CONSIDERATIONS RELATING TO THE CALCULATION OF SOUND VELOCITY

by

C.C. Leroy
Laboratoire Marine d'Acoustique
Arcueil, Paris, France

The various equations for the calculation of the speed of sound in sea water that can be found in the literature may be grouped into three main categories:

(a) Equations of the first investigators (see, for example, Wood [Ref. 1]). These were simple but are now too inaccurate.

(b) Equations to fit the tables of Kuwahara [Ref. 2] and Matthews [Ref.3]. The tables came from computations based on physical formulae and data. The equations that were developed in the early stages of electronic computers were already complex. The pressure effect was taken into account. We can mention Mackenzie's equation [Ref. 4].

(c) Equations to fit directly measured values of sound velocities.

The equations appeared progressively with the measurements and were made possible by the development of sound velocimeters.

One finds the following equations:

1. By Del Grosso [Ref. 5] for sea water at atmospheric pressure. Simple but now inaccurate.

2. By Greenspan and Tschiegg [Ref. 6] for fresh water from 0^{o} to 100^{o} C. Few terms of some complexity and excellent agreement between formula and data points (but there is only one parameter: temperature).

3. The various Wilson equations [Ref. 7]. For fresh water under pressure up to 14 000 lb/in^2: 20 terms, 7 significant figures in each coefficient. For sea water, up to 14 000 lb/in^2 pressure, June 1960. Fits data points of limited salinity: 21 terms, 4 to 6 significant figures. This equation is not applicable outside the investigated domain of salinity [see below Fig. 7].

4. Another equation by Wilson [Ref. 7] of October 1960, for sea water. It fits previous data points + fresh water data points, + new data points at 10, 20, 30‰ salinity. Again complex equations (22 terms, 4 to 6 significant figures).

Wilson's second equation [Ref. 7] was the last to have been published when the present author, trying to develop a simple equation for use in the limited oceanographic conditions encountered in the Red Sea, realised how easy it was to make a simple formula once the data points to be fitted were reduced.

At the same time, the author started to investigate simple ways of computing the pressure at depth, in order to apply Wilson's equation [Ref. 7] easily and his own (unpublished) equation for the Red Sea. It turned out that pressure could be computed from depth and latitude only, without having to consider temperature or salinity. One universal equation was developed which only needed to be replaced by other ones for the Black Sea and the Baltic. This work has been published in Ref. 8.

The possibility then appeared of developing a universal equation for sea water using depth as a parameter.

An investigation of the existing water masses of the world demonstrated that Wilson [Ref. 7] had used, to establish his formula, data points that were made outside realistic values [Figs. 1 and 2]. By limiting the domain of validity to existing waters, a much simpler formula than Wilson's could be developed [Fig. 3] that could approximate his second equation to within $\pm$ 0.1 to $\pm$ 0.2 m/s [Fig. 4]. This equation is built up with progressive terms that have either to be omitted or taken into account, according to the ranges of temperatures and salinities involved.

A further step was to try using the original data points, belonging to the realistic domain, for making a new equation. This equation differs only slightly from the previous one. Both have been published in Ref. 9.

The data taken into account were all data points from Wilson [Ref.7] inside the defined domain except those for fresh water at atmospheric pressure, which were replaced by Greenspan & Tschiegg's [Ref. 6]. The last data of Wilson [Ref. 7] (high salinity and high temperature) were also considered.

Again the formula was dictated by the following considerations (besides the choice of the data):

(a) Make it simple so that it could be programmed on desk computers or even by hand.

(b) Make it susceptible to simplification in certain cases (progressive terms for high temperature, unusual depths or salinities, etc.)

(c) Use depth instead of pressure.

The equation published proved to fit Wilson's data better [Ref. 7] in the useful domain [Fig. 5]. An example of comparison between various equations is given in Fig. 6, the author's second equation being taken as the reference.

Since the time of its publication no other equation has been proposed but a number of new data appeared:

(a) It was agreed generally that Greenspan & Tschiegg [Ref.6] were too high by 0.35 m/s in the entire range of temperature (instrumental error).

(b) Extremely precise measurements of sound speed were performed by Vincent Del Grosso [Ref. 5] who kindly sent his results to the author.

For the moment only data at atmospheric pressure are available, but further values at pressure should be published.

