean bottom would lead one to suspect that the effects of bottom scatter would be suppressed in a long-range experiment.* Surface scatter effects, however, could be significant and need to be incorporated in an extended model. In applying our model we shall assume that we can isolate, and then suppress, the portion of the acoustic signal that interacts with the surface.

The volume scatter effects incorporated in the model arise due to fluctuations in the sound speed, which are caused by mixing of an inhomogeneous temperature field by dynamic ocean processes; e.g., by internal waves or by ocean turbulence.

The sound field measure in which the model is formulated is the mutual coherence function, which is defined for cw signals according to

$$\{\hat{\Gamma}(\underset{\sim}{x}_1, \underset{\sim}{x}_2, \nu)\} \equiv \{\hat{p}(\underset{\sim}{x}_1, \nu)\hat{p}^*(\underset{\sim}{x}_2, \nu)\} \tag{1}$$

where $\hat{p}(\underset{\sim}{x}, \nu)$ denotes the complex acoustic pressure field; ν, the signal frequency; an asterick, complex conjugation; and the braces, an averaging. The information of interest in the mutual coherence function can be summarized as follows:

i) The collapsed coherence function in which the two points are coincident gives the averaged intensity field, which enables our estimating transmission loss.

ii) The averaged power output of the linear sum of outputs of an (phased) array of hydrophones is given by the coherence function. Thus, it enables the estimation of array signal gain.

iii) The directional decomposition of the energy flux received by an array is directly given by a spatial transform of the coherence function. Thus, it enables an estimation of the loss of resolution due to volume scattering.

Introducing z to denote the range coordinate, the propagation model is expressed in terms of the coherence function measured at two points in the same range plane; i.e.,

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*This need not be true if the receiving array is bottom-mounted.

$$\{\hat{\Gamma}(\mathbf{x}_{\perp 1}, \mathbf{x}_{\perp 2}, z)\} = \{\hat{p}(\mathbf{x}_{\perp 1}, z)\,\hat{p}^{*}(\mathbf{x}_{\perp 2}, z)\}\,, \tag{2}$$

where $\mathbf{x}_{\perp 1}, \mathbf{x}_{\perp 2}$ denote transverse coordinates. (We henceforth suppress explicit dependence on $\bar{\nu}$ in our notation). The form of the model is, then, written

Rate of Change of $\{\hat{\Gamma}\}$ with Range =

Diffraction Term
\+ Refraction Term
\+ Scattering Terms

Specifically, the mathematical model is given by the partial differential equation

$$\begin{aligned}\frac{\partial}{\partial z}\{\hat{\Gamma}\} = {} & \frac{i}{\langle \bar{k}(z)\rangle}\left(\frac{\partial^2}{\partial p_x \partial s_x} + \frac{\partial^2}{\partial p_y \partial s_y}\right)\{\hat{\Gamma}\} \\ & + i\left[\bar{k}(p_y + s_y/2) - \bar{k}(p_y - s_y/2; z)\right]\{\hat{\Gamma}\} \\ & + \bar{\sigma}_2(\mathbf{s}; p_y; z)\{\hat{\Gamma}(s_x, s_y = 0, \mathbf{p}, z)\} - \bar{\sigma}_2(0; p_y; z)\{\hat{\Gamma}\}\end{aligned} \tag{3}$$

In writing Eq. (3), we have introduced the average and difference transverse coordinates; i.e.,

$$\begin{aligned}\mathbf{p} &= \tfrac{1}{2}(\mathbf{x}_{\perp 1} + \mathbf{x}_{\perp 2}) \\ \mathbf{s} &= \mathbf{x}_{\perp 1} - \mathbf{x}_{\perp 2}\end{aligned} \tag{4}$$

Further, $\bar{k}(y,z)$ denotes the (ensemble) averaged signal wavenumber; i.e., $2\pi\bar{\nu}/\{c\}$; which can vary with range and depth. Also, $\langle \bar{k}(z)\rangle$ denotes the average of $\bar{k}$ taken over the depth. The fluctuations in the sound speed are described by $\bar{\sigma}_2(\mathbf{s}, p_y, z)$, which is given in terms of the correlation function defined on the fluctuating wavenumber by

$$\bar{\sigma}_2(\mathbf{s}, p_y, z) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{k}^3}{4} \int_0^{\infty} \frac{\cos\left(\frac{\bar{k} s_y^2}{2 s_z} - \frac{\pi}{4}\right)}{(\bar{k} s_z)^{1/2}} \left(\int_{-\infty}^{\infty} \sigma(s_x, s_y', s_z; p_y, z)\, ds_y'\right) ds_z \tag{5}$$

The statistics of the sound speed fluctuations can also be range and depth dependent.

