STATUS OF RAY THEORY DEVELOPMENT AT NAVAL UNDERSEA RESEARCH AND DEVELOPMENT CENTER

by

D.F. Gordon
Naval Undersea Research and Development Center
San Diego, California, U.S.

INTRODUCTION

The first part of this paper discusses NUC (Naval Undersea Research and Development Center) work on the accuracy and validity of ray theory. By comparing the range to convergence zones as indicated by experiment and by theory, we have found which techniques are required to make accurate ray computations.

By comparing computations done by ray theory and normal-mode or wave theory, we can determine the limits to the accuracy of ray theory at low frequencies.

The final portion of this paper discusses new developments in ray theory.

CONVERGENCE ZONE RANGE

The range to the leading edge of the convergence zone can be determined very accurately experimentally. This is partly because the average travel time to the leading edge of the zone is very insensitive to minor variations in the velocity profile and can be used to measure range accurately, and partly because of the rather abrupt increase in sound pressure at the zone. The range at which the propagation loss decreases to less than 95 dB has been used to indicate the leading edge of the zone because at frequencies of a few kiloHertz this loss is clearly distinguishable from bottom returns. It is, therefore, interesting to compare this range with the range to the first zonal caustic of ray theory.

Pedersen and Anderson gave a paper on this topic at the 28th Naval Symposium on Underwater Acoustics. Figure 1 is a summary of portions of that paper. The figure indicates average results from a number of Pacific locations. The jagged line indicates a possible experimentally observed convergence zone edge. The three vertical lines represent computed losses at caustics with their characteristic shape and indicate the range relative to the true zone. This leading caustic is formed by rays which travel downward from the source and upward to the receiver.

Early attempts to compute the range to this caustic, used sound velocities computed from Kuwahara's tables. Several trials gave ranges which averaged 1.5 kyd short of the zone, as indicated in Fig. 1. Altering the profile to simulate the effect of earth curvature shortened the range an additional 600 yd as shown.

The advent of Wilson's equations for the computation of sound velocity increased the range to computed caustics and the correction for earth curvature became an asset rather than a liability. The caustic line labelled "Wilson" indicates the average relative position of computed caustics from 12 different locations in the Pacific. This average position is about 500 yd beyond the true convergence zone. However, by applying a caustic correction taken from Brekhovskikh, the difference between theory and experiment is reduced by half.

The final step in Pedersen and Anderson's investigation was to make an adjustment in sound velocities to fit a portion of Wilson's data which includes those ranges of temperature, salinity, and pressure found in the Pacific. This gave sound velocities which were smaller at shallow depths and larger at greater depths than those obtained from Wilson's equations by amounts up to 1 ft/s. The median difference between computed and experimental convergence zone ranges became zero. This is shown by the diffraction curve labelled "Wilson adjusted". These results indicate in a statistical sense that the current methods for computing convergence zone ranges have no significant bias. Although the average of the differences is zero, their scatter is not. Fifty percent of the differences were less than 360 yd. The largest difference was over 4 kyd.

RAY AND NORMAL-MODE THEORY

Figure 2 shows two profiles of special form for which both ray and normal-mode computations can be made. Comparisons will be made between computations made by the two theories. On the left is an Epstein layer which is a five-parameter function of hyperbolic cosines and tangents. It has been fitted to an Indian Ocean velocity profile. The curve has two vertical asymptotes, one at 1636 and one at 1753 yd/s. To simplify certain aspects of the problem, computations were done without the surface, so the profile is shown extending above the surface.

On the right side of the figure is a four-layer approximation to an Atlantic profile in which the squared index of refraction is linear in each layer. Normal-mode computations for this profile at 10 Hz and 30 Hz were published by Tolstoy and Clay in JASA in 1960.

Figure 3 compares propagation loss as computed in three different ways for the Epstein layer. Since no surface or bottom is included in this profile model, only energy trapped in the duct by diffraction is seen at the zones. Only two caustics appear at each zone. With surface reflection, three additional caustics would appear. The source and receiver are at depths of 33 yd and 100 yd and the frequency is 30 Hz. The channel axis is at 1589-yd depth.