In view of this, the author tried to compare the value given by his equation simply corrected by -0.35 m/s (the instrumental error of Greenspan & Tschiegg [Ref. 6]) with the new data of Del Grosso. The comparison was made with tabulated values of Del Grosso (that agree within 2 or 3 cm/s with the data points) and with the data of Greenspan & Tschiegg [Ref. 6] corrected by -0.35 m/s.

The comparison with Del Grosso's new data was made at 35‰, and at the extreme ranges of salinity investigated, viz. 31‰ and 39‰. The overall results are quite encouraging.

Figure 7 (taken from Del Grosso) shows the difference between his new data and equation and the previous equations of Del Grosso and Wilson [Ref. 7]. If plotted on that curve, the values of sound speed from the author's corrected second formula would stay within -0.045 and + 0.11 m/s of Del Grosso's results in the entire range of temperature from 0 to 35°C, which is perfectly acceptable and much better than any other formula valid also at depth.

At 31‰ the difference with Del Grosso [Ref. 5] was found to be between - 0.10 and - 0.20 m/s from 0 to 30°C, and at 39‰ this difference varied from + 0.17 to + 0.32 (the maximum being at 10°C). This less good agreement at 39‰ comes from the paucity of high salinity data that could be used for the author's equation.

At zero per unit salinity (fresh water) the agreement with Greenspan & Tschiegg [Ref. 6] lies between - 0.18 m/s and + 0.30 m/s in the useful range of temperature.

In all it appears that the author's second equation simply corrected by the constant figure of - 0.35m/s, is appropriate to approximate the latest accurate figures. This does not mean that some effort should not still be made, but in my view it would be better to wait until new data are published concerning the speed of sound at various hydrostatic pressures.

An interesting result may also be drawn from the above. Most often the comparisons of sound speed values have been made from the results of the equations. One of the discrepancies illustrated in Fig. 7 is between the new Del Grosso equation (very accurate) and Wilson's equations [Ref. 7] (not too accurate). As the author's formula was approximating Wilson's data, and as it fits well with Del Grosso's this means that, in the end, Wilson's data points were not as far from Del Grosso's as one would think. This is also a good reason for having confidence for the moment in the pressure effect unless there should be some inaccuracy in the measurements of pressure itself.

To conclude, the author wishes to stress the importance for the next equations to appear, that they should be

(a) more concerned with realistic values of temperature, salinity and depth;

(b) as simple as possible with not too complicated coefficients although this is less critical now;

(c) capable of simplifications by the removal of well-defined terms in the polynomial development;

(d) using depth directly instead of pressure.

REFERENCES

1. A.B. Wood, "A Textbook of Sound". Bell, London, (First Edition, 1930) Third Edition, 1955, p. 261.

2. Susumu Kuwahara, "The Velocity of Sound in Sea Water and Calculation of the Velocity for Use in Sonic Sounding". The Hydrographic Review, Monaco, Vol. 16, No. 2, November 1939.

3. D.J. Matthews, "Tables of the Velocity of Sound in Pure Water and Sea Water". 2nd Edition, London, British Admiralty, Hydrographic Department, 1939.

4. K.V. Mackenzie, "Formulas for the Computation of Sound Speed in Sea Water". J. Acoust. Soc. Am., Vol. 32, No. 1, January 1960, p. 100.

5. V.A. Del Grosso, "The Velocity of Sound in Sea Water at Zero Depth." Naval Research Laboratory, NRL Report 4002 - June 1962.

6. M Greenspan & C.E. Tschiegg, "Tables of the Speed of Sound in Water." J. Acoust. Soc. Am., Vol. 31, No. 1, January 1959, p. 75.

7. W.D. Wilson, "Speed of Sound in Distilled Water as a Function of Temperature and Pressure." J. Acoust. Soc. Am., Vol. 31, No. 8, August 1959, p. 1067.

   W.D. Wilson, "Ultrasonic Measurement of the Velocity of Sound in Distilled Water and Sea Water." U.S. Naval Ordnance Laboratory, NAVORD Report 6746, January 1960.

   W.D. Wilson, "Speed of Sound in Sea Water as a Function of Temperature, Pressure and Salinity." J. Acoust. Soc. Am., Vol. 32, No. 6, June 1960, p. 641.