The assumptions made in developing Eq. (3) and the concomitant restrictions on the validity of the model are complex. For details, the reader is referred to a series of papers and reports [1-5]. Briefly, the model requires the validity of the parabolic approximation. (In [4], we show that the equation, without the scattering terms, follows directly from the parabolic equation on the plane wave amplitude field). Thus, the model is restricted to experiments in which the angular distribution of energy flux is limited to "narrow" angles relative to the range direction. Clearly, then, the model requires the scattering to be a narrow-angled scattering. This is shown [1] to be the case for ocean acoustic experiments so long as the acoustic wavelength is short relative to all significant correlation lengths for the sound speed fluctuations measured in a horizontal plane. Further, the model requires that the effects of scattering, diffraction and defraction are all uncoupled over limited ranges. Over extended ranges, the effects are of course coupled. In addition, the model requires that the amount of scattering over limited ranges be small. This is assured by virtue of the weakness of the sound speed fluctuations. Finally, the specific form of the scattering terms is intimately related to the details of the angular distribution of the locally scattered energy. In the ocean acoustic model, which we term an anisotropic model, the sound speed fluctuations are taken such that correlation lengths measured in a horizontal plane are orders of magnitude larger than those measured in the depth direction. The frequency of the acoustic signal is taken to be such that the acoustic wavelenth is of the same order as or larger than the significant correlation lengths measured in the depth direction. The wavelength, as mentioned previously, is assumed to be small relative to horizontal correlation lengths. For much higher frequencies, the signal wavelength will become small relative to correlation lengths measured in the depth direction as well as those in the horizontal plane. In this case, the form of the scattering terms changes and the model becomes that which has been extensively studied in the optics literature [6,7]. This latter model, we refer to as an isotropic model. It is noteworthy that the solutions of the two models are qualitatively different in the multiple scatter region.

In the remainder of this paper we shall briefly outline some special cases in which analytic solutions to the model have been obtained. In particular we discuss three situations.

(1) Homogeneous statistics; absence of a sound speed profile; plane wave incidence: In this case the model is used to determine the spatial correlation for distances separated along a horizontal line. Detailed calculations are carried out for sound speed fluctuations which, when sampled in a horizontal plane, obey a minus two power spectrum.

(2) Homogeneous statistics; a range independent sound speed profile; point and finite coherent sources: In this case the model is used to determine the averaged (taken over the depth) spatial correlation for distances separated along a horizontal line. Thus, the effect of diffraction on the above results is seen.

(3) We briefly consider a solution algorithm based on the model with the scattering term neglected. The algorithm indicates the possibility of predicting beam spreading by appending side calculations to a ray trace program. It should also prove useful for predicting the propagation of signals emanating from partially coherent, or noisy, sources.

Plane Wave Source in a Statistically Homogeneous Fluctuating Ocean

For the case of plane wave incidence and a statistically homogeneous ocean, the diffraction and refraction terms vanish. Equation (3) thus reduces to an ordinary differential equation in the range coordinate, in which the separation coordinates (S_x, S_y) are parameters. The solution is readily obtained and is written

$$\{\hat{\Gamma}(s_x,s_y,z)\} = \hat{I}\Big[\frac{\bar{\sigma}_2(s_x,s_y)}{\bar{\bar{\sigma}}_2(s_x,0)}\exp\left(-[\bar{\sigma}_2(0,0)-\bar{\sigma}_2(s_x,0)]z\right) + \left(1-\frac{\bar{\sigma}_2(s_x,s_y)}{\bar{\bar{\sigma}}_2(s_x,0)}\right)\exp[-\bar{\sigma}_2(0,0)z]\Big] \tag{6}$$

where $\hat{I}$ denotes the intensity of the incident plane wave. For studying the horizontal resolution of a line array positioned orthogonal to the range corrdinate we make use of $\{\hat{\Gamma}(s_x,0,z)\}$ which is given by

$$\{\hat{\Gamma}(s_x,0,z)\} = \hat{I}\exp\left(-[\bar{\sigma}_2(0,0)-\bar{\sigma}_2(s_x,0)]z\right) \tag{7}$$

As indicated in the Introduction, the environment is described by the term. In order to obtain a propagation model that requires as input, a few easily measured environmental paramaters we must make a number of assumptions pertaining to the environment.

Included among these are

$$\text{i)} \int_{-\infty}^{\infty} \sigma(s_x, s_y', s_z)\, ds_y' = \ell_{yM}\, \sigma(s_x, 0, s_z)$$

$$\text{ii)} \quad \sigma(s_x, 0, s_z) = \sigma_H\left(\sqrt{s_x^2 + s_z^2}\right) \tag{8}$$

The first of these assumption leads to a propagation model in which the vertical structure of the fluctuations field is described by a single length scale, ℓ_{yM}. The second assumption is that the horizontal structure is isotropic. Taken together, the environment description required is given by horizontal tow data plus a measure of the length scale for the vertical structure.