The normal-mode theory gives the most accurate solution for this idealized duct. The inability of the simple ray theory to compute diffraction effects is apparent. In the modified ray theory Brekhovskikh's caustic correction has been applied to each caustic and the results, which are Airy functions, have been added in random phase. A possible explanation for the difference between the mode and modified ray theory results is that the caustic corrections were not added in phase.

The next four figures will compare ray and mode theory for the four-layer Atlantic profile. Mode theory will be shown for 10 Hz, 30 Hz, and 100 Hz. Figure 4 is a ray diagram for a source at 500-yd depth and the upper 500 yd is shown. Rays are drawn at each 1° in source angle with the rays that just penetrate into the surface duct and just graze the bottom included. The range is to 100 kyd and includes one convergence zone.

The leading caustic runs from 54 upward to 62-kyd range before it encounters the surface. A similar caustic is formed by the rays which start upward at the source. Three additional caustics are formed by the surface-reflected rays, the last between 70 kyd and 73 kyd.

Figure 5 shows propagation loss contours as computed by normal modes for precisely the same situation as was used on the ray diagram, except that an extra 100 yd in depth is shown. The frequency is 10 Hz. The two refracted caustics and the final surface-reflected caustic from the previous figure are shown by broken lines. Note that the leading caustic from the ray diagram from 54 kyd to 57 kyd approximately parallels the 80 and 90-dB contours. Note also the surface-image effect which depresses the 90-dB contour in the zone deeper than 50 yd from the surface.

The 110-dB contour appears to be influenced by the surface duct which has a depth of 153 yd. However, judging from the next figure, this is not a result of the surface duct which is too small at this frequency to have any large effect upon the loss.

Figure 6 is the same as Fig. 5 but for 30 Hz. Here the leading caustic lies partly within the 80-dB contour. The second refracted caustic lies near the string of 80-dB contours. The 90-dB contour comes within about 25 yd of the surface at this frequency.

At 10-Hz frequency the 110-dB contour in the near field extended to 28 kyd range. Here it reaches only 21 kyd. It seems more reasonable to attribute this difference to differences in diffraction into the shadow zone than to attribute it to propagation in the surface duct which should be stronger at the higher frequency.

Figure 7 shows the contoured field at 100 Hz. Here definite surface duct propagation is seen. This surface duct can trap one mode at 100 Hz so this propagation is not unexpected. The effect of the surface duct can be seen in the 90, 100, and 110-dB contours in and following the direct field and in the 100 and 110-dB contours following the zone. The zone itself, as outlined by the 90-dB contour, is only slightly larger than the ray theory zone bounded by the first refracted caustic and the last surface-reflected caustic between 54 and 73-kyd range.

Some phase interference or Lloyd-mirror beats can be seen near 10 kyd. They were at somewhat shorter range for the lower frequencies.

These figures have shown several limitations of ray theory at low frequencies. Diffraction from caustics and shadow zones is important as is the interaction between ducts such as the SOFAR duct and surface duct. This interaction between ducts can remain important at higher frequencies. The surface image or surface decoupling effect must be considered.

NEW TECHNIQUES

Three items under current development at NUC are generalized velocity functions, two-dimensional velocity variation, and numerical quadrature.

In March 1968, Pedersen published his generalized ray theory in JASA. This theory uses depth as a function of velocity to represent the velocity profile. The function can be a polynomial, a power series, or a series in non-integral powers of velocity. This makes it possible to fit velocity profiles directly with polynomials of any required degree or to use standard profile forms by expanding velocity as a power series in depth and then inverting the power series. By using non-integral powers of velocity, Pedersen was able to develop a theory of the axial ray published in JASA in January 1969. This ray theory requires the use of elliptic integrals. However, new developments have determined the range and travel time as a power series, making elliptic integrals unnecessary. A report by Pedersen and White on this development was given at the recent International Acoustic Congress in Budapest.

In another new development, White and Keir at NUC have developed a method of determining ray fields with two-dimensional velocity variations. This is done by transformations on the depth and range axes. This technique gives theoretical examples of two-dimensional velocity variation which can approximate various realistic situations and also can give models to test numerical ray tracing programs.

In May 1971, Mr Edward R. Floyd of NUC published an article in JASA on ray tracing by Gaussian quadrature. This method again allows a polynomial of arbitrary degree to be fitted to all given velocity points and thereby avoids false caustics. It is not yet clear whether this method can give sufficient accuracy for computing intensities from detailed velocity profiles. However, it appears to be well suited for quick approximations to range and travel time.