   W.D. Wilson, "Equations for the Computation of the Speed of Sound in Sea Water." U.S. Naval Ordnance Laboratory, NAVORD Report 6906.

   W.D. Wilson, "Equation for the Speed of Sound in Sea Water." J. Acoust. Soc. Am., Vol. 32, No. 10, October 1960, p. 1357.

   W.D. Wilson, "Extrapolation of the Equation for the Speed of Sound in Sea Water." J. Acoust. Soc. Am., Vol. 34, No. 6, June 1962, p. 866.

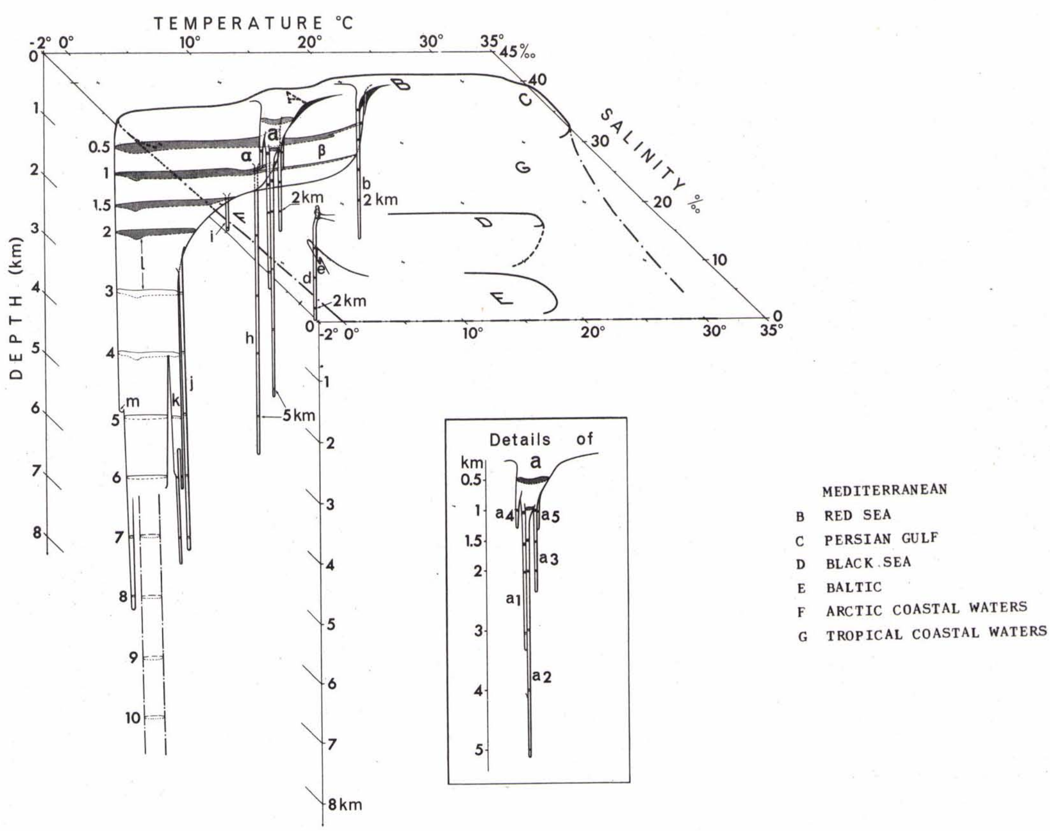
8. C.C. Leroy, "Formulas for the Calculation of Underwater Pressure in Acoustics." J. Acoust. Soc. Am., Vol. 44, 1968, pp. 651-653.

9. C.C. Leroy, "Development of Simple Equations for Accurate and more Realistic Calculations of the Speed of Sound in Sea Water." J. Acoust. Soc. Am., Vol. 46, 1969, pp. 216-226.

DISCUSSION

The author was asked whether the low-salinity correction, recommended for salinities below 30‰ but having an (S-35) dependence, could be used without too much error from salinities of 35‰ on down; the advantage being that no discontinuity at S=30 is thus introduced. The answer was affirmative, but with the warning that for salinities above 35‰ the error so introduced might become unacceptable.

A discussion followed concerning the importance of the effect of pressure on sound speed. There were certain experimental anomalies which indicated the need for more extensive information at depth. In fact, because some commonly used STD systems use pressure as a measure of depth, it might be advisable to develop both depth and pressure sound-speed formulae.