The horizontal tow data is more usually presented in terms of the spatial power spectrum, which is given by the Fourier transform of $\sigma_H\left(\sqrt{s_x^2 + s_z^2}\right)$. We denote this spectrum by $\Phi_1(p)$, where p, here, denotes the transform variable. Our final assumption is to assume that $\Phi_1(p)$ obeys a minus two power law over that portion of the spectrum that contributes significantly to the scattering. It is important to realize that a justification for choosing a functional form for $\Phi_1(p)$ requires some knowledge of which length scales defined by the fluctuating sound speed field are important. This, in turn, depends on the acoustic experiment that is of interest. That is, the justification can only be argued, a posteori. For the low frequency experiments for which the model was developed, we have demonstrated [3] that the important length scales are moderately large (500-5000 m.), for which the minus two law has attained a degree of acceptance.

With the above assumptions of the environment, we can reduce the exponent in Eq. (7) to a simple algebraic form,

$$\{\hat{\Gamma}(s_x, 0, z)\} = \hat{I} \exp\left[-E(\bar{k}s_x)^{3/2}(\bar{k}z)\right] \tag{9}$$

The environment is described for Eq. (9) by a single nondimensional parameter, E, which is given in terms of measured data according to

$$E = 1.76 \times 10^{-5} A_T^2 \ell_{yM} \tag{10}$$

where A_T^2 is obtained by fitting a minus two law to horizontal tow data of temperature fluctuations. The units of A_T^2 is $(^{\circ}C)^2$/length.

Equation (9) represents a particularly simple expression for estimating the loss of spatial coherence due to scattering from large scale ocean temperature fluctuations. A comparison has been made between predictions made by it and archival experimental data and the results are very encouraging. The classification of some of the archival data presents my showing these results at this session. The comparison will be available, however, in an article to be submitted to JUA [8].

Effect of Finite Source Size and a Depth Dependent Sound Speed Profile

We consider next a calculation in which the sound speed is taken to vary with depth and the source is taken to have a finite size. The fluctuations in sound speed are still taken to be statistically homogeneous. The quantity to be calculated is

$$\langle\{\hat{\Gamma}(p_x,s_x,z)\}\rangle = \frac{1}{H}\int_0^H \{\hat{\Gamma}(p_x,s_x,p_y,0,z)\}\,dp_y \tag{11}$$

where H is envisioned to span the sound channel. The quantity $\langle\{\hat{\Gamma}\}\rangle$ is interpreted to be an averaged coherence function for two points located on the same horizontal line that is orthogonal to the principal propagation direction. Thus, it provides a measure of an averaged loss of horizontal resolution.

For $\bar{\sigma}_2$ independent of P_y, we can average Eq. (3) and obtain the following equation on $\langle\{\hat{\Gamma}\}\rangle$ [3]

$$\frac{\partial\langle\{\hat{\Gamma}\}\rangle}{\partial z} = \frac{i}{\langle\bar{k}\rangle}\frac{\partial^2\langle\{\hat{\Gamma}\}\rangle}{\partial p_x\,\partial s_x} + \langle\bar{k}^2\rangle\left[\bar{\sigma}_2(s_x,0)-\bar{\sigma}_2(0,0)\right]\langle\{\hat{\Gamma}\}\rangle \tag{12}$$

where $\langle\bar{k}\rangle$ is the spatially averaged, mean wavenumber. Equation (12) shows that there is no effect of a depth dependent sound speed on the averaged (taken over the depth) horizontal resolution. We note that the presence of depth dependent statistics for the sound speed fluctuations cannot be treated so easily.

In [2], we consider solutions of Eq. (12). The "source" for these calculations is located in the z=0 plane, which is taken to be a constant phase plane. The intensity distribution across the source is taken to be Gaussian. We calculated $\langle\{\hat{\Gamma}(0,s_x,z)\}\rangle$, which provides an estimate of the loss of signal coherence along the beam center. The results differ in the numerical factor in the definition of E. For a point source (a Gaussian intensity distribution in which the horizontal beam width is zero), the numerical factor is 0.7×10^{-5}; for a finite source the factor is smaller still. Thus, the coherence of a signal emanating from a point source decays more slowly than that from a plane wave source; the coherence of a signal emanating from a finite source decays more slowly than those from either of the two limits.

$\langle\{\hat{r}(p_x, 0, z)\}\rangle$

on the averaged intensity measured across the sound channel. This enables estimates of the dependence of beam spread on diffraction and on scattering. In the limits in which one or the other of these mechanisms is predominant, simple analytic results were obtained. Thus, for conditions in which diffraction effects are predominant, the beam width increases with range as $(a^2 + z^2/k^2\, a^2)^{1/2}$, where "a" is the initial beam width. For conditions in which scattering effects are predominant, the length scale for observing variations in intensity is of the order of $(E^{2/3}\, k^{2/3}\, Z^{5/3})$. For an intermediate case in which scattering and diffraction effects are both important, it is necessary to resort to automatic computation. The interested reader is referred to [2].