SUMMARY

The range to convergence zones can be accurately computed if accurate velocity profiles which are independent of range are known, and if earth curvature and diffraction from the caustic are taken into consideration.

The technique of comparing ray and mode solutions for identical velocity profiles gives valuable information on the validity of ray theory for finite wavelengths.

New work includes power series expansions for a general class of velocity profiles, velocity-depth transformations to simulate two-dimensional velocity variation, and numerical quadrature.

DISCUSSION

Bartberger had also encountered convergence difficulties using Gaussian quadrature even with 25 points. The author felt, however, that numerical techniques were now available which might make the method usable.

In reply to a question concerning the continued use of the random-phase addition of modes, the author said that certain results to be found in Brekhovskikh's work now made this unnecessary.

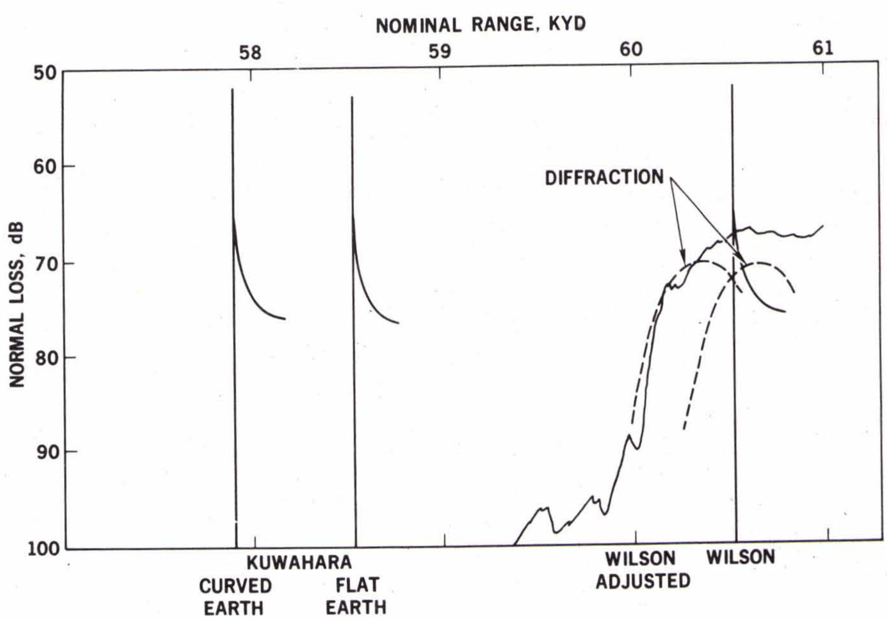


FIG. 1

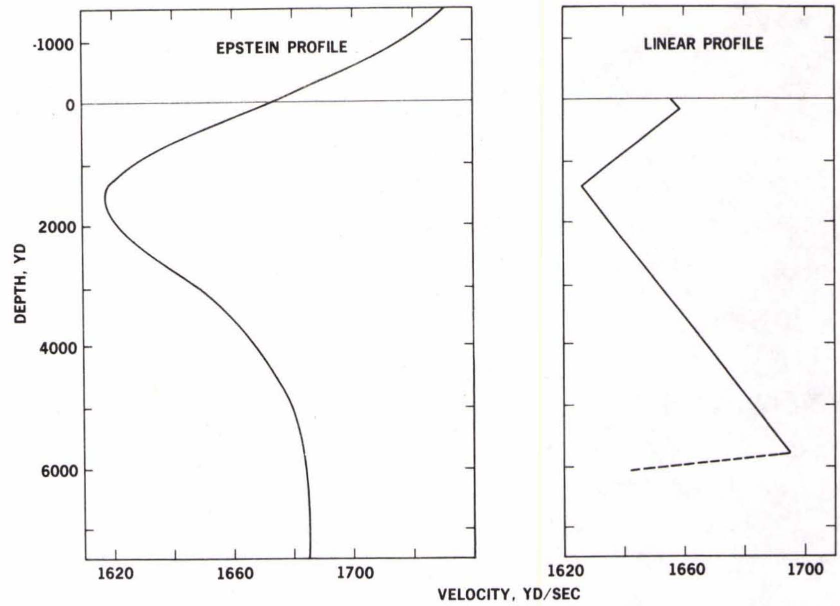


FIG. 2

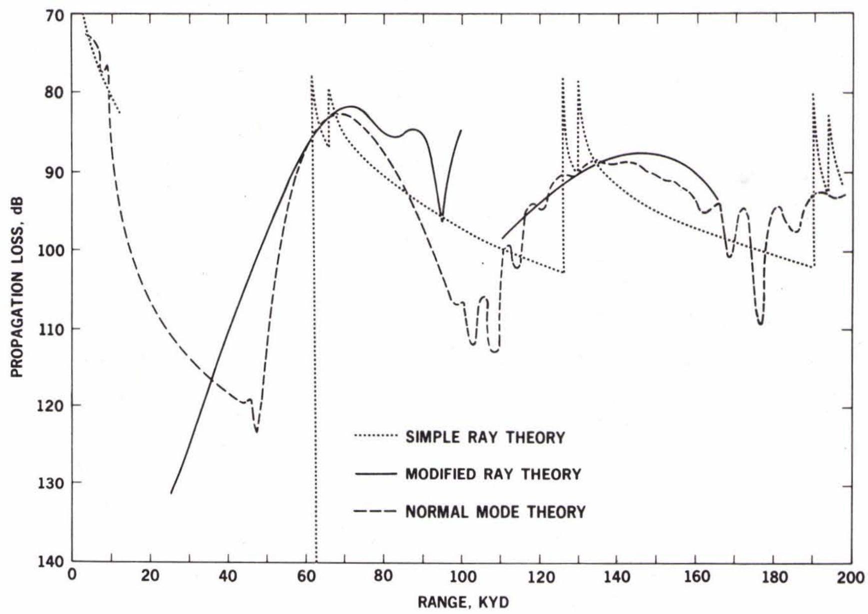


FIG. 3

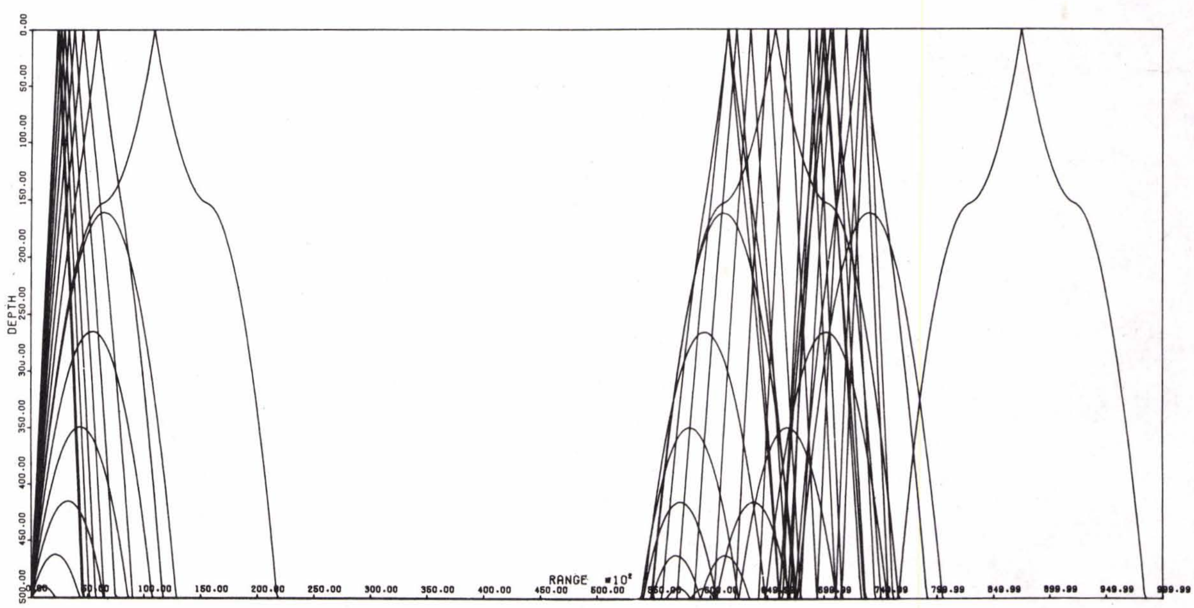
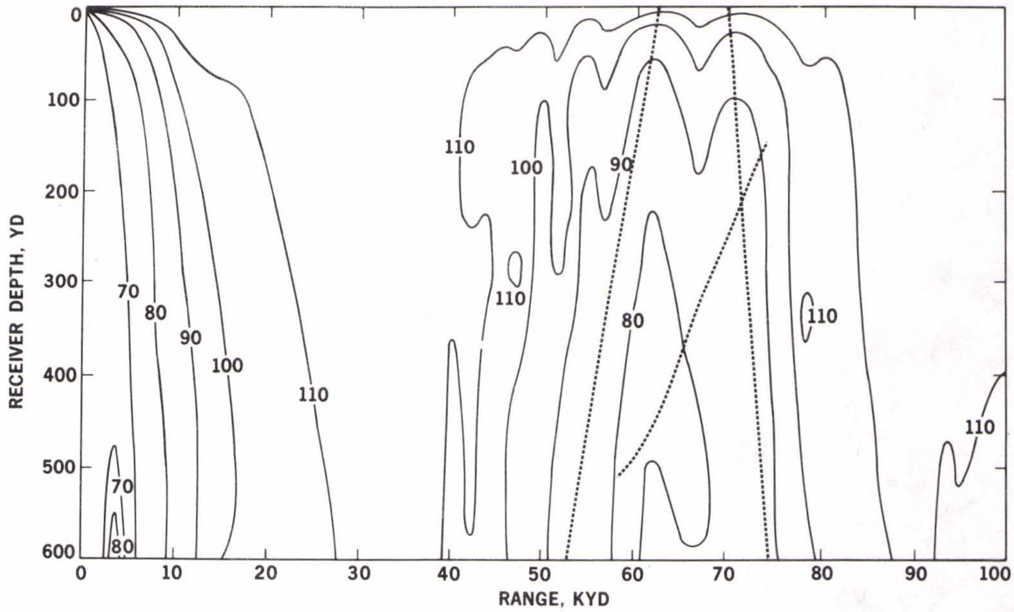


FIG. 4

FIG. 5



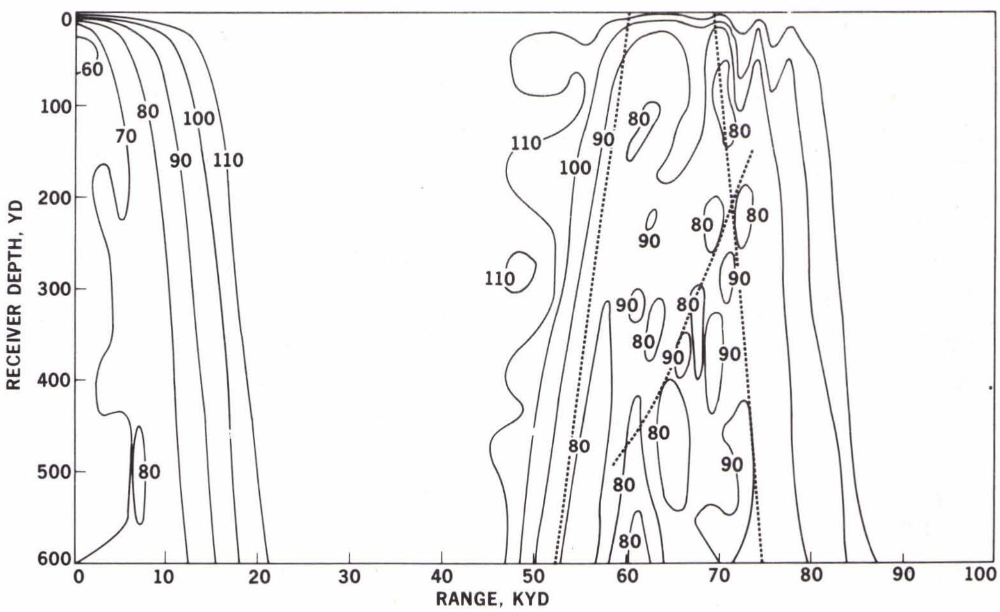


FIG. 6

FIG. 7