THREE-DIMENSIONAL REPRESENTATION OF THE TEMPERATURE-SALINITY-DEPTH VOLUME ENCLOSED BY REAL WATERS

FIG. 1

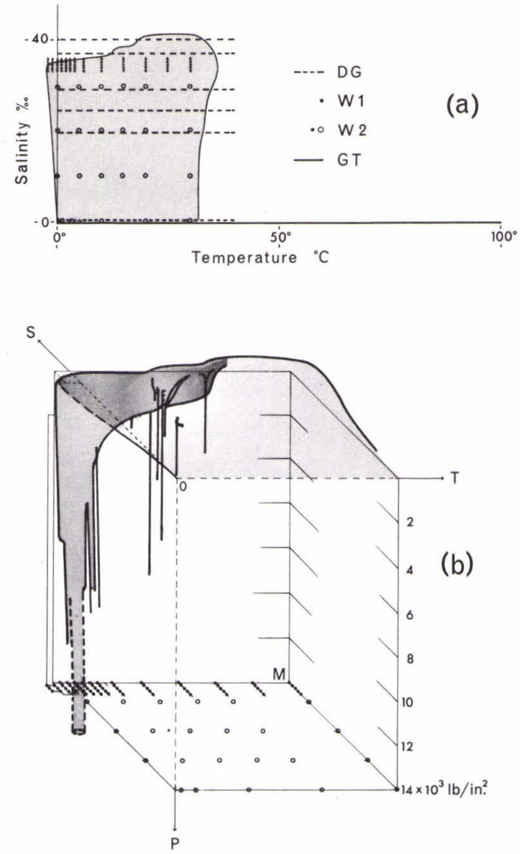


FIG. 2

COMPARISON OF REAL TEMPERATURE-SALINITY-PRESSURE COMBINATIONS WITH THOSE FOR WHICH SOUND SPEED MEASUREMENTS HAVE BEEN MADE

(a) AT SURFACE PRESSURE

(b) AT DEPTH

FORMULA FOR THE CALCULATION OF SOUND VELOCITY IN SEA WATER

COMPLETE	$V = V_o + V_a + V_b + V_c + V_d$
BASIC	$V = V_o + V_a + V_b$
SIMPLIFIED	$V = V_o$

in which

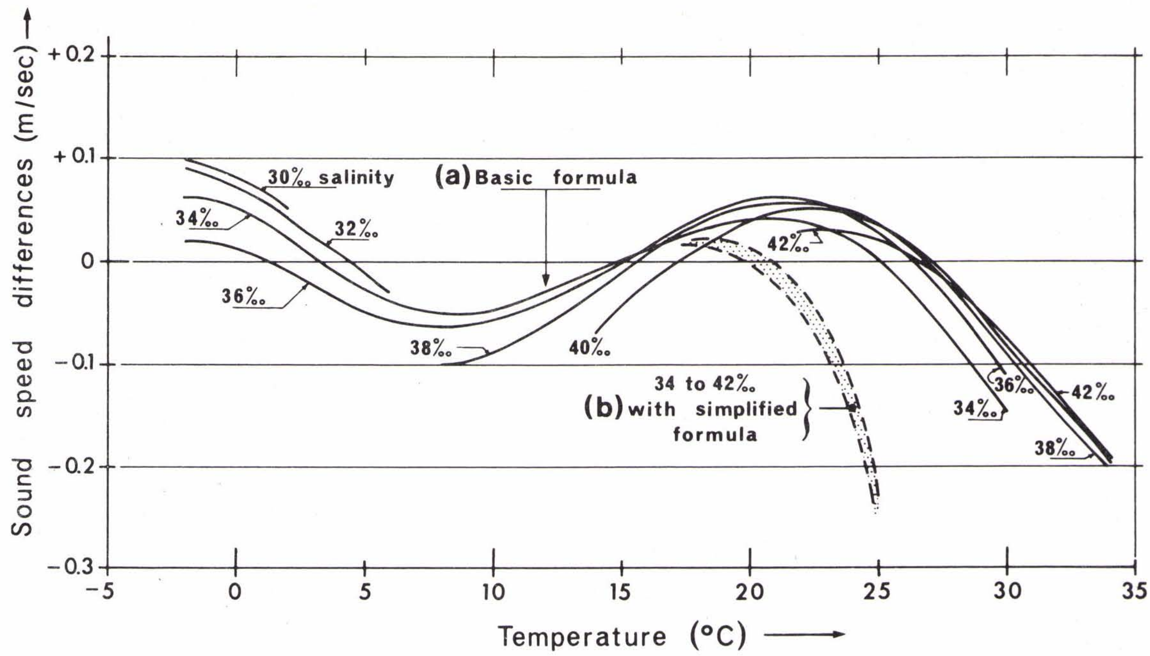
$V_o = 1492.9 + 3(T - 10) - 6 \times 10^{-3}(T - 10)^2 - 4 \times 10^{-2}(T - 18)^2 + 1.2(S - 35) - 10^{-2}(T - 18)(S - 35) + Z/61$
$V_a = +10^{-1}\zeta^2 + 2 \times 10^{-4}(T - 18)^2 + 10^{-1}\zeta\Phi/90$
$V_b = +2 \times 10^{-7}T(T - 10)^4$
$V_c = -5 \times 10^{-4}\zeta^2(\zeta - 6)^2$
$V_d = +1.5 \times 10^{-3}(S - 35)^2(1 - \zeta)$

Where
- V is the sound velocity in m/s
- T is the temperature in °C
- S is the salinity in ‰
- Z is the depth in m, and $\zeta = Z/1000$, the depth in km
- Φ is the latitude in degrees

Note: V_o can also be written:

$$V_o = 1449.34 + 4.56T - 0.046T^2 + 1.2(S - 35) - 10^{-2}(T - 18)(S - 35) + Z/61$$

FIG. 3



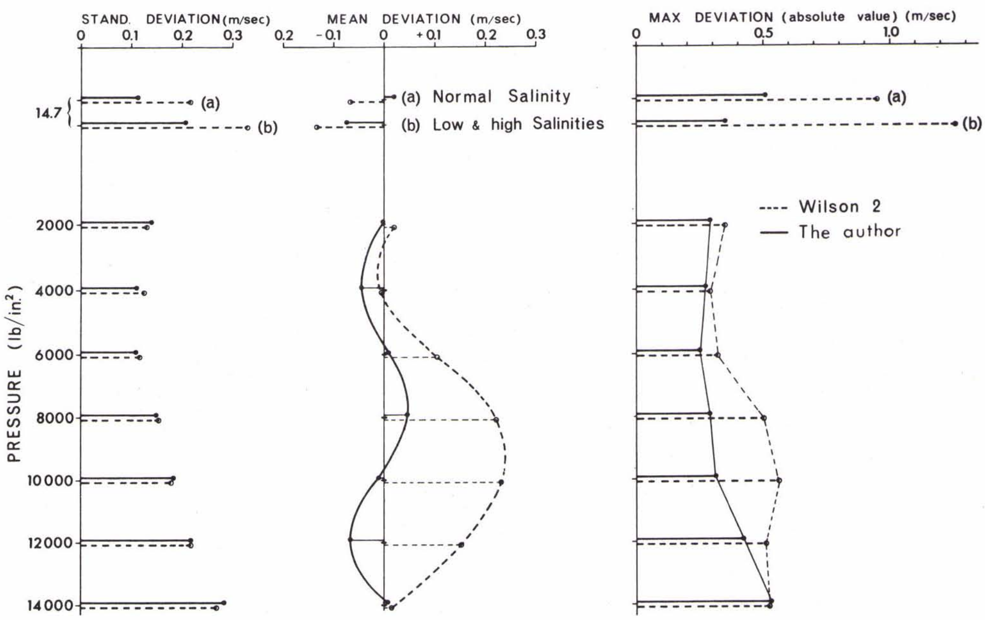
DIFFERENCES IN SOUND VELOCITY AT SEA LEVEL BETWEEN RESULTS OF

(a) Basic and Wilson's formulae ———

(b) Simplified and Wilson's formulae – – – –

AS FUNCTIONS OF TEMPERATURE AND AT DIFFERENT SALINITIES

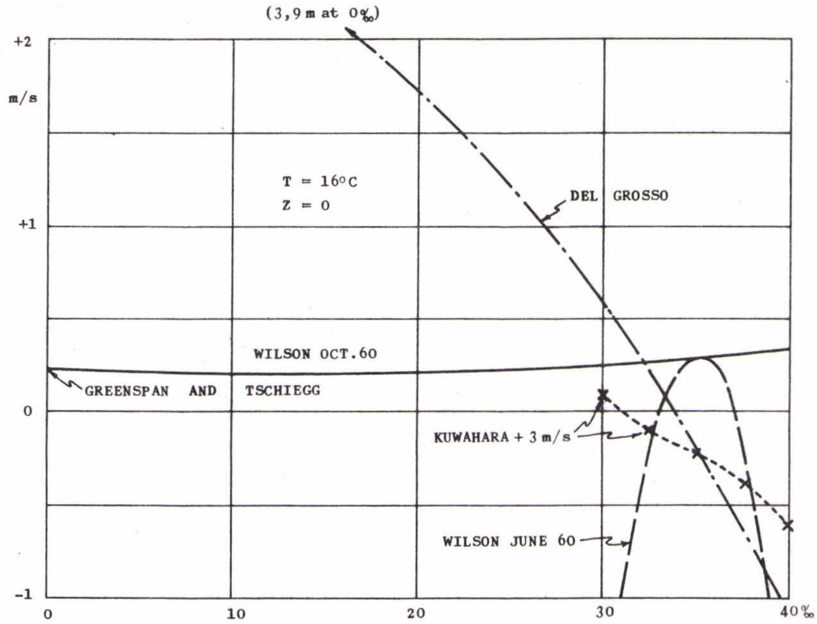
FIG. 4



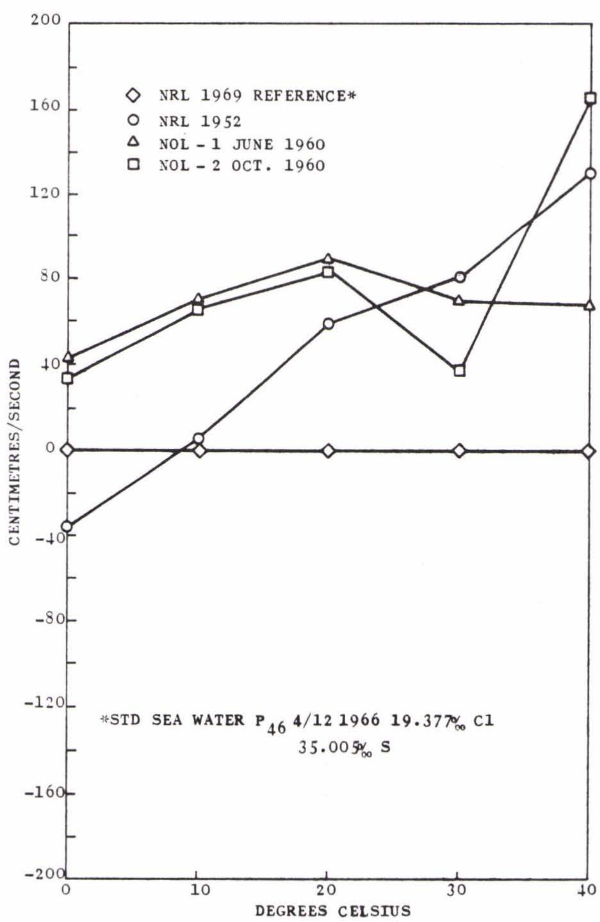
COMPARISON OF THE STANDARD, MEAN AND MAXIMUM DEVIATIONS OF WILSON'S SECOND EQUATION FOR SEA WATER AND THOSE OF THE AUTHOR

FIG. 5

FIG. 6



(3,9 m at 0‰)
+2
m/s
+1
0
-1
T = 16°C
Z = 0
DEL GROSSO
WILSON OCT.60
GREENSPAN AND TSCHIEGG
KUWAHARA + 3 m/s
WILSON JUNE 60
0
10
20
30
40‰



NRL 1969 REFERENCE*
NRL 1952
NOL - 1 JUNE 1960
NOL - 2 OCT. 1960
CENTIMETRES/SECOND
200
160
120
80
40
0
-40
-80
-120
-160
-200
*STD SEA WATER P_{46} 4/12 1966 19.377‰ Cl
35.005‰ S
0
10
20
30
40
DEGREES CELSIUS

FIG. 7