Propagation in Nonfluctuating Media

Equation (3) minus the last two terms serves as the basis of an acoustic model for experiments in which volume scatter is not significant. In [4], we show the equation to follow directly from a parabolic equation written on the plane wave amplitude. Any "randomness" of the radiation field for this deterministic medium problem enters via the source. Thus, for example, one might be interested in sound emanating from a partially coherent, or noisy source. Or, one might be interested in sound that has passed through a fluctuating region of an otherwise deterministic medium. Problems of stochastic sources have received a good deal of attention in treating electromagnetic radiation fields. An equation similar to Eq. (3) with no scattering terms could be used to formulate the theory of partial coherence as it is termed in this latter literature.

Further, since a deterministic radiation field can be interpreted as a limiting case of a stochastic radiation field, in which all manifestations of the experiment are identical, Eq. (3) minus the scattering terms could be applied to a purely deterministic problem. With reference to this, however, we note that the dimensions of the space on which Eq. (3) is defined are five in number, whereas the purabolic equation on the plane wave amplitude itself is defined on a three dimensional space. Thus, in general, it would appear to be easier, in treating a deterministic problem, to first solve for the pressure field and then, if desired, calculate the intensity field or the "coherence" field. The value of a model based on Eq. (3) for treating deterministic fields would then rest simply in the unification that it brings to treating a spectrum of problems. As indicated below, however, the value of a model based on Eq. (3) may, for a class of deterministic problems, be more extensive than this.

Consider Eq. (3), with no scattering terms, and introduce an approximation based on expanding and truncating the $k(P_y \pm S_{y/z};Z)$ terms appearing therein. A truncation after the quadratic term leads to the following approximate equation,

$$\frac{\partial\{\hat{\Gamma}\}}{\partial z} = \frac{1}{\langle\bar{k}(z)\rangle}\left(\frac{\partial^2}{\partial p_x \partial s_x} + \frac{\partial^2}{\partial p_y \partial s_y}\right)\{\hat{\Gamma}\} + i\,\frac{d\bar{k}(p_y)}{dp_y}\,s_y\,\{\hat{\Gamma}\}. \tag{13}$$

In [4], we discuss in some detail the validity of Eq. (13) for both the stochastic and deterministic source problems. Briefly, for the former the requirement is that spatial extent of the region over which the resulting field is coherent must be small relative to the length scales on which $k\ (P_y)$ vary. For narrow beamed deterministic signals the validity can be argued on the assumption that the beam width is small on the length scales on which $k\ (P_y)$ vary. For broad beamed deterministic signals, the validity can still be argued over limited ranges.

To solve Eq. (13), we first introduce a spatial Fourier transform of the separation coordinate. Denoting by S, the transform variable, the equation on $\{\tilde{\Gamma}(p, \varsigma, z)\}$ is then written

$$\frac{\partial\{\tilde{\Gamma}\}}{\partial z} = \frac{1}{\langle\bar{k}(z)\rangle}\left(\varsigma_x\frac{\partial}{\partial p_x} + \varsigma_y\frac{\partial}{\partial p_y}\right)\{\tilde{\Gamma}\} + \frac{d\bar{k}(p_y)}{dp_y}\,\frac{\partial\{\tilde{\Gamma}\}}{\partial \varsigma_y} \tag{14}$$

This equation is recognized as a first order partial differential equation. Thus, a general solution procedure exists for its inversion, which requires that we first construct its characteristics. Then, an ordinary differential equation (in this case of first order) is written on the change of $\{\hat{A}\}$ with distance measured along the characteristic and is solved. To determine $\{\hat{A}\}$ requires that we transform back to s space. Excluding the algorithms required by the transformation into and out of s space, the principal numerical task in carrying out the above procedure is the construction of the characteristics. These, moreover, can be identified with the rays of the geometric theory suggesting the possibility of appending the needed added calculations to an existing ray trace program. In this way, diffraction effects can be incorporated in a geometric theory program by carrying along some side calculations. The effects of scattering apparently cannot be introduced in such a simple manner.

We have considered a number of problems for which Eq. (13) is exact, within the context of the parabolic approximation, and for which we could carry out the above described solution procedure analytically. The solutions obtained exhibit the qualitative behavior expected; e.g., beam bending, beam spreading due to diffraction, beam focusing in a sound channel, a characteristic oscillatory behavior in the intensity field in the neighborhood of caustics, etc. These analytic solutions are being used to test some of our numerical routines that are nominally based on a parabolic approximation.